Growth with a Nonrenewable Resource:
Substitution, Investment, and Income

Henry Thompson

Auburn University

May 2008

A nonrenewable resource is added to capital and labor inputs in a neoclassical model of production and growth with a focus on the instantaneous adjustment of factor prices, per capita income, and input ratios. Optimal depletion implies a rising resource price and resource depletion is expected to decline but may rise due to investment or labor growth that would increase resource productivity. Increased resource input in spite of its rising price may be favored by substitutes or complements in production. A critical issue is whether investing all income from the resource is sufficient to maintain constant per capita income as in the model with only capital and resource inputs.

Key words: Resource depletion, substitution, production, growth

JEL: D2, D5, D9, Q3

Thanks go to Geir Ashem for substantive remarks, and to Aimee Chin, Telemachos Efthimiadis, Trellis Green, Mark Klined, Mostafa Malki, David Pappel, Roy Ruffin, Charlie Sawyer, Ingmar Schumacher, Kaz Miyagiwa, and Santanu Chattarjee.

Contact information: Economics, Comer Hall, Auburn University, AL 36849, USA, 334-844-2910, fax 5639, *thomph1@auburn.edu*

Growth with a Nonrenewable Resource:
Substitution, Investment, and Income

The present general equilibrium production model includes a nonrenewable resource along with capital and labor inputs, integrating concepts from economic growth with resource economics and production theory. The focus is on input substitution in the face of a depleting nonrenewable resource in an instantaneous comparative static model. The resource introduces its own behavioral adjustment and expands the role of substitution in production.

The comparative static model solves for instantaneous changes in the resource price, wage, capital return, and income per capita as well as the capital/labor and resource/labor input ratios. Depletion may rise even with an increasing resource price due to increasing resource productivity from investment and labor growth. The pattern of substitutes or complements between the three inputs may also favor increased resource input.

The Hartwick (1977) rule of investing all resource rent to maintain per capita income proves inadequate with the three inputs. Conditions necessary for capital deepening to maintain the wage and per capita income are developed in this paper.

Production with three inputs opens a variety of possible adjustment patterns and the full model is not tractable. The paper proceeds in steps, first reviewing solution of the capital/labor and capital/resource models. The three input model with a constant depletion rate is then solved followed by a section on the Cobb-Douglas model. The simple model with labor a specified as a substitute for both capital and the resource is then analyzed. Solutions for the tragedy of the commons and monopoly resource ownership are also examined.

1. A fixed depletion rate in the neoclassical model

Output $Y = f(K, L)$ is produced with constant returns of inputs of capital K and labor L in the neoclassical growth model of Solow (1956) and Swan (1956). National income Y in the competitive economy is exhausted by payments to productive factors $Y = rK + wL$ and Y changes according to $Y' = rK' + wL'$ where primes represent time derivatives in a first order approximation.

For a closed economy, investment equals saving S and assuming no depreciation $K' = S$. For convenience assume a constant marginal propensity to save σ, perhaps the golden saving rate, implying $K' = \sigma Y$. It is straightforward to introduce intertemporal utility optimization with an endogenous saving rate but the constant σ is more straightforward and the present focus is on comparative statics along the growth path. If the labor force grows at the constant rate $\lambda \equiv L'/L$ the model is closed and $Y' = r\sigma Y + w\lambda L$.

The capital/labor ratio k evolves according to $k' = (K/L)' = \sigma Y/L - nk = \sigma y - \lambda k$. Per capita income $y \equiv Y/L$ adjusts according to $y' = Y'/L - \lambda y = r\sigma y + \lambda w - \lambda y = r\sigma y - \lambda(y - w)$. In the steady state $k' = 0$ and solving for steady state per capita income $y^s = \lambda k^s/\sigma$ with saving exactly offsetting labor growth to maintain k^s. If capital depreciates $y^s = (\lambda + \delta)k^s/\sigma$ and saving has to offset depreciation δ as well as labor growth. If $\lambda = \sigma = 0$ there is no unique y^s. The model converges to the steady state since $\text{sgn}\,(k^s - k) = \text{sgn}\,k'$.

The following production models focus on conditions that lead to the intergenerational equity of Solow (1974) with $y' = 0$. The focus is on the comparative statics of production and there is no discount of future consumption as in Ramsay (1928) models. The present models lay the foundation of production adjustment with the additional input.

Add a nonrenewable resource N to the neoclassical production function, $Y = f(K, L, N)$. The third input introduces its own behavioral dynamics as well as increased scope for substitution. Extraction diminishes the resource stock S according to $N = -S'$. If a fixed portion α of the stock is depleted each period $N = \alpha S$ and changes in N are directly related to its level $N' = \alpha S' = -\alpha N$ implying constant decreasing depletion, $N'/N = -\alpha$. The present paper assumes a known fixed stock although the stock can be made sensitive to price as Farzin (2001) shows for US oil reserves.

The resource relates to income dynamics like "negative labor" and $Y' = r\sigma Y + w\lambda L - n\alpha N$. In per capita terms $Y'/L = r\sigma y + w\lambda - n\alpha h$ where $h \equiv N/L$. Changes in per capita income are $y' = (Y/L)' = Y'/L - \lambda y$ implying $y' = r\sigma y + \lambda w - \alpha nh - \lambda y$. Income is the sum of factor payments, $y = w + rk + nh$ implying $y' = r(\sigma y - \lambda k) - (\lambda + \alpha)nh$.

The steady state would be $y^s = \lambda k^s/\sigma + (\lambda + \alpha)n^s h^s/\sigma r^s$ but the model converges with resource exhaustion to $h^s = 0$ and the KL steady state $y^s = \lambda k^s/\sigma$. Some N input is required given the Inada condition but a backstop resource can be introduced. At any rate, the present focus is on instantaneous adjustment rather than long term growth.

The resource/labor ratio evolves according to $h' = -(\lambda + \alpha)h$. Convergence is monotonic since $h > h^s = 0$ implies $h' = -(\lambda + \alpha)h < 0$ and the same holds for k. Since $k' = \sigma y - \lambda k$ the change in per capita income reduces to

$$y' = rk' + nh'. \tag{1}$$

If $y' = 0$ it follows that h' and k' have opposite signs. Isoquant $y' = 0$ has negative slope $h'/k' = -f_k/f_h = -r/n$ and convexity implies h falls at a decreasing rate as k rises. Constant per capita income $y' = 0$ requires $y = [(\lambda + \alpha)nh + \lambda rk]/r\sigma$, not a unique condition. There is no unique steady state and existence depends on initial conditions.

There are various patterns of income distribution but one presumption is that capital deepening with a rising k lowers the capital return r and raises the wage w. Increased capital input also raises resource productivity and the rising resource price n can increase capital demand. If labor and the resource are weak substitutes or complements, the rising resource price lowers labor demand and possibly the wage w.

Marginal extraction cost MEC would affect the net return to the resource but assume for simplicity that MEC = 0 making resource rent equal to revenue nN. Rising MEC complicates the analysis but does not affect intuition.

Total consumption is $C = Y - K'$ or in per capita terms $c = y - K'/L = y - (k' + \lambda k)$ which changes according to $c' = y' - (\sigma y - \lambda k)' - \lambda k' = (1 - \sigma)y'$ implying c' and y' have the same signs.

If all capital rent is invested, $K' = nN$ or $k' = nh - \lambda k$ and from (1) $y' = rk' + nh' = r(nh - \lambda k) + nh'$. Constant consumption per capita c occurs only if $h' = r(\lambda k - nh)/n = \lambda rk/n - rh$. Given $h' = -(\lambda + \alpha)h$ it follows that c is constant only given the accidental condition $n = \lambda rk/(r - \alpha - \lambda)h$.

Hartwick (1977) shows that investing resource rent implies constant consumption per capita in the two factor capital/resource model with Cobb-Douglas production but that level of investment is insufficient in the present three factor model.

2. Optimal depletion with three factors

The resource stock is an asset and resource owners with perfect foresight deplete to satisfy the Hotelling (1931) condition equalizing returns from owning either the resource or capital. Solow (1974) and Stiglitz (1974) show that per capita income falls in the steady state with exponential labor growth. Mitra (1983) shows constant per capita income is feasible if the growth rate of labor diminishes quasi-arithmetically. Dasgupta and Heal (1979) examine optimal growth paths, Robinson (1980) develops classical properties of the model, Faber and Proops (1993) provide some background on resource rent, and Tilton (1996) focuses on sustainability.

Maintaining the assumption of zero marginal extraction cost, the return to holding the resource equals the rate of change in its price,

$$n'/n = r. \qquad (2)$$

This no-arbitrage condition implies asset holders would be indifferent between holding capital and resource assets. Resource rent is $\pi = nN$ and its rate of change is $\pi'/\pi = (nN)'/nN = n'/n + N'/N = r + N'/N$.

This optimal depletion condition appears reasonable examining the difference between the percentage change in the real price of US crude oil and the real rate of return to S&P index since 1950 in Figure 1. The mean is 0.004 and covariance 0.094, and the series is trend stationary due to the relatively stable price of oil up to the early 1970s. Using 1972 as a break the variance is constant and the difference $n'/n - r$ is white noise. Two outlier years 1974 and 2000 do not affect these conclusions.

* Figure 1 *

In related two factors models, Garg and Sweeney (1978) derive optimal growth paths in a Cobb-Douglas model focusing on initial endowments, Farzin (2003) examines sustainability, Mikesell (1989) looks at discounting and intergenerational equity, and Mainardi (1995) develops a model with technology diffusion.

Per capita income evolves according to (1) as $y' = rk' + nh' = r(\sigma y - \lambda k) + nh'$. Stationary per capita income $y' = 0$ implies k would have to rise if h is falling. Adjustment in k is stationary but nothing can be said about stability of h or the steady state k^s.

Hartwick (1977) shows that investing resource rent results in constant consumption per capita with optimal depletion and Cobb Douglas production in the model with capital and resource inputs. The Hartwick investment rule is developed by Dixit, Hammond, and Hoel (1980), Hamilton (1995), Withagen and Asheim (1998), and Sato and Kim (2002). With Cobb Douglas production there is a constant factor share $\theta_N = nh/y$ and $y' = 0$ only if $y'/y = n'/n + h'/h$ with the rising resource price exactly offsetting depletion.

In a tragedy of the commons, resource owners drive price down to MEC. At the other extreme, a monopoly resource owner restricts depletion so marginal revenue product equals MEC. These two market structures are analyzed in the model with substitution.

3. Instantaneous substitution with a nonrenewable resource

Substitution contributes to determining the instantaneous direction of income distribution in the three factor growth model. Allen (1938) and Takayama (1993) develop the underlying production theory and Takayama (1982), Jones and Easton (1983), and Thompson (1985) focus on production with three inputs. In the resource market, endogenous depletion equals resource demand. Let a_N be the cost minimizing input of the resource per unit of output, and the resource market clears with $N = a_N Y$ or in per capita terms $h = a_N y$. Shephard's lemma implies a_N is the partial derivative of the cost function $c(r, w, n; Y)$ with respect to the resource price n. Given constant returns $a_N(r, w, n)$ is homogeneous of degree zero in factor prices.

Employment of the resource is captured in the adjustment $h' = ya_N' + a_N y'$ or $h' = N_r r' + N_w w' + N_n n' + a_N y'$. Cross price substitution terms are $N_r \equiv (\delta a_N/\delta r)y$ for resource input with respect to the capital return r. The cross wage substitution term N_w is similar. The own price substitution term N_n gauges the response of N input to its own price n. Capital substitution terms are (K_r, K_w, K_n) and for labor (L_r, L_w, L_n). These substitution terms describe the local surface of the production isoquant.

Concavity of the cost function implies $\delta a_N/\delta n = N_n < 0$ and similarly $K_r < 0$ and $L_w < 0$. Shephard's lemma and Young's theorem imply symmetry of cross price substitution terms: $N_r = K_n$, $N_w = L_n$, and $L_r = K_w$. Substitutes have positive cross substitution terms but one pair of inputs may be complements making K_n, L_n, or K_w negative. There is an extensive literature on the issue of whether capital and energy are substitutes or complements, and the following analysis focuses on capital/resource substitution K_n.

The substitution elasticity between resource input and the capital return is $\varepsilon_{Nr} \equiv (r/a_N)\delta a_N/\delta r = rK_n/a_N$. With homogeneous cost functions, substitution elasticities sum to zero weighted across factor prices, $n\varepsilon_{in} + r\varepsilon_{ir} + w\varepsilon_{iw} = 0$ for i = N, K, L. Own substitution terms can then be expressed in terms of cross terms: $N_n = -(r/n)K_n - (w/n)L_n$, $K_r = -(n/r)K_n - (w/r)K_w$, and $L_w = -(n/w)L_n - (r/w)K_w$.

Employment conditions for the resource in the resource/labor ratio h and similar conditions for the capital/labor ratio k′ and labor (L/L)′ = 0 lead to the first three equations of the general equilibrium production model (3).

Per capita income y = w + rk + nh adjusts according to y′ = w′ + kr′ + hn′ + rk′ + nh′ and output per worker adjusts through the production function according to y′ = rk′ + nh′ in (1). Income equals output in a closed economy implying w′ + kr′ + hn′ = 0, the fourth equation in (3).

Per capita investment $k' = \sigma y - \lambda k$ is the last equation in (3). The optimal depletion condition n′ = rn from (2) is included eliminating n′ as an endogenous variable. The change in consumption per capita $c' = (1 - \sigma)y'$ can be derived from y′. The system (3) solves for endogenous instantaneous changes in the capital return r, wage w, per capita income y, resource/labor ratio h, and capital/labor ratio k in terms of substitution terms, input levels, and model parameters,

$$\begin{pmatrix} K_n & L_n & a_N & -1 & 0 \\ K_r & K_w & a_K & 0 & -1 \\ K_w & L_w & a_L & 0 & 0 \\ k & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{bmatrix} r' \\ w' \\ y' \\ h' \\ k' \end{bmatrix} = \begin{pmatrix} -rnN_n \\ -rnK_n \\ -rnL_n \\ -rnh \\ \sigma y - \lambda k \end{pmatrix} \tag{3}$$

Focus on qualitative dynamics and rescale inputs and output to unit values, 1 = K = L = N = Y implying $h = k = y = a_K = a_L = a_N = 1$. National income Y = wL + rK + nN implying 1 = w + r + n making factor prices equal to factor shares. Own substitution terms are written in terms of cross terms. The scaling implies a positive determinant, $\Delta = 2K_w - L_w - K_r = (K_w + rnL_n + wnK_n)/wr$. Two inputs may be complements but regularity conditions imply $\Delta > 0$.

Solve the scaled system (3) for the instantaneous change in the capital return $r' = -(\sigma - \lambda) - (rn/w)[(w + r)K_w + (w + n)L_n - wK_n]/\Delta$. There is a presumption that $r' < 0$ with capital deepening $\sigma - \lambda > 0$ but a high degree of K_n substitution favors a rising capital return. With the resource price rising, capital/resource substitution implies an increase in capital demand possibly offsetting the increased supply from investment. In the limiting case of no substitution for the depleting resource $K_n = L_n = 0$ but there is no assurance that $r' < 0$ as in the neoclassical model.

The instantaneous change in the wage $w' = (\sigma - \lambda) - n[(w + r)K_w + (n + r)K_n - rL_n]/\Delta$ depends on substitution although there is a presumption the wage rises with capital deepening. Labor/resource complements favor falling labor demand and a decrease in the wage with the rising resource price. Capital/resource complements favor a falling wage. In a limiting case with no substitution for the depleting resource it is not necessary that $w' > 0$ as in the neoclassical model.

The signs of y', h', and k' are more difficult to analyze due to the possibilities of substitution. The next section turns to the simpler instantaneous adjustment model with two factors.

4. Two factor instantaneous adjustment models

The present section illustrates the instantaneous comparative static model with a pair of two factor models. The scaled model capital/labor in (3) simplifies to

$$\begin{pmatrix} K_r & K_w & 1 & -1 \\ K_w & L_w & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} r' \\ w' \\ y' \\ k' \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ \sigma - \lambda \end{pmatrix}. \tag{4}$$

The positive determinant Δ is the same as in (3). Homogeneity eliminates the own substitution terms according to $L_w = -rK_w/w$ and $K_r = -wK_w/r$. With scaling $w + r = 1$ implies $\Delta = K_w/wr$. A higher degree of substitution implies a larger Δ and smaller instantaneous adjustments as the economy moves along a less convex growth curve. The familiar diminishing instantaneous changes with growth occur because $K_w = (\delta a_K/\delta w)y$ and Δ increases in y.

The endogenous vector $(r', w', y', k') = (-(\sigma - \lambda)wr/K_w, (\sigma - \lambda)wr/K_w, (\sigma - \lambda)r, \sigma - \lambda)$ has sign pattern (- + + +) if $k' = \sigma - \lambda > 0$ switching if $k' < 0$. More substitution reduces adjustments in w and r but the two effects balance and there is no effect on y′. Consumption per capita is constant only in the steady state.

The Hartwick rule is proven in the capital/resource model with no labor input,

$$\begin{pmatrix} K_n & 1 & -1 & 0 \\ K_r & 1 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} r' \\ Y' \\ N' \\ K' \end{pmatrix} = \begin{pmatrix} -rnN_n \\ -rnK_n \\ -rn \\ \sigma \end{pmatrix} \tag{5}$$

and the determinant is $\Delta_H = 1$. Endogenous instantaneous changes are $(r', Y', N', K') = (-rn, \sigma - nK_n, \sigma - K_n, \sigma)$ given the homogeneity conditions $K_r = -nK_n/r$ and $N_n = -rK_n/n$ and the scaling $n + r = 1$.

The capital return r falls as the resource price rises since $r' = -n'$. The resource price accelerates if $r > n$ since $n'' = (rn)' = nr' + rn' = -rn^2 + r^2n = rn(r - n)$ but decelerates as n rises and r falls. The fall in the capital return dampens as it falls and n rises with $r'' = (-rn)' = nr(n - r)$.

Depletion N rises if $\sigma > K_n$. Without substitution, depletion would rise along with capital from saving. A higher saving rate increases depletion as the added capital increases resource demand.

Income Y′ is constant iff $\sigma = nK_n$. With no substitution income rises with the increasing resource price. A higher saving rate and lower resource price also favor rising income. More substitution implies more of an increase in capital demand in the face of the rising resource price. As n rises Y is destined to fall.

There are three possible signs of (Y′, N′) depending on the saving rate relative to the degree of substitution. If $\sigma < nK_n < K_n$ with low saving relative to substitution (Y′, N′) has signs (- -) with declining depletion and the lack of saving a drag on output. With higher saving and intermediate substitution $nK_n < \sigma < K_n$ and the sign is (- +) as Y increases due to investment. With high saving and little substitution $nK_n < K_n < \sigma$ and the sign is (+ +) with N rising as the added capital increases resource demand.

The capital/resource ratio $\kappa \equiv K/N$ changes according to $\kappa' = K'/N - (N'/N)\kappa = [\sigma - \kappa(\sigma - K_n)]/N = [(1 - \kappa)\sigma + \kappa K_n]/N$. A higher degree of substitution favors a rising κ with increased capital demand in the face of the rising resource price and N may fall. With a low degree of substitution and rising depletion, κ would nevertheless increase due to the added capital.

With Cobb-Douglas substitution $K_n = 1$ implying $N' = \sigma - 1 < 0$ and $Y' = \sigma - n$. Depletion falls but income rises if $\sigma > n$. As n rises income must eventually fall. Investing resource rent $\sigma = nN = n$ implying the Hartwick (1977) result $Y' = 0$.

5. The Cobb-Douglas model

Cobb-Douglas (CD) production is a relatively simple case to analyze with moderate substitution between the three inputs. The CD Allen elasticities equal 1 implying unit substitution terms with the present scaling, $K_n = K_w = L_n = 1$. Homogeneity implies $N_n = (n - 1)/n < 0$, $K_r = (r - 1)/r$, and $L_w = (w - 1)/w$. Substitution elasticities simplify to factor shares as in the elasticity between the resource and capital $\varepsilon_{Nr} = rK_n/a_N = r$. The positive determinant for the three factor CD economy in (3) simplifies to $\Delta_{CD} = (1 - n)/wr > 0$. Production with constant elasticity of substitution (CES) is a generalization of CD and the model is similar with a higher degree of CES substitution dampening instantaneous adjustments.

The resource/labor ratio h falls according to $h' = [(\sigma - \lambda) - 1]/(1 - n) < 0$ since $\sigma < 1$. The capital return adjusts according to $r' = -r[w(\sigma - \lambda) + rn]/(1 - n)$ and $r' < 0$ unless $k' = \sigma - \lambda < -rn/w < 0$. Capital deepening $k' > 0$ implies $r' < 0$. The capital return may rise even with capital deepening depending on K/N substitution.

The wage adjusts according to $w' = rw[(\sigma - \lambda) - n]/(1 - n)$ which is positive if $k' > n$. The wage falls along with the capital return, however, when the increase in k is insufficient to raise labor's marginal productivity more than its increasing supply. A higher resource price favors a falling wage. The condition for a constant wage is $k' = n$ or $rk' = n'$ with the change in the value of investment per capita matching the change in the resource price.

Per capita income adjusts according to $y' = [(\sigma - \lambda) - rn]/(1 - n)$. The condition for constant consumption per capita is $k' = \sigma - \lambda = rn = n'$ with the change in the investment per capita matching the change in the resource price and both r and w falling. The investment rule to maintain per capita income in the three factor growth model is to keep the capital/labor ratio apace with the rising resource price regardless of the source of the investment income.

The signs of r', w', and y' can be analyzed with respect to capital accumulation k' as in Figure 2. A high level of capital deepening with $k' > n$ implies the expected fall in r and increase in w and y with capital accumulation pulling up the wage. The wage falls if $rn < k' < n$ even with per capita income rising as income is distributed to resource owners and away from labor and capital. If $-rn/w < k' < rn$ income per capita also falls. A fall in the capital/labor ratio with $k' < -rn/w$ implies a rising return to capital with the wage and per capita income falling. The same outcomes can be reached with variation in levels of inputs, outputs, or factor prices.

* Figure 2 *

Investing resource rent does not ensure constant per capita income. The change in capital would be $K' = k' + \lambda = nN = n$ implying $k' = n - \lambda$. Income is distributed according to $r' = -r[w(n - \lambda) + rn]/(1 - n)$, $w' = -\lambda rw/(1 - n) < 0$, and $y' = [n(1 - r) - \lambda]/(1 - n)$. With any labor growth the wage falls. Higher labor growth leads to falling per capita income but a rising capital return due to increased capital productivity. Per capita income falls if $\lambda > n(1 - r)$ and the capital return rises if $\lambda > n(1 - n)/w$ and these switch points are ranked $n(1 - n)/w = n(r + w)/w > n(1 - r)$ since $n(r + w) > wn(1 - r)$.

With an increase in λ starting from zero in Figure 3, the low labor growth has little impact on capital productivity and r falls but the rising resource price n pulls up per capita income. Per capita income falls y if $\lambda > n(1 - r)$ and income is spread over more workers. Finally if $\lambda > n(1 - n)/w$ the capital return r rises as the added labor increases capital productivity enough to raise its return.

* Figure 3 *

With no labor growth, investing resource rent implies $k' = n = n'/r$ or $rk' = n'$ and the value of investment keeps up with the resource. The rising resource price pulls up per capita income with $w' = 0$ and $r' < 0$. The investment rule to maintain constant per capita income is $\sigma - \lambda = k' = rn = n'$ keeping the change in the capital/labor ratio up with the rising resource price.

6. Capital/resource substitution

Consider the optimal depletion model (3) with various degrees of substitution between capital and the resource with labor a substitute with both capital and the resource, $K_w = L_n = 1$. Various degrees of capital/resource substitution are examined with the substitution term K_n. The positive determinant of (3) simplifies to $\Delta = 5 + K_n$.

Substitution between capital and energy is a fundamental topic in energy economics and estimates depend on model specification, estimation of capital input, location, and econometric technique. The time frame of the data allowing for input adjustment also affects estimates with capital and energy substitutes given enough time to adjust to rising energy prices.

The derived change in the capital return $r' = [-(\sigma - \lambda) + rn(K_n - 4)]/\Delta$ is positive if $K_n > [(\sigma - \lambda) + 4rn]/rn$. Capital/resource substitution increases capital demand with the rising resource price. Capital thinning with a negative $k' = \sigma - \lambda$ favors a rising capital return. Complementary resource and capital inputs with a negative K_n favor a falling capital return as demand for capital falls along with the reduced resource input.

The wage adjusts according to $w' = [(\sigma - \lambda) - rn(1 + 2K_n)]/\Delta$ and the wage would fall with enough K_n substitution since $w' < 0$ iff $K_n > [(\sigma - \lambda) - rn]/2rn$. If $k' = \sigma - \lambda < 0$ any K_n substitution implies $w' < 0$. Capital/resource substitution favors capital over labor with a rising resource price leading to falling labor demand while complements favor a rising wage as demand for labor increases to replace the declining resource and capital inputs.

Per capita income adjusts according to $y' = [3(\sigma - \lambda) - rn(4 + 6K_n)]/\Delta$. Per capita income would be constant iff $K_n = [3(\sigma - \lambda) - 4rn]/6rn = (.5k' - .67n')/rn$, a purely accidental condition. A higher degree of capital/resource substitution favors falling per capita income with the wage falling as the capital return potentially rises and more investment is required to maintain per capita income. With no K_n substitution $y' = 0$ iff $k' = 4n'/3$ with the increase in the capital/labor ratio more than matching the increase in the resource price. Capital/resource complements favor rising per capita income.

The resource/labor ratio h adjusts according to $h' = [4(\sigma - \lambda) - 9rn + (18rn - (\sigma - \lambda))K_n]/\Delta$. If $18rn = 18n' > \sigma - \lambda = k'$ a higher degree of K_n substitution implies a higher h'. With no K_n substitution and no change in the capital/labor ratio, h falls. Constant y implies $4n' = 3k'$ and $18n' > k'$ and a higher degree of K_n substitution would imply a higher h'. The resource/labor ratio is constant iff $K_n = (4k' + 9n')/(k' -18)$.

In summary, more capital/resource substitution favors an increase in the capital return but a decrease in the wage and a decrease in per capita income as depletion declines. Without capital/resource substitution and capital deepening, the return to capital falls as the wage and per capita income rise. Capital/resource complements favor a decrease in the capital return but increases in the wage and per capita income.

7. Tragedy of the commons

Common pool ownership leads to the tragedy of the commons with over depletion of the nonrenewable energy resource. The tragedy is a simple analytical situation with zero profit from resource extraction as common pool resource owners drive price down to marginal extraction cost. Maintaining the assumption that MEC = 0 results in the clear tragedy $n = n' = 0$. The comparative static model remains (3) with the transposed exogenous vector $(0, 0, 0, 0, \sigma, -\lambda)$.

Changes in the capital return and wage are symmetric with $r' = -w' = -(\sigma - \lambda)/\Delta = -k'/\Delta$. Capital deepening implies $w' > 0 > r'$ and $y' = (\sigma - \lambda)(K_w - L_w)/\Delta > 0$ even if capital and labor are complements. Tragedy depletion bolsters per capita income by raising labor and capital productivities and the resource price has already fallen as far as it can.

The resource/labor ratio h depends on substitution with $h' = (\sigma - \lambda)(K_w + L_n - L_w - K_n)/\Delta$. There is a presumption h rises but K_n substitution favors a falling h as production turns toward capital. With Cobb-Douglas production $K_w = L_n = K_n = 1$ and $L_w = (w - 1)/w$ implying $y' = h' = (\sigma - \lambda)/\Delta w > 0$ with capital deepening. The rising h is a symptom of the tragedy. With no labor substitution $K_w = L_n = L_w = 0$ and $K_n > 0$ implying $h' < 0$ and $y' = 0$ while r' and w' are magnified by the small determinant Δ. With no substitution for the depleting resource $K_n = L_n = 0$ and $y' = h' > 0$. If $k' < 0$, it follows that $r' > 0$ while $w' < 0$ and $y' < 0$ reversing the discussion of the resource/labor ratio.

A tragedy of the commons is characterized by over depletion and a rising resource/labor ratio even with labor growth except possibly with strong substitution between capital and the resource. Depletion itself rises if $h' > -\lambda h$ since $N' = (h' + \lambda h)L$.

8. Monopoly resource owner

A monopoly resource owner maximizes transitory profit by setting marginal revenue MR equal to MEC disregarding the asset characteristic of the resource. Total resource revenue is nN implying $MR = \delta(nN)/\delta N = n + N(\delta n/\delta N) = n + N(n'/N')$. If MEC = 0 it follows that $N'/N = -n'/n$ with depletion offsetting the resource price to maintain constant marginal revenue. Constant MEC implies constant marginal profit.

The change in the resource/labor ratio is $h' = N'/L - \lambda h$ and monopoly extraction $N' = -n'N/n$ implies $h' = -(n'/n + \lambda)$. The resource/labor ratio offsets the increase in the resource price and labor growth. Without labor growth the resource input offsets its price implying a constant resource income share. Labor growth implies a falling resource share of income.

Adding the endogenous n' and the monopoly extraction condition $nh' + n' = -\lambda n$ to (3) results in a model that requires assumptions on parameters to arrive at qualitative conclusions. Focus on capital/resource substitution assuming unit labor cross substitution $L_n = K_w = 1$ with factor prices $w = .6$, $r = .3$, and $n = .1$. Own substitution terms are $N_n = -(r/n)K_n - (w/n)L_n = -6 - 3K_n$, $K_r = -(n/r)K_n - (w/r)K_w = -2 - K_n/3$, and $L_w = -(n/w)L_n - (r/w)K_w = -2/3$. Assuming $\sigma = .2$ and $\lambda = .02$, $k' = \sigma y - \lambda k = .18$.

Adjustments when $K_n = 1$ are $(n', r', w', y', h') = (.07, -.07, -.01, -.02, -.74)$. The capital return falls as does the wage with increased labor supply. Per capita income falls with the wage and capital return in spite of the rising resource price. The resource/labor ratio falls as anticipated and depletion falls by $N' = h' + \lambda = -.72$. There is a net decrease in resource demand due to substitution with the capital return and wage falling.

With capital/resource complements $K_n = -1$ adjustments are $(n', r', w', y', h') = (-.02, -.02, .05, .07, .18)$. The monopolist suffers a falling resource price and extracts heavily to maximize profit. The resource/labor ratio rises as does depletion with $N' = .20$. Labor productivity increases raising labor demand and the wage, pulling up per capita income.

The monopolist may not conserve the resource more than a competitive market with optimal depletion as typically assumed in partial equilibrium resource economics. The monopoly resource price is tied to resource demand rather than the capital return. In the optimal depletion model under the present assumptions when $K_n = 1$ the adjustment vector is $(r', w', y', h') = (-.02, -.82, -.79, .17)$ and $N' = .19$ even with the resource price increasing at $n' = .03$. The monopolist is more conservative at $N' = -.72$. Roles are reversed, however, when $K_n = -1$ with the optimal depletion adjustment vector $(r', w', y', h') = (-.03, -.02, .02, -.70)$ and $N' = -.68$, more conservative than the monopolist at $N' = .20$.

9. Conclusion

A nonrenewable energy resource added to the neoclassical growth model introduces its own dynamics and increases the potential role of substitution in economic growth and income distribution. Depletion may rise even with a higher resource price due to increased resource productivity from investment and labor growth as well as substitution toward the resource. The wage may fall even with capital deepening if there is enough substitution toward capital.

The present three factor model of production and growth lays the foundation for further analysis of economic growth in resource rich countries. The related model of a small open economy facing exogenous international prices for the nonrenewable resource or capital can be developed, as can the endogenous growth model with optimal consumption and an endogenous saving rate. In applications, simulated adjustment paths based on calibrated data should be revealing. Switch points to backstop resources can be introduced. The model with a renewable resource can also be examined.

References

Allen, R.G.D. (1938) *Mathematical Analysis for Economists*, New York: St. Martin's Press

Barro, Robert and Xavier Sala-i-Martin (1999) *Economic Growth*, Cambridge: The MIT Press

Dasgupta, P.S. and Heal, G.M. (1974) The optimal depletion of exhaustible resources, *Review of Economic Studies*, 3–28

Dixit, A., P. Hammond, and M. Hoel (1980) On Hartwick's rule for regular maximin paths of capital accumulation and resource depletion, *Review of Economic Studies* 47, 551-6

Faber, Malte and John Proops (1993) Natural resource rents, economic dynamics and structural change: a capital theoretic approach, *Ecological Economics* 8, 17-44

Farzin, Y.H. (1999) Optimal saving policy for exhaustible resource economies, *Journal of Development Economics* 58, 149-84

Farzin, Y.H. (2001) The impact of oil price on additions to US oil reserves, *Resource and Energy Economics* 23, 271-91

Garg, Prem and James Sweeney (1978) Optimal growth with depletable resources, *Resources and Energy* 1, 43-56

Hamilton, Kirk (1995) Sustainable Development, the Hartwick Rule, and Optimal Growth, E*nvironmental and Resource Economics* 5, 393-411

Hartwick, J. (1977) Intergenerational equity and the investing of rents from exhaustible resources, *American Economic Review* 66, 972-74

Hartwick, J. (1990) Natural resources, national accounting and economic depreciation, *Journal of Public Economics* 43, 291-304

Hotelling, H. (1931) The economics of exhaustible resources, *Journal of Political Economy* 39, 137-75

Jones, Ron and Stephen Easton (1983) Factor intensities and factor substitution in general equilibrium, *Journal of International Economics* 15, 65-99

Mikesell, Raymond (1989) Depletable resources, discounting and intergenerational equity, *Resources Policy* 15, 292-6

Mitra, T. (1983) Limits on population growth under exhaustible resource constraints, *International Economic Review* 24, 155–168

Robinson, T. J. C. (1980) Classical foundations of the contemporary economic theory of nonrenewable resources, *Resources Policy* 6, 278-89

Ramsey, Frank (1928) A mathematical theory of saving, *Economic Journal* 38, 543-59.

Sato, Ryuzo and Youngduk Kim (2002) Hartwick's rule and economic conservation laws, *Journal of Economic Dynamics and Control* 26 437-49

Solow, Robert (1956) A contribution to the theory of economic growth, *Quarterly Journal of Economics* 70, 65-94

Solow, Robert (1974) Intergenerational allocation of natural resources, *Review of Economic Studies* 41, 29-45

Stiglitz, Joseph (1974) Growth with exhaustible natural resources: Efficient and optimal growth Paths, *Review of Economic Studies*, 123–137

Swan, Trevor (1956) Economic growth and capital accumulation, *Economic Record* 32, 234-61

Takayama, Akira (1982) On theorems of general competitive equilibrium of production and trade: A survey of recent developments in the theory of international trade, *Keio Economic Studies* 19, 1-38

Takayama, Akira (1993) *Analytical Methods in Economics*, Ann Arbor: University of Michigan Press

Thompson, Henry (1985) Complementarity in a Simple General Equilibrium Production Model, *Canadian Journal of Economics* 18, 616-21

Tilton, John (1996) Exhaustible resources and sustainable development: Two different paradigms, *Resources Policy* 22, 91-7

Withagen, Cees and Geir B. Asheim (1998) Characterizing sustainability: The converse of Hartwick's rule, *Journal of Economic Dynamics and Control* 23, 159-63

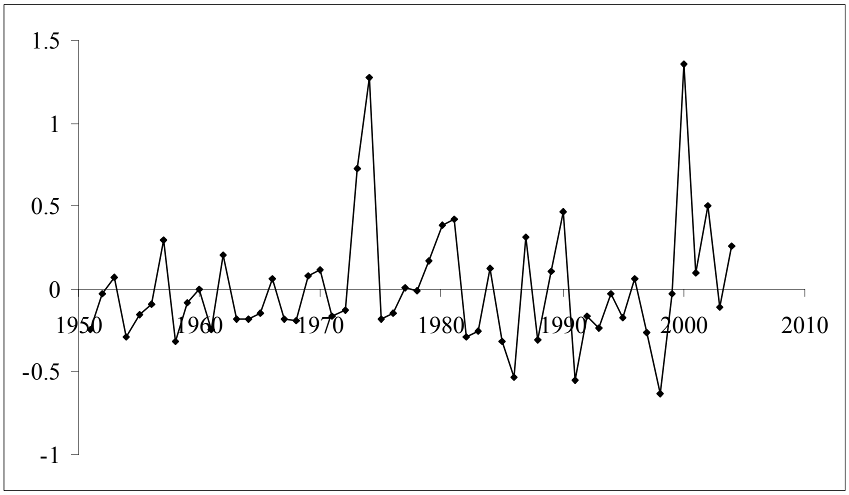

Figure 1. Return to real crude oil less S&P return

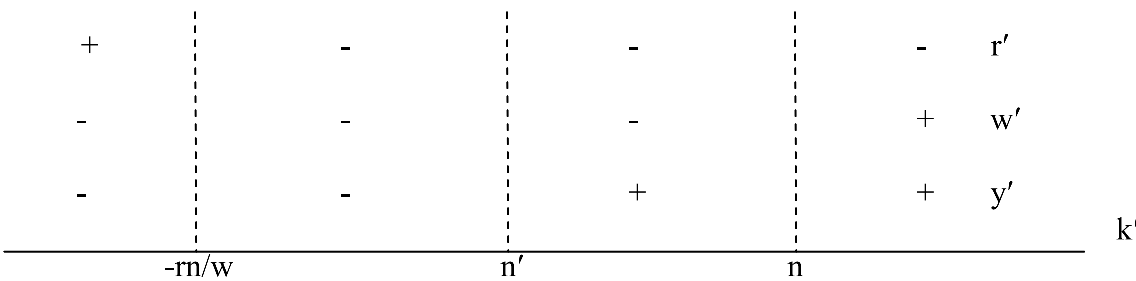

Figure 2. Cobb-Douglas capital accumulation and income distribution

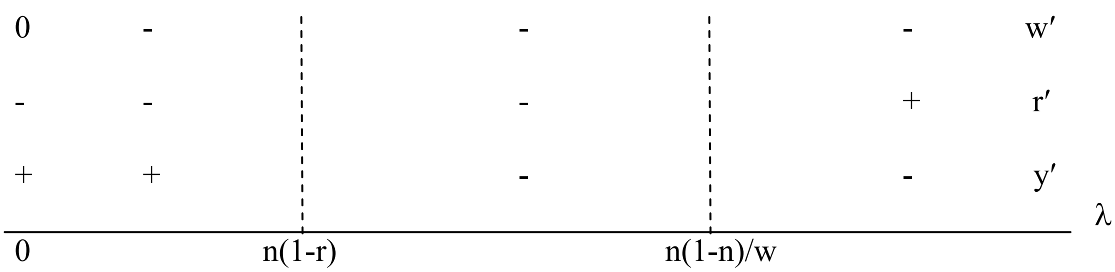

Figure 3. Labor growth and income distribution with Hartwick investment