Fixed Factor Proportions Production and Trade

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A "fixed factor proportions" model with fixed coefficients of various inputs bridges the classical constant cost model with a single input and the neoclassical factor proportions model. Factor abundance or technology can determine the direction of trade in the fixed factor proportions FFP model. The trade theorems of factor proportions theory are identical in spite of the lack of substitution in the FFP model. The FFP model with various numbers of factors and products is analyzed.

Keywords factor proportions, trade, fixed inputs

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Complete specialization characterizes the classical fixed opportunity cost model based on Ricardo as reviewed by Sraffa (1951) with constant opportunity cost along a linear production frontier. Comparative advantage, the opportunity cost of the single factor of production, determines the direction of trade. Chipman (1965), Morishima (1989), Maneschi (1992), and Ruffin (2002) develop and extend the constant cost model.

Partial specialization characterizes the neoclassical factor proportions trade model based on Heckscher and Ohlin as reviewed by Flam and Flanders (1991). The production frontier is concave and relative factor endowments determine the direction of trade given similar preferences across countries. Samuelson (1953), Chipman (1966), and Jones and Scheinkman (1977) formalize the neoclassical factor proportions model based on cost minimization and input substitution.

Jones (1973) develops the model with fixed unit inputs and inequality employment constraints. Ruffin (1988) develops a model with fixed unit inputs each producing on their own. The present fixed factor proportions FFP model is novel in that it imposes full employment with all factors required in production. Either factor abundance or technology can determine the direction of trade in these models.

The FFP model introduces the basic factor proportions trade theorems without the mechanics of neoclassical cost minimization. The four basic theorems (factor price equalization, Stolper-Samuelson, Rybczynski, Heckscher-Ohlin) have identical comparative static properties in the FFP model. Tariffs or changing prices, however, do not affect outputs due to the fixed unit inputs and full employment. In a temporal context, the FFP model captures the immediate effects of exogenous price and endowment changes before input substitution takes place. When prices change, full adjustment of factor prices occurs immediately while outputs adjust over time with substitution.

The FFP model is introduced in the first section below followed by a section on comparative advantage. The algebraic model is developed in a third section followed by a section on assumptions that lead to solutions of uneven FFP models with different numbers of factors and products. The model with more products than factors is tractable assuming arbitrarily small markup pricing. The model with more factors than products can be solved with an arbitrarily small degree of substitution between any inputs. The Ricardian factor endowment model of Ruffin (1988) is reviewed in Section 6.

1. The FFP Model

The FFP production diagram with two factors and two products in Figure 1 has Leontief right angled isoquants with inputs of factors v_1 and v_2. Scale outputs to $p_j = 1$ and unit value isoquants represent one unit of numeraire where $x_j = 1/p_j$. Competitive pricing and factor mobility imply the single isocost line $p_j = c_j = 1 = a_{1j}w_1 + a_{2j}w_2$.

* Figure 1 *

Full employment determines outputs x_j along expansion paths with product 1 using factor 1 intensively in the ranking $a_{11}/a_{21} > a_{12}/a_{22}$. Diversified production requires an endowment point E = (v_1, v_2) inside the production cone in the condition $a_{11}/a_{21} > v_1/v_2 > a_{12}/a_{22}$. Changes in E inside the cone with constant prices alter outputs in a linear fashion following factor intensity. An increase in factor v_1 raises x_1 and lowers x_2 as both factors leave industry 2 and outputs adjust along the Rybczynski (1955) line, identical to changes in the factor proportions model developed by Kemp (1964). Increased v_1 leads toward specialization in product 1 and beyond the expansion path becomes redundant, the factor proportions problem of Eckaus (1955). Comparing two countries with different endowments, the Heckscher-Ohlin theorem follows with each country exporting the product using its abundant factor intensively.

There is local and global factor price equalization in Figure 1. A changing endowment inside the production cone with constant prices has no effect on factor prices, identical to the factor proportions model.

Stolper-Samuelson (1941) effects of price changes on factor prices are also identical to the factor proportions model. Price changes shift the unit isocost line and factor prices adjust. The slope of the isocost line is the relative factor price w_2/w_1. An increase in the price of product 1 shifts the unit value isoquant $1/p_1$ toward the origin representing less physical output. The result is a lower slope w_2/w_1 in an adjustment identical to the factor proportions model with convex isoquants and any degree of substitution.

Two countries that differ only in endowments export the product using their abundant factor intensively, the Heckscher-Ohlin theorem. Two countries that differ only in technologies export according to factor intensity. Two countries that differ only in utility functions export the product with a lower marginal rate of substitution.

Figure 2 illustrates the equilibrium of production and trade. The autarky relative price p_2/p_1 is determined by the marginal rate of substitution of the indifference curve passing through production point A. If the world relative price of product 2 is lower at the terms of trade p_2^*/p_1^* the country imports product 2. Home consumption with free trade at point C_1 has higher utility than autarky consumption at point A.

* Figure 2 *

A tariff does not alter production but utility maximizing consumers face tariff distorted prices. The relative price of imports rises to $p_2^*(1 + t)/p_1^*$ in Figure 2 where t is the tariff rate. There is decreased consumption of the imported product along the terms of trade line to point C_2 where the marginal rate of substitution equals the distorted domestic relative price. The tariff lowers both utility and real income measured in autarky prices.

Output distortions in the factor proportions model imply larger net losses. The partial equilibrium deadweight loss of the tariff is larger due to the fixed output and no offsetting gain in producer surplus.

2. Comparative Advantage in the FFP model

Technology differences alone may lead to trade between two FFP economies. In the classical model, lower opportunity cost of the single input is equivalent to comparative advantage and implies the direction of trade. In the FFP model with more than a single input, lower opportunity cost of an input implies imports. Consider the following two country model.

Full employment $v_i = a_{i1}x_1 + a_{i2}x_2$ implies output levels $x_1 = (a_{22}v_1 - a_{12}v_2)/b$ and $x_2 = (a_{11}v_2 - a_{21}v_1)/b$ where $b \equiv a_{11}a_{22} - a_{12}a_{21} > 0$ as in Figure 1. A positive b follows from the factor intensity ranking

$$a_{11}/a_{21} > a_{12}/a_{22} \tag{1}$$

with factor 1 (2) the intensive input in product 1 (2). Similar conditions hold in the foreign country.

Focus on technology differences across countries and suppose there are identical endowments as in Figure 3 with expansion paths of each country spanning endowment point $E = (v_1, v_2) = E^* = (v_1^*, v_2^*)$. With no loss in generality, rescale factor 1 to the unit input $a_{11} = 1$ and product 1 to $a_{11}^* = 1$. Similarly rescale factor 2 and product 2 to $a_{22} = a_{22}^* = 1$.

* Figure 3 *

Product 1 uses factor 1 intensively and the foreign country has an intensity bias in factor 1,

$$a_{11}^*/a_{21}^* > a_{11}/a_{21} > a_{12}^*/a_{22}^* > a_{12}/a_{22}, \tag{2}$$

or with the rescaling $1 > a_{12}^* > a_{12}$ and $1 > a_{21} > a_{21}^*$. The home output ratio $x_1/x_2 = (1 - a_{12})/(1 - a_{21})$ must be higher than the foreign output ratio $x_1^*/x_2^* = (1 - a_{12}^*)/(1 - a_{21}^*)$. The home country must then export product 1 given identical homothetic preferences and free trade.

The home country has an intensity bias in factor 2 and imports the product using that factor intensively. The home country exports product 1although it has a higher opportunity cost for that product in both factors, $a_{i1}/a_{i2} > a_{i1}^*/a_{i2}^*$. The point is that factor intensity consumes more of a factor and leads to lower relative output of the product using that factor intensively. Low opportunity cost favors imports when there is more than a single factor of production.

Endowment differences and factor intensities determine the direction of trade. As pointed out in communication by Yutaka Horiba, the direction of trade with identical factor intensities for one product would be determined by comparison of endowment differences and factor intensities of the other product.

The other situation to consider with two products and two countries is when one production cone spans the other making the output ratio between countries ambiguous. Suppose the home cone spans the foreign cone, $1 > a_{12} > a_{12}^*$ and $1 > a_{21}^* > a_{21}$. Output ratios and exports then depend on degrees of factor intensity. A country would be more likely to export a product using a factor intensively with less intensity of that factor in the other product. There may be no trade at all as with technology $(a_{12}\ a_{21}\ a_{12}^*\ a_{21}^*) = (.7\ .4\ .8\ .6)$ implying $x_1/x_2 = x_1^*/x_2^*$.

3. Comparative Static FFP Model

The comparative static model clarifies properties of the 2x2 FFP model and illustrates how to set up higher dimensional models. Competitive pricing of product j is stated $p_j = a_{1j}w_1 + a_{2j}w_2$. With exogenous world prices p_j the static solution for factor prices is $w_1 = (a_{22}p_1 - a_{21}p_2)/b$ and $w_2 = (a_{11}p_2 - a_{12}p_1)/b$. Positive factor prices require that factor intensity span the relative price in the condition $a_{11}/a_{21} \geq p_1/p_2 \geq a_{12}/a_{22}$.

Differentiate the full employment and competitive pricing conditions to find $dv_i = a_{i1}dx_1 + a_{i2}dx_2$ and $dp_j = a_{1j}dw_1 + a_{2j}dw_2$ and combine these four equations into the comparative static system

$$0 \quad 0 \quad a_{11} \quad a_{12} \qquad dw_1 \qquad\qquad dv_1$$

$$\begin{pmatrix} 0 & 0 & a_{21} & a_{22} \\ a_{11} & a_{21} & 0 & 0 \\ a_{12} & a_{22} & 0 & 0 \end{pmatrix} \begin{pmatrix} dw_2 \\ dx_1 \\ dx_2 \end{pmatrix} = \begin{pmatrix} dv_2 \\ dp_1 \\ dp_2 \end{pmatrix}. \qquad (3)$$

The positive determinant of this block recursive matrix is $\Delta = b^2$. Cramer's rule leads to partial derivative solutions for endogenous dw_i and dx_j with respect to exogenous dv_i and dp_j.

Endowment changes do not affect factor prices in the factor price equalization result $\delta w_i/\delta v_k = 0$. Any substitution between inputs would enter the upper left quadrant of the system matrix but would be cancelled by zeros in the lower right quadrant in the cofactors of $\delta w_i/\delta v_k$ terms.

Price changes do not affect outputs due to the absence of substitution, $\delta x_j/\delta p_m = 0$. Outputs cannot change given their input mix and full employment but an arbitrarily small degree of substitution in the upper left quadrant of the system matrix would imply output adjustments.

The other partial derivatives in (3) are reciprocal,

$$\begin{aligned} \delta w_1/\delta p_1 &= \delta x_1/\delta v_1 = a_{22}/b && > 0 \\ \delta w_1/\delta p_2 &= \delta x_2/\delta v_1 = -a_{21}/b && < 0 \\ \delta w_2/\delta p_1 &= \delta x_1/\delta v_2 = -a_{12}/b && < 0 \\ \delta w_2/\delta p_2 &= \delta x_2/\delta v_2 = a_{11}/b && > 0, \end{aligned} \qquad (4)$$

identical to the factor proportions model with any degree of substitution. As Thompson (1995) points out, factor intensity plays a larger role than factor substitution in general equilibrium models of production. Price changes affect factor prices but not outputs. A higher p_1 increases the demand for its intensive factor 1 raising w_1. Factor price adjustments are a Jones (1965) magnified effect of price changes. With substitution, factor 1 would be bid into industry 1 and its output would expand. Output adjustments in the factor proportions models are not related to the factor price adjustments.

Endowment changes cause adjustments in output but not factor prices. Firms hire inputs in their ratio with one industry contracting as the other expands in a linear Rybczynski adjustment. An increase in the factor 1 endowment causes industry 1 to absorb all of that additional input and attract both factors from the other industry. The output changes for $dv_1 = 1$ are $dx_2 = -a_{21}/b$ and $dx_1 = a_{22}/b$ implying the Rybczynski line slope $dx_1/dx_2 = -a_{22}/a_{21}$.

Adjustment to the endowment involves temporary factor price changes inducing factor movements. An increase in v_1 raises the marginal productivity and return to factor 2 in industry 1 explaining its attraction to industry 1. The return to factor 1 is temporarily higher in industry 1 with its marginal productivity stimulated by the incoming factor 2. Both factor prices return to their original levels as output adjustments absorb the full effects of the endowment change.

4. Uneven FFP Models

If there are more products than factors, the FFP model is underdetermined as in the classical 1x2 Ricardian model. Any output combination is consistent with full employment along the linear production frontier. The zero system determinant does not allow solution of the underdetermined system. In the 2x3 FFP model of an expanded (3) the null matrix in the lower right hand corner dominates the system matrix.

One way to close models with more products than factors is to introduce markup pricing as a function of output $\mu_j(x_j)$ as in Thompson (2003). Markup pricing enters the pricing condition as $p_j = a_{1j}w_1 + a_{2j}w_2 + \mu_j(x_j)$. Differentiate to find $dp_j = a_{1j}dw_1 + a_{2j}dw_2 + \mu_j'dx_j$ where μ_j' is the derivative of $\mu_j(x_j)$. The μ_j' terms in the lower right quadrant of the system expanded from (3) lead to a nonzero determinant and allow comparative static solution. For small degrees of markup ricing as $\mu_j' \rightarrow 0$ the Stolper-Samuelson elasticities are identical to the model with competitive pricing. It is underappreciated in the literature that the model with more products than factors is fully determined with any arbitrarily small degree of markup pricing for any product.

With more factors than products, the FFP model is overdetermined given the fixed unit inputs and arbitrary endowments. As an example, in the 2x1 model the expansion path a_{11}/a_{21} might not match the endowment v_1/v_2. Any degree of substitution, however, leads to tractable results with substitution terms entering the upper left quadrant of the system matrix similar to (3).

Substitution terms are defined as $s_{ik} \equiv \sum_j x_j \delta a_{ij}/\delta w_k$. Suppose there is substitution between factors 1 and 2 with $s_{12} = s_{21} = s$ in the 3x2 model of Thompson (1985). With homogeneity and scaling, the own substitution terms can be written $s_{11} = s_{22} = -s$. In the comparative static $\delta w/\delta p$ and symmetric $\delta x/\delta v$ results, the term s is factored out of the cofactors and signs of these Stolper-Samuelson and Rybczynski results are independent of substitution. In the limit as $s \rightarrow 0$ with arbitrarily small substitution, comparative static $\delta x/\delta p$ production possibility terms become large while $\delta w/\delta v$ terms approach zero. Any degree of substitution between any pair of inputs leads to tractable results with more factors than products.

5. The Ricardian Factor Endowment Model

Similar to the present FFP model, the Ricardian factor endowment (RFE) model of Ruffin (1988) integrates concepts of constant cost and factor proportions models. Fixed unit inputs produce independently in the RFE model and the opportunity cost (comparative advantage) of a factor between industries determines its employment within a country. An industry will employ the other factor only if demand exceeds its ability to produce with the factor that has a lower opportunity cost.

Assume product 1 use factor 1 intensively, $a_{11}/a_{12} > a_{21}/a_{22}$ as in (1). Factor 1 has a lower opportunity cost in product 2. Factor intensity involves relatively high unit inputs, while opportunity cost involves relatively low ones.

The production frontier in the RFE model has two flat regions that connect at a hinge with each factor employed by its comparative advantage industry. The production point A in Figure 4 has specialized outputs $x_1 = v_2/a_{21}$ and $x_2 = v_1/a_{12}$. The output of x_1 would increase by v_1/a_{11} moving from point A to complete specialization, making the absolute value of the slope of the segment a_{12}/a_{11}. The lower section has slope a_{22}/a_{21}.

* Figure 4 *

Autarky production would take place at point A for a range of preferences that determine the domestic relative price p_2/p_1 given $a_{22}/a_{21} > p_2/p_1 > a_{12}/a_{11}$. If preferences switch toward x_1 the relative price would fall as far as $p_2/p_1 = a_{12}/a_{11}$ along the upper section of the production frontier. Prices are lower with lower opportunity cost inputs, $p_1 = w_2a_{21} < w_1a_{11}$ and $p_2 = w_1a_{12} < w_2a_{22}$ implying the relative factor price is limited by technology according to $a_{22}/a_{12} > w_2/w_1 > a_{21}/a_{11}$.

The move from autarky to free trade may not shift production. Let p_2/p_1 be the domestic relative price and suppose $p_2/p_1 > p_2^*/p_1^* > a_{12}/a_{11}$ with a higher exogenous international relative price of x_1 below the opportunity cost of factor 1. The economy exports product 1 trading to point C_1 with higher utility than at point A but no change in production.

With the move to free trade, factor price adjustments depend directly on prices. The price of factor 1 is tied to production of the import competing product, $w_1 = p_2/a_{12}$. Free trade lowers p_2 to p_2^* and w_1 falls. With unspecialized trade w_1 would fall while w_2 rises from p_1/a_{21} to p_1^*/a_{21}. Prices of the same factor converge across trading countries but fall short of factor price equalization.

Specialization occurs if the world relative price of product 1 is higher than the opportunity cost of factor 1, $p_2^*/p_1^* < a_{12}/a_{11}$. Factor 1 is then more valuable in industry 1 and $x_1 = v_2/a_{21} + v_1/a_{11}$. Trade moves consumption to utility maximization at point C_2.

Changes in factor endowments shift the hinge point and affect factor prices. An increase in factor 1 increases x_2 but x_1 is unchanged, lowering autarky p_2/p_1 and w_1/w_2 given homothetic demand. If technology is identical between two economies with identical preferences, endowment differences lead to trade. If $v_1 > v_1^*$ and $v_2 < v_2^*$ they could trade with no change in production. This foreign production point A* would be northwest of home point A in Figure 4 with the home country exporting product 2 based on its abundant factor 1. Factor owners effectively trade with each other, the country with a relative abundance exporting that lower opportunity cost product.

Ruffin (1992) develops the related two country model with differences in technology, endowments, or preferences determining trade. Two countries with identical endowments and preferences might trade based on opportunity costs between factors within each country as well as the opportunity cost of factors across countries. Endowment differences can also lead to trade as in factor proportions models with cost minimizing inputs.

6. Conclusion

The present fixed factor proportions model provides a link between classical and factor proportions models of production and trade, stressing that factor intensity is the foundation of the basic trade theorems. Factor intensity proves the most robust concept for predicting the direction of trade. For application, an advantage of the present fixed factor proportions model is that tests of trade theorems do not have to assume factor price equalization.

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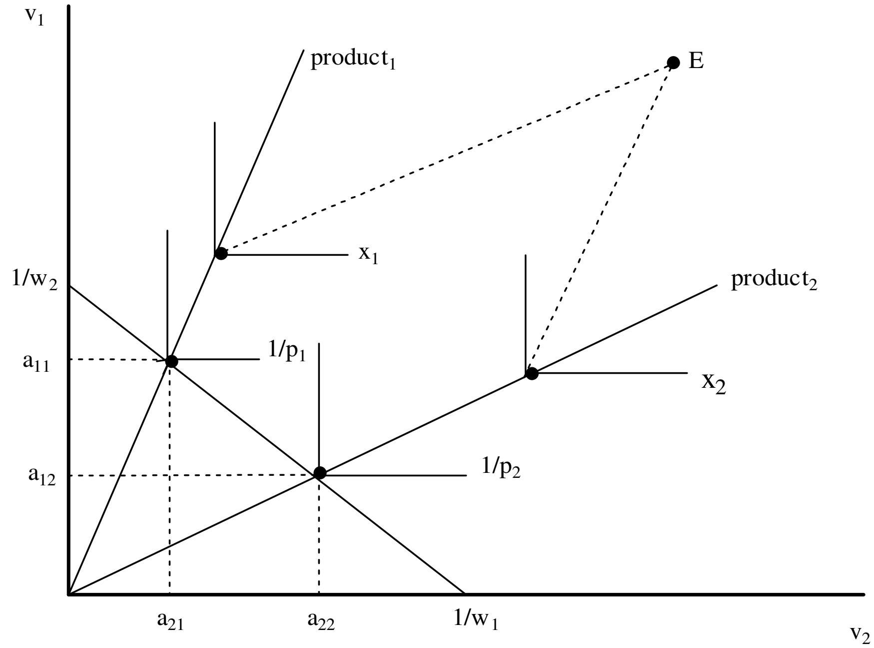

Figure 1. Production in the FFP Model

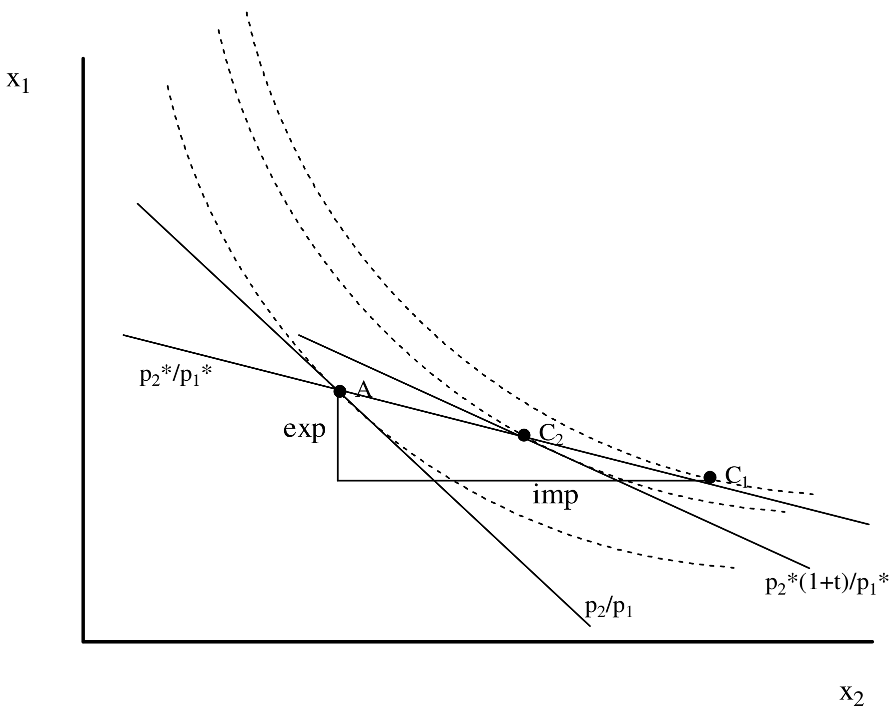

Figure 2. Trade in the FFP Model

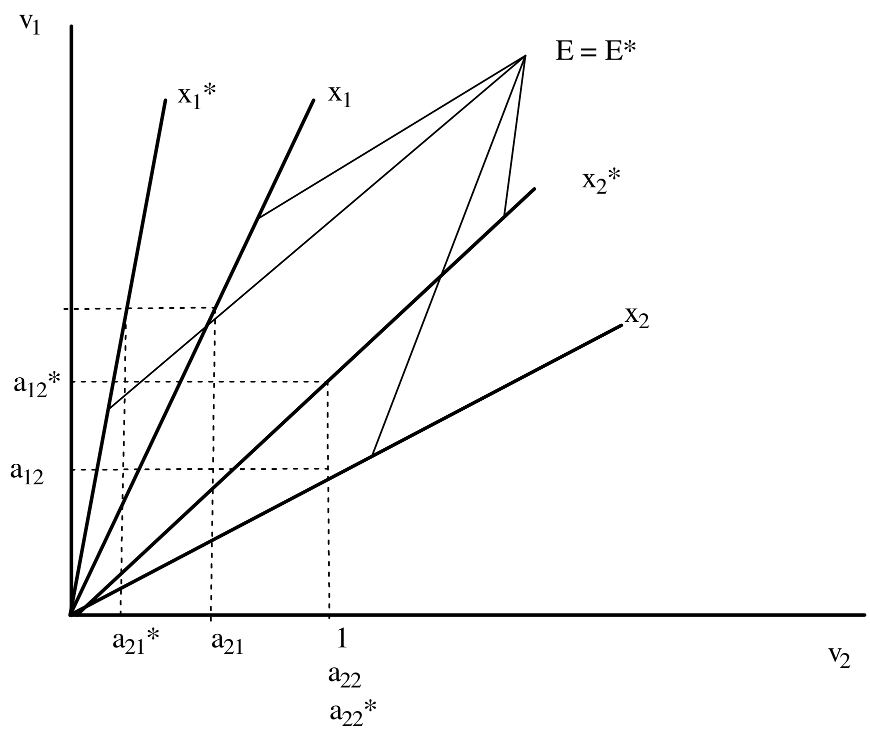

Figure 3. Technology Induced Trade in the FFP Model

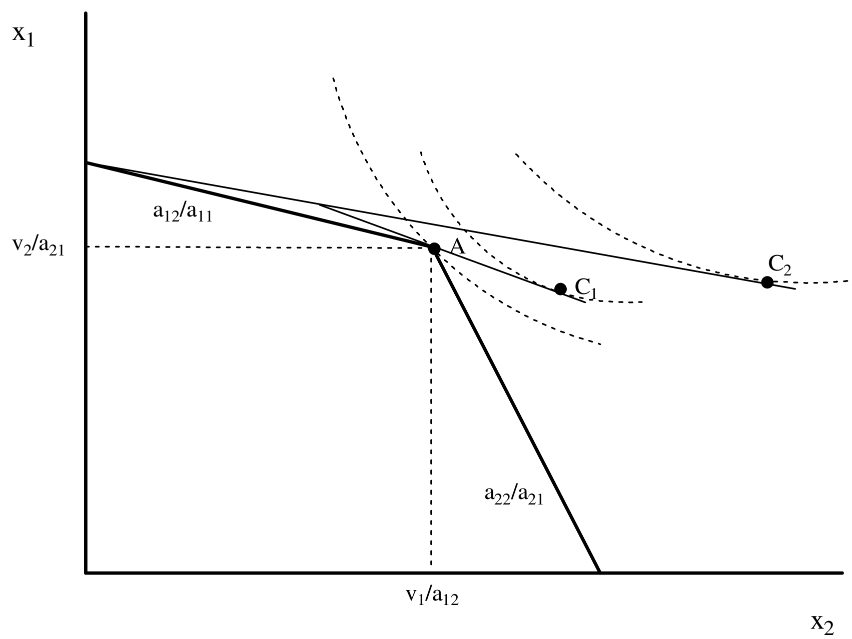

Figure 4. The Missing Link Model