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Abstract Rights to a free resource lead to distributional deadweight losses in partial equilibrium. The present paper examines related distortions in a general equilibrium model of production with output prices constant for the small open economy. The free resource can result in lower output than a market with weak substitution in the other sector. The free resource also leads to a convex production frontier implying a price increase lowers output in the sector. Regarding policy, an import tariff, export subsidy, or price support would lower sector output. These general equilibrium distortions increase the incentives to favor resource markets over rights.

Keywords Free resource · Water rights · General equilibrium

JEL Classification Q20 · Q30 · D50

1 Introduction

The deadweight loss due to open access resource rights in partial equilibrium are cited in favor of resource markets. The present paper develops related distortions in a neoclassical general equilibrium model of production for a small open economy. The sector with a free resource is more resource intensive than it would be facing a resource market. Its factor mix is insensitive to relative factor prices. Market transition lowers sector output if there is weak substitution to the falling relative resource price in the other sector. The production frontier is convex, implying a price increase lowers output. An import tariff, export subsidy, or price support lower sector output.

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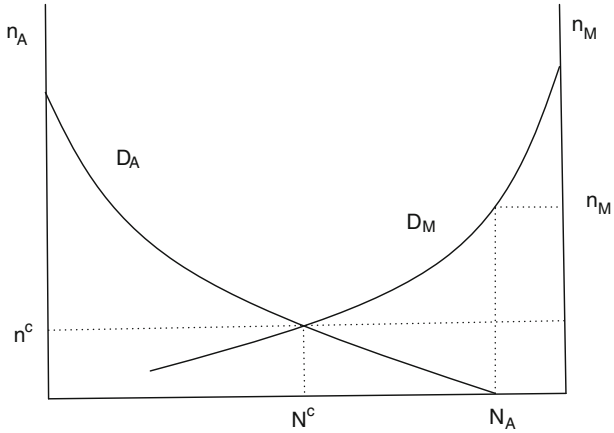

Fig. 1 Partial equilibrium resource market

Water rights in agriculture provide the motivating example of a free resource input although resource rights have broad applications. Sovereign resource rights may avoid markets. The present results imply water rights can lower agricultural output in a small open economy with insufficient substitution in the rest of the economy. A tariff for import competing agriculture, a subsidy for exported agriculture, or agricultural price supports lower agricultural output. These distortions increase incentives to favor water markets over water rights.

The following sections develop the general equilibrium model of production for the small open economy with prices exogenous at world levels. Resource rights imply a zero price for the resource input. Labor is the other factor of production. The paper starts with a static model, followed by a comparative static model that examines adjustments to changes in output prices as well as endowments of the resource and labor. A final section briefly presents a three factor version of the model adding capital input.

2 A Free Resource for a Sector

There is a rich literature on partial equilibrium water rights starting with Burness and Quirk (1979) and Anderson and Johnson (1986). Provencher and Burt (1993, 1994) examine the inefficiencies of free water input. Zilberman et al. (1994) analyze a potential water market for California agriculture. Rose (1990) and Tarlack (2001) take a look at the legal aspects of prior appropriation rights. Huffaker et al. (2000) take a look at various water right issues. Deacon (1994) analyzes the welfare loss due to water rights. Groom and Swanson (2003) analyze losses due to water rights in a general equilibrium model of temporal distribution.

The present general equilibrium model of production assumes the economy produces two outputs A and M with inputs of a natural resource N and labor L. Prices p_A and p_M are exogenous at the world level for the small open economy. Sector A has rights to free N input while sector M pays the resource price n_M. Labor is mobile and efficiently distributed between the two sectors at the competitive wage w.

Figure 1 shows the partial equilibrium resource market with total endowment N on the horizontal axis. Sector demands D_A and D_M are marginal revenue products. Sector A has resource rights and consumes N_A at a zero price leaving the residual $N - N_A$ for sector M. Demand D_M determines the residual resource price n_M for sector M.

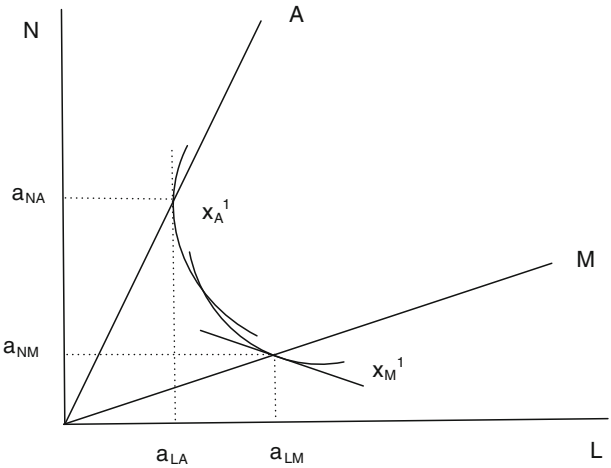

Fig. 2 Cost minimization

Introducing a resource market, the competitive market price n_C would efficiently distribute the resource at N_C. Resource demands shift, however, in the general equilibrium as labor moves between sectors and outputs adjust. These adjustments in the general equilibrium of production have not been considered.

The present model relates to the literature on input subsidies in general equilibrium models of production and trade for the small open economy. Input subsidies are analyzed by Johnson and Miezkowski (1970), Jones (1971), Herberg and Kemp (1971), Bhagwati and Srinivasan (1971), Magee (1971), and Thompson (2003). The present model extends this line of analysis to an infinite subsidy for a single sector.

Figure 2 illustrates cost minimization and factor intensity with inputs N and L measured along either axis. Assume homothetic production with linear expansion paths. Production functions determine neoclassical unit isoquants x_A^1 and x_M^1 representing one unit of output. Industries minimize cost implying the first order condition that the slope of the isocost line equals the marginal rate of substitution.

The resource has a zero price $n_A = 0$ in sector A implying saturation and the infinite slope w/n_A of its isocost line. Input occurs where marginal product is zero and the isoquant switches to positive slope. Holding labor input constant, an increase in resource input at any point on expansion path A lowers output with labor spent replacing the lost output.

Unit cost in sector A is $c_A = wa_{LA}$ where a_{LA} is the cost minimizing labor input per unit of output. Competitive pricing implies price equals cost, $p_A = c_A$. Unit cost in manufacturing is $c_M = a_{LM}w + a_{NM}n_M$ where the a_{iM} are cost minimizing unit inputs. The unit isoquant x_M^1 is tangent to the relative input price w/n_M with the slope of the unit isocost line $a_{NM} = c_M/n_M - (w/n_M)a_{LM}$ equal to the marginal rate of substitution. Competitive pricing implies $p_M = c_M$.

Cost minimization occurs at the input mix (a_{NA}, a_{LA}) and (a_{NM}, a_{LM}). Sector A is assumed resource intensive, $a_{NA}/a_{LA} > a_{NM}/a_{LM}$. Changes in the wage have no effect on the cost minimizing input mix in sector A. Input prices in the other sector affect that input mix with a_{NM}/a_{LM} increasing in the relative wage w/n_M.

Figure 3 illustrates the static equilibrium and limits to production for given endowments of the resource and labor. The parallelogram from the endowment point (N,L) to the expansion paths determines outputs with full employment. The dotted line from (N, L) parallel to

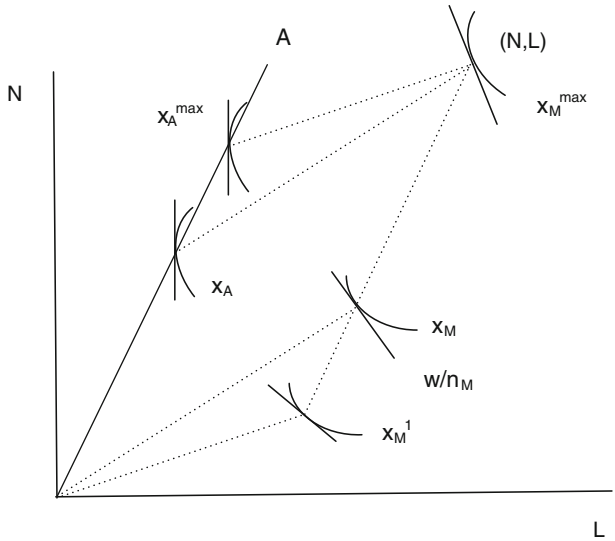

Fig. 3 Limits to production

expansion path A extends to unit isoquant x_M^1. Assuming this minimum single unit of output, production of M takes place along this dotted line.

The static equilibrium occurs at isoquants x_A and x_M in Fig. 3. The expansion path for M extends from the origin to isoquant x_M. The input price ratio w/n_M is steeper at x_M than at the unit isoquant x_M^1 implying the relative wage increases with output of labor intensive M. The highest relative wage occurs at maximum output x_M^{max} with all of both inputs employed in M. The wage is too high to support production of A even with its free resource input. Maximum output x_A^{max} occurs with one unit of output M at the lowest relative wage.

In a market transition, sector A becomes less resource intensive. Sector M becomes more resource intensive in the face of the rising relative wage. Weak substitution in sector M can result in increased output in sector A as shown in the following algebraic model.

3 Market Transition in the Static Model

The algebraic static model depicts equilibrium with full employment and competitive pricing. The full employment conditions are

$$N = a_{NA}x_A + a_{NM}x_M \tag{1}$$

$$L = a_{LA}x_A + a_{LM}x_M \tag{2}$$

Cost minimizing unit inputs a_{ij} are functions of input prices. Competitive pricing implies price equals cost in each sector,

$$p_A = a_{LA}w \tag{3}$$

$$p_M = a_{LM}w + a_{NM}n_M, \tag{4}$$

where p_A and p_M are exogenous world prices. Both sectors pay labor the common competitive wage w but only sector M pays for the resource. Combine (1) through (4) into the recursive block system

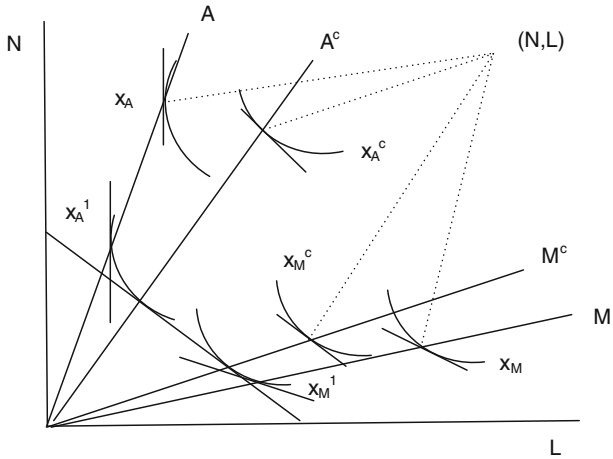

Fig. 4 Market transition

$$\begin{pmatrix} 0 & 0 & a_{NA} & a_{NM} \\ 0 & 0 & a_{LA} & a_{LM} \\ 0 & a_{LA} & 0 & 0 \\ a_{NM} & a_{LM} & 0 & 0 \end{pmatrix} \begin{pmatrix} n_M \\ w \\ x_A \\ x_M \end{pmatrix} = \begin{pmatrix} N \\ L \\ p_A \\ p_M \end{pmatrix} \tag{5}$$

The determinant $\Delta = -a_{LA}a_{NM}\alpha$ is negative assuming A is resource intensive, $\alpha \equiv a_{NA}a_{LM} - a_{LA}a_{NM} > 0$.

Solutions for the endogenous variables are

$$\begin{aligned} n_M &= \left(a_{LA}p_M - a_{LM}p_A\right)/a_{LA}a_{NM} & w &= p_A/a_{LA} \\ x_A &= (a_{LM}N - a_{NM}L)/\alpha & x_M &= (a_{NA}L - a_{LA}N)/\alpha \end{aligned} \tag{6}$$

Positive outputs x_A and x_M require $a_{NA}/a_{LA} > N/L > a_{NM}/a_{LM}$ with the expansion paths in Fig. 3 spanning the endowment point (N,L).

A positive resource price n_M requires

$$a_{LA}/a_{LM} > p_A/p_M. \tag{7}$$

Labor's opportunity cost of A must be greater than the relative price of A.

In a competitive resource market where superscript c refers to the market equilibrium, $w^c = (a^c_{NM}p_A - a^c_{NA}p_M)/a^c_{LA}a^c_{NM}$. Positive factor prices imply

$$a^c_{LA}/a^c_{LM} > p_A/p_M > a^c_{NA}/a^c_{NM}. \tag{8}$$

Factor opportunity costs must span the relative output price, assumed constant in the small open economy.

Figure 4 illustrates a market transition. The resource/labor ratio a_{NA}/a_{LA} in sector A falls from A^R to A^c. The manufacturing input ratio a_{NM}/a_{LM} rises from M to M^c with the increased relative wage and substitution along unit isoquant x^1_M. With the degree of substitution in Fig. 4 x_A rises and x_M falls. The substitution in M is insufficient to generate a decrease in x_A. Output rises in A even as it becomes less resource intensive. Output falls in M as it attracts only a small amount of N and loses L. With stronger substitution, there is more of an increase in a_{NM}/a_{LM} and output M increases. The stronger substitution in sector M would lead to

decreased output in A. The two products are traded at exogenous world prices with excess supply exported and excess demand imported.

In the related resource market of Fig. 1 resource demand D_A increases with the incoming labor as sector A expands. Resource demand D_M falls due to its lost labor. Resource input increases for A in the market transition of Fig. 4.

4 The Comparative Static Model with Resource Rights

An exogneous price change for sector A in (3) generates adjustments according to the total differential $dp_A = wda_{LA} + a_{LAdw}$. Unit inputs do not change implying $da_{LA} = 0$ and

$$dp_A = a_{LA}dw. \tag{9}$$

The price of sector M in (4) changes according to $dp_M = wda_{LM} + n_M da_{NM} + a_{LM}dw + a_{NM}dn_M$. Cost minimization implies the slope of the isoquant equals the negative of the input price ratio $dN/dL = da_{NM}/da_{LM} = -/wn_M$ leading to the envelope condition $wda_{LM} + n_M da_{NM} = 0$ that implies

$$dp_M = a_{LM}dw + a_{NM}dn_M. \tag{10}$$

The resource is fully utilized by the two sectors, $N = a_{NA}x_A + a_{NM}x_M$ from (1). Changes in the resource endowment occur according to $dN = a_{NA}dx_A + a_{NM}dx_M + x_A da_{NA} + x_M da_{NM}$. The fixed unit input implies $da_{NA} = 0$ while $da_{NM} = (\partial a_{LM}/\partial w)dw + (\partial a_{LM}/\partial n_M)dn_M$. Substitution enters in the negative own labor substitution term $S_{LL} \equiv x_M(\partial a_{Lj}/\partial w)$ for j = A,M, negative own resource substitution $S_{NN} \equiv x_M(\partial a_{LM}/\partial n_M)$, and positive cross price substitution $S_{NL} \equiv x_M(\partial a_{NM}/\partial w)$. Cross price substitution terms S_{NL} and S_{LN} are equal due to Shephard's lemma and the Taylor rule.

Changes in the resource endowment relate to outputs and input prices according to

$$dN = a_{NA}dx_A + a_{NM}dx_M + S_{NL}dw + S_{NN}dn_M \tag{11}$$

Similarly, changes in the labor force in (2) are

$$dL = a_{LA}dx_A + a_{LM}dx_M + S_{LL}dw + S_{LN}dn_M. \tag{12}$$

Employment conditions (11) and (12) combine with pricing conditions (9) and (10) in the comparative static system

$$\begin{pmatrix} S_{NN} & S_{NL} & a_{NA} & a_{NM} \\ S_{LN} & S_{LL} & a_{LA} & a_{LM} \\ 0 & a_{LA} & 0 & 0 \\ a_{NM} & a_{LM} & 0 & 0 \end{pmatrix} \begin{pmatrix} dn_M \\ dw \\ dx_A \\ dx_M \end{pmatrix} = \begin{pmatrix} dN \\ dL \\ dp_A \\ dp_M \end{pmatrix} \tag{13}$$

Cramer's rule solves for the partial derivative adjustments in factor prices and outputs to exogenous changes in factor endowments and prices. The negative determinant of (13) is identical to that of the static system (5).

5 Comparative Statics of Endowment Changes

Exogenous endowments changes have the following endogenous adjustments from (13),

$$\begin{aligned} \partial n_M/\partial N &= \partial w/\partial N = 0 & \partial n_M/\partial L &= \partial w/\partial L = 0 \\ \partial x_A/\partial N &= a_{LM}/\alpha > 0 & \partial x_A/\partial L &= -a_{NM}/\alpha < 0 \\ \partial x_M/\partial N &= -a_{LA}/\alpha < 0 & \partial x_M/\partial L &= a_{NA}/\alpha > 0. \end{aligned} \tag{14}$$

Null effects of endowment changes on factor prices are the factor price equalization property of Lerner (1952) and Samuelson (1949). Changes of the endowment point in Fig. 3 do not affect factor prices or slopes of isocost lines in spite of the different in the resource price between sectors.

Changing endowments affect outputs in the Rybczynski (1955) intensity pattern with a positive effect on their intensive output and a negative effect on the other output. Two similar economies with free trade would have identical unit value isoquants and input prices. The Heckscher-Ohlin theorem would then hold with the resource abundant country exporting the product with resource rights.

An implication of factor price equalization is that an increase in the resource endowment would not affect its price. Its higher supply is offset by the increased relative output of A. Sector A bids away labor from M but there is offsetting increased labor supply. If the resource endowment increased to expansion path A in Fig. 3 then sector M would shut down. Any further increase in the resource endowment would lie idle in the factor proportions problem of Eckaus (1955).

Increased labor endowment raises M output. Labor departs sector A where decreased output exactly offsets the higher marginal product. In expanding sector M the incoming labor offsets increased demand neutralizing the wage effect.

6 Comparative Statics of Price Changes

Changes in the price of the sector with resource rights have the following general equilibrium effects from (13),

$$\begin{aligned} \partial n_M/\partial p_A &= -a_{LM}/a_{LA}a_{NM} < 0 & \partial w/\partial p_A &= 1/a_{LA} > 0 \\ \partial x_A/\partial p_A &= (a_{NM}S_1 + a_{LM}S_2)/\Delta < 0 & \partial x_M/\partial p_A &= -(a_{NM}S_3 + a_{LM}S_4)/\Delta > 0 \end{aligned} \tag{15}$$

with the positive terms $S_1 \equiv a_{LM}S_{NL} - a_{NM}S_{LL}$, $S_2 \equiv a_{NM}S_{NL} - a_{LM}S_{NN}$, $S_3 \equiv a_{LA}S_{NL} - a_{NA}S_{LL}$, and $S_4 \equiv a_{NA}S_{NL} - a_{LA}S_{NN}$. The two inputs are substitutes with a positive cross price term S_{NL}.

These effects are opposite to the Stolper and Samuelson (1941) theorem, breaking the reciprocity with Rybczynski endowment/output effects in (15). An increase in p_A raises the wage and lowers the price n_M of the intensive input in the other sector. Output A falls as output M rises along the convex production frontier. The fixed factor intensity in A accounts for these distortions.

The lower relative resource price induces sector M to become more resource intensive. The expansion path for M becomes steeper in Fig. 4. Declining production of resource intensive x_A releases the resource relative to labor leading to the falling n_M.

In the distribution box of Fig. 1 the increase in p_A initially raises demand but expanding x_M attracts labor leading to the net decrease. Resource utilization falls in sector A as demand in M rises due to the increased productivity from incoming labor. Increased resource demand does not completely offset increased supply.

A tariff for import competing sector A would lower output x_A while the wage and x_M increase. Output in sector A is shrunk by its own tariff. Effective price supports for sector A would lower output.

Adjustments to changes in p_M in (13) are opposite to the competitive model,

$$\begin{aligned} \partial n_M/\partial p_M &= 1/a_{NM} > 0 & \partial w/\partial p_M &= 0 \\ \partial x_A/\partial p_M &= -a_{LA}S_2/\Delta > 0 & \partial x_M/\partial p_M &= a_{LA}S_4/\Delta < 0, \end{aligned} \tag{16}$$

with no effect on w as n_M rises. Output effects again indicate a convex production frontier. Constant cost in sector A implies the lack of wage adjustment. Increased resource input in sector A raises labor demand while in sector M reduced resource input lowers labor demand. The net result is no wage change, a razor edge result not necessary in the three factor model.

7 The Three Factor Model with Resource Rights

Adding capital to the model, a pair of inputs may be complements and capital/labor substitution affects the resource market. Thompson (1985) analyzes this three factor, two good production structure. To simplify, focus on resource intensity in the input matrix

$$\begin{pmatrix} a_{NA} & a_{NM} \\ a_{LA} & a_{LM} \\ a_{KA} & a_{KM} \end{pmatrix} = \begin{pmatrix} a_{NA} & 1 \\ 1 & 1 \\ 1 & 1 \end{pmatrix} \tag{17}$$

Sector A is resource intensive relative to labor and capital if $a_{NA} > 1$. Also assume all inputs are uniform cross price substitutes with the unit substitution terms $S_{NK} = S_{NL} = S_{LK} = 1$. Scaling to unit factor prices, the symmetric substitution matrix similar to (13) has 1 for cross price terms and -2 for own price terms on the main diagonal. The system determinant $\Delta = 2(3a_{NA} - 1)$ is positive if $a_{NA} > 1/3$.

Changes in the price of sector A affect the resource price in sector M according to $\partial n_M/\partial p_A = 6(1-a_{NA})/\Delta$. If $a_{NA} > 1$ the resource price falls. If sector M is labor and capital intensive relative to resource, $a_{NA} < 1$ then n_M increases. The wage adjusts according to $\partial w/\partial p_A = (1+3a_{NA})/\Delta$ and the capital return according to $\partial r/\partial p_A = -(7+3a_{NA})/\Delta$. These two opposite effects depend on the sign of Δ. The possible factor price adjustments are developed by Jones and Easton (1983) and Thompson (1993).

Output of A adjusts according to $\partial x_A/\partial p_A = -27/\Delta$ that also depends on the sign of Δ. A higher p_A may lower x_A even with substitution between three inputs. If sector M is resource intensive with a negative Δ then x_A rises with p_A while x_M adjusts in the opposite direction according to $\partial x_M/\partial p_A = 9(2 + a_{NA})/\Delta$. The production frontier is convex for positive Δ.

In summary, the uneconomic properties due to the free resource may arise in the three factor model and in any higher dimensional model with more outputs or inputs.

8 Conclusion

The distinctive property of a sector with a free resource input is its exaggerated resource intensity that is insensitive to relative input prices. The free resource lowers output for the sector relative to a market unless there is sufficient substitution in the rest of the economy. Another uneconomic property due to the free resource is that import tariffs, export subsidies, or price

supports lower output. The benefits of transition to a resource market are underestimated by not considering these distortions in the general equilibrium of production.

The present model can be extended to include utility maximization for a closed economy, and trade between two such large economies with differing factor endowments. In both cases, output prices are endogenous in the general equilibrium. Allowing investment and capital accumulation, a three factor growth model with a free resource input can be developed.

The present results relate to a wide range of resource issues including potential markets for clean air and ecosystem services. Avoiding these markets generates uneconomic behavior with losses beyond the partial equilibrium. The properties and results also apply to binding price controls on any input, increasing the incentives to favor market allocation of resources.

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