A Nonrenewable Resource in the Heckscher-Ohlin Model

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This paper examines the intertemporal equilibrium of a small open economy producing a nonrenewable resource intensive export in a Heckscher-Ohlin framework. Labor is assumed to grow at a steady rate, and capital with investment out of income. Optimal depletion implies the resource price rises at the rate of the capital return. The model solves for intertemporal factor prices, depletion, outputs, and income. The effects of a depletion tax, import tariff, and export subsidy are examined. Cobb-Douglas simulations illustrate endogenous variable paths. A constant depletion rate, tragedy of the commons, and myopic resource owner are also discussed.

Key words: resource depletion, trade, general equilibrium

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Exports of many countries are based on nonrenewable resources, motivating the present Heckscher-Ohlin model of a small open economy producing a resource intensive export. Labor grows at a steady rate while capital grows with investment based on income. Optimal depletion implies the resource price rises at the rate of the capital return turning attention to the intertemporal equilibrium involving depletion, production, and factor prices.

The wage and capital return depend on factor intensity only while the two outputs depend on substitution, investment, labor growth, and the state of the economy as well. Effects of a depletion tax, import tariff, and export subsidy are examined. Simulations with Cobb-Douglas production functions illustrate the paths of depletion, domestic factor prices, outputs, and income. Depletion diminishes in the general equilibrium regardless of demand for the resource and even if export production rises based on capital and labor growth.

The first section introduces the model followed by a review of substitution in production. The model is presented in the third section. The fourth section analyzes taxes and subsidies. Simulations with Cobb-Douglas production functions in the fifth section illustrate endogenous variable paths. A final section considers the alternative assumptions of a constant depletion rate, tragedy of the commons, and myopic resource owner.

1. Dynamics of labor, capital, and the resource

The present small open economy produces an export and an import competing good with a nonrenewable resource, capital, and labor. Optimal depletion of a nonrenewable resource in partial equilibrium is developed by Dixit, Hammond, and Hoel (1980), Hamilton (1995), Withagen and Asheim (1998), and Sato and Kim (2002). The resource owner treats the resource as an asset and depletes to keep price rising at the rate equal to the capital return. In the present general equilibrium model the endogenous capital return presents a moving target. The literature assumes the level of depletion diminishes over time, not an obvious property in the present general equilibrium model with capital and labor inputs growing and the two outputs adjusting.

Stiglitz (1974), Dasgupta and Heal (1979), and Solow (1986) develop the growth model with optimal depletion of a nonrenewable resource but capital the only other input. In principle, investment can replace the decreasing input of the nonrenewable resource to maintain income. Thompson (2013) extends this model to include a growing labor force and substitution between the three inputs.

The present Heckscher-Ohlin factor proportions model with three factors has its roots in classical economics as discussed by Robinson (1980). The interplay of factor intensity and substitution in this 3x2 model is developed by Ruffin (1981), Jones and Easton (1983), and Thompson (1985).

In the present model, the labor force L_t is assumed to grow at the constant rate $\lambda \equi L_t'/L_t$ where the ʹ represents a time derivative. The change in capital $K_t' = dK_t/dt$ equals saving ζ_t assuming no depreciation or foreign investment. A share of output is transformed into capital at no cost as in neoclassical growth theory. Assuming a constant saving rate σ it follows that $K_t' = \zeta_t = \sigma Y_t$.

Capital K_t and labor L_t are fully employed in the two sectors, $K_t = \Sigma_j K_{jt}$ and $L_t = \Sigma_j L_{jt}$, $j = 1, 2$. The resource is also utilized by the two sectors, $N_t = \Sigma_j N_{jt}$. National income is $Y_t = r_t K_t + w_t L_t + n_t N_t$ where r_t, w_t, and n_t are competitive factor prices equal to marginal revenue products between the two sectors. Income is equivalently the value of output $Y_t = \Sigma_j p_j x_{jt}$ determined by outputs x_j at exogenous world prices p_j.

Depletion N_t diminishes the resource stock S_t according to $N_t = -S_t'$. Optimal depletion leads to the Hotelling (1931) condition treating the resource stock as an asset with the rate of return on the resource stock n_t'/n_t equal to the capital return r_t. The model assumes adequate stock to make the planning horizon irrelevant. The intertemporal equilibrium is bound by the asset market clearing condition,

$$n_t' = r_t n_t. \tag{1}$$

2. Substitution between the three factors

Substitution determines the direction and strength of adjustments in cost minimizing inputs due to changing factor prices. Allen (1938) and Takayama (1982) lay the foundation applied to three factors by Thompson (2006). Constant returns production functions are $x_{jt} = x_j(K_{jt}, L_{jt}, N_{jt})$ where x_{jt} is the output of good j = 1, 2 at time t.

Resource depletion equals demand in the condition $N_t = \Sigma_j a_{Nj} x_{jt}$ where a_{Nj} is the cost minimizing input per unit of output. Depletion adjusts according to $N_t' = \Sigma_j x_{jt} a_{Nj}' + \Sigma_j a_{Nj} x_{jt}'$. With homothetic production a_{Nj}' depends only on factor prices,

$$N_t' = S_{NK} r_t' + S_{NL} w_t' + S_{NN} n_t' + \Sigma_j a_{Nj} x_{jt}', \tag{2}$$

where the S_{ik} are substitution terms. The cross price substitution of the resource relative to the capital return r is $S_{NK} \equiv \Sigma_j x_{jt}(a_{Nj}'/r_t')$. The cross price substitution relative to the wage is S_{NL} and the own price effect S_{NN}. Capital substitution S_{Ki} and labor substitution S_{Li} terms are similar for i = K, L, N.

Cross price terms are positive between substitutes but in the three factor model one pair of inputs may be complements. Cost minimization and Shephard's lemma imply the cost minimizing unit inputs a_{ij} are partial derivatives of the unit cost function $c_j(r_t, w_t, n_t)$. Young's theorem implies substitution terms are symmetric. The a_{ij} are homogeneous of degree zero assuming unit cost functions homogeneous of degree one in factor prices,. Concave cost functions imply negative own price substitution.

3. The intertemporal general equilibrium

The first equation in the system (3) below is capital employment similar to (2) with the change in the capital input from saving, $K_t' = \sigma Y_t$. The second equation is the change in labor employment where $L_t' = \lambda L_t$. The third equation is the resource market clearing condition in (2) with n_t' replaced by $r_t n_t$ from (1).

The last two equations in (3) represent competitive pricing of the two goods. Price equals cost, $p_{jt} = a_{Kj}r_t + a_{Lj}w_t + a_{Nj}n_t$ for j = 1, 2. Differentiating, the envelope condition of cost minimization implies $p_{jt}' = a_{Kj}r_t' + a_{Lj}w_t' + a_{Nj}n_t'$.

The model solves for the intertemporal equilibrium r_t', w_t', N_t', and x_{jt}' with Cramer's rule in the system

$$\begin{pmatrix} S_{KK} & S_{KL} & 0 & a_{K1} & a_{K2} \\ S_{KL} & S_{LL} & 0 & a_{L1} & a_{L2} \\ S_{KN} & S_{LN} & -1 & a_{N1} & a_{N2} \\ a_{K1} & a_{L1} & 0 & 0 & 0 \\ a_{K2} & a_{L2} & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} r_t' \\ w_t' \\ N_t' \\ x_{1t}' \\ x_{2t}' \end{pmatrix} = \begin{pmatrix} \sigma Y_t - S_{KN}r_t n_t \\ \lambda L_t - S_{LN}r_t n_t \\ -S_{NN}r_t n_t \\ p_1' - a_{N1}r_t n_t \\ p_2' - a_{N2}r_t n_t \end{pmatrix}. \qquad (3)$$

Adjustments in capital K_t' and income Y_t' are derived separately. Given exogenous world prices $p_1' = p_2' = 0$ but import tariffs and export subsidies are considered. The negative determinant of the system (3) is $\Delta = -(a_{K1}a_{L2} - a_{L1}a_{K2})^2$.

4. The intertemporal equilibrium

The intertemporal capital return and wage in (3) are

$$r_t' = -a_{NL}r_tn_t/a_{KL} \quad (4)$$

$$w_t' = a_{NK}r_tn_t/a_{KL}.$$

These adjustments are independent of the growth in capital and labor inputs due to the factor price equalization property. Smaller capital/labor intensity a_{KL} leverages these factor price adjustments as capital and labor become a more similar input.

Assume the resource is extreme in export production in the factor intensity condition

$$a_{N1}/a_{N2} > a_{K1}/a_{K2} > a_{L1}/a_{L2}. \quad (5)$$

Factor intensity is summarized by the positive terms $a_{KL} \equiv a_{K1}a_{L2} - a_{L1}a_{K2} > 0$, $a_{NK} \equiv a_{N1}a_{K2} - a_{K1}a_{N2} > 0$, and $a_{NL} \equiv a_{N1}a_{L2} - a_{L1}a_{N2} > 0$. Intensity terms $A \equiv (a_{KL}\ \ a_{NL}\ \ a_{NK})$ imply $(r_t'\ \ w_t')$ has the signs $(-\ \ +)$. As the resource price rises, the price of the other extreme factor labor rises while the price of the middle factor capital falls. The falling capital return and rising wage are consistent with capital deepening in growth theory. The capital/labor ratio K/L rises if $Y/K > \lambda/\sigma$.

If the resource were extreme in the import competing sector in the ranking $a_{K1}/a_{K2} > a_{L1}/a_{L2} > a_{N1}/a_{N2}$ intensity terms would have signs $A = (+\ \ -\ \ -)$ implying $(r_t'\ \ w_t') = (+\ \ -)$. The rising resource price increases the price of extreme capital but lowers the price of middle labor. If the resource were the middle factor in the ranking $a_{K1}/a_{K2} > a_{N1}/a_{N2} > a_{L1}/a_{L2}$ then $A = (+\ \ +\ \ -)$ and $(r_t'\ \ w_t') = (-\ \ -)$. Prices of both extreme factors would fall with the rising price of the middle factor. These two alternative rankings exhaust possibilities assuming $a_{K1}/a_{K2} > a_{L1}/a_{L2}$.

Solving (3) for intertemporal depletion,

$$N_t' = -r_tn_t\Delta_{32}/\Delta < 0, \quad (6)$$

where Δ_{32} is the negative determinant of the 3x2 model. Depletion declines in the general equilibrium as the resource price rises implying downward sloping *mutatis mutandis* resource demand. This property is not apparent given the growing capital and labor inputs and the flexibility of producing two goods.

The intertemporal output of the resource intensive export is

$$x_{1t}' = [a_{KL}\kappa - r_t n_t(a_{NK}S_1 + a_{NL}S_2 + a_{KL}S_3)]/a_{KL}^2, \quad (7)$$

where $S_1 \equiv a_{L2}S_{KL} - a_{K2}S_{LL} > 0$, $S_2 \equiv a_{K2}S_{KL} - a_{L2}S_{KK} > 0$, $S_3 \equiv a_{L2}S_{KN} - a_{K2}S_{LN}$, and $\kappa \equiv a_{L2}\sigma Y_t - a_{K2}\lambda L_t$. There is a presumption that $x_{1t}' < 0$ but increased production is favored by higher investment, lower labor growth, weaker substitution, and lower levels of r_t and n_t. The mirror image presumption is increasing import competing production, $x_{2t}' > 0$. Both outputs may rise due to investment and labor growth, or both may fall due to declining depletion.

Intertemporal income $Y_t' = K_t r_t' + L_t w_t' + N_t n_t' + n_t N_t'$ becomes

$$Y_t' = r_t n_t[a_{KL}(a_{NK}L_t - a_{NL}K_t) + a_{KL}^2 N_t + n_t\Delta_{32})]/a_{KL}^2, \quad (8)$$

from (4) and (6). Rising income is favored by higher labor and resource inputs but lower capital input given the falling capital return. Rising income is favored by weaker substitution with Δ_{32} closer to zero implying a smaller N_t'. Income per capita rises if $Y_t' > \lambda Y_t$.

Intertemporal income in (8) reflects the gains from trade. Income rises if increased import competing production outweighs decreased export production with the economy moving to a higher terms of trade line if $-p_1x_{1t}' < p_2x_{2t}'$. The economy may fall to a lower terms of trade line. Regardless, the level of trade would fall assuming homothetic utility.

Figure 1 illustrates intertemporal trade with homothetic utility implying a constant consumption ratio c_1/c_2. Equilibrium production occurs at point P_0 on the production frontier with utility maximizing consumption at C_0. The terms of trade line tt connecting P_0 and C_0 reflects the income level.

* Figure 1 *

Presumed output changes imply the production point moves northwest. Income increases if the new production point is above the tt line in Figure 1. The trade triangle shrinks with the level of trade. Income falls if the new production point is below the tt line. The level of trade would increase if both outputs rise due to more of an increase in relative output x_1/x_2 than in relative consumption.

5. Depletion taxes, import tariffs, and export subsidies

A depletion tax raises the price of the resource to $(1 + t_N)n_t$ amplifying the intertemporal dynamics. Depletion N′ in (6) becomes more negative with the rising resource price. Adjustments in the capital return and wage in (4) are amplified with labor benefiting from the shift toward import competing production. Stronger substitution S_{LK} between labor and capital in (7) favors this shift as do lower saving σ and higher labor growth λ. The depletion tax amplifies income adjustment in (8).

Import tariffs and export subsidies add their effects through p_j' in (3). A tariff reinforces the two factor prices according to

$$r_t'/p_2' = -a_{L1}/a_{KL} + r_t' < 0 \tag{9}$$

$$w_t'/p_2' = a_{K1}/a_{KL} + w_t' > 0,$$

where r_t' and w_t' are the underlying adjustments in (4). The tariff favors intensive labor in the import competing industry. Smaller differences in capital/labor intensity leverage the effects in (9). A tariff affects depletion according to

$$N_t'/p_2' = -(S_4 a_{NK} + S_5 a_{NL} + S_6 a_{KL})/a_{KL}^2 + N_t', \tag{10}$$

where $S_4 \equiv a_{L1}S_{KL} - a_{K1}S_{LL} > 0$, $S_5 \equiv a_{K1}S_{KL} - a_{L1}S_{KK} > 0$, $S_6 \equiv a_{L1}S_{KN} - a_{K1}S_{LN}$, and $N_t' < 0$ in (6). The two terms S_4 and S_5 increase as depletion declines. A negative S_6 with labor a strong substitute for the resource weakens the effect on depletion in (10).

An export subsidy affects the intertemporal capital return according to

$$r_t'/p_1' = a_{L2}/a_{KL} + r_t', \qquad (11)$$

where $r_t' < 0$ is the decrease in (4). The higher export price temporarily raises the capital return if $r_t n_t < a_{L2}/a_{LN}$ but the intertemporal decline would resume. The subsidy affects the wage according to

$$w_t'/p_1' = -a_{K2}/a_{KL} + w_t', \qquad (12)$$

where $w_t' > 0$ in (4). The subsidy offsets the rising wage, lowering the wage if $r_t n_t < a_{K2}/a_{NK}$. The subsidy temporarily raises N_t in an expression similar to (10) but diminishing depletion resumes.

Output changes due to an import tariff or export subsidy involve substitution. For instance, the effect of an export subsidy on its production is

$$x_{1t}'/p_1' = (2a_{K2}a_{L2}S_{KL} - a_{K2}^2 S_{LN} - a_{L2}^2 S_{KK}) + x_{1t}', \qquad (13)$$

where x_{1t}' is the intertemporal adjustment in (7). Stronger substitution favors a temporary increase in x_{1t}. The export subsidy unambiguously lowers x_{2t} in a similar expression. In the simulations, the export subsidy leads to overshooting the new trends.

6. Model simulations

The following simulations assume Cobb-Douglas production with the export produced according to $x_{1t} = K_{1t}^{0.6}L_{1t}^{0.1}N_{1t}^{0.3}$ and import competing production $x_{2t} = K_{2t}^{0.4}L_{2t}^{0.5}N_{2t}^{0.1}$. The factor intensity consistent with (5) implies a rising wage and falling capital return as in (4).

The model is simulated over 10 time periods starting at initial values of K_1 = 100,000 and L_1 = 100. Exogenous world prices of the two outputs are 1. The saving rate is $\sigma = 0.25$ and labor growth rate $\lambda = 0.01$.

The initial equilibrium is determined assuming resource price $n_1 = 24$ implying depletion $N_1 = 9.5$. The resource price increases according to $n_{t+1} = (1 + r_t)n_t$ determining depletion N_{t+1} and the intertemporal adjustments in (3). Variables are rescaled for display in the following Figures.

This economy in Figure 2 strongly trends toward production of import competing good 2 with depletion N_t decreasing at a decreasing rate. Income Y_t falls but only slightly as the rising wage w_t and resource price n_t nearly offset the declining capital return r_t. Income falls as does the level of trade.

* Figure 2 *

Figures 3 pictures the economy with higher saving $\sigma = 0.40$ and lower labor growth $\lambda = 0.005$. Good 1 output x_1 increases as capital growth more than offsets the slower decline in depletion N_t. Import competing production x_2 expands more slowly. Factor price paths of w_t and r_t are identical to Figure 2 due to factor price equalization. Income Y_t increases with the wage w_t and resource price n_t. Consumption of both goods increases as could the level of trade.

* Figure 3 *

Figure 4 shows an economy with the original saving rate $\sigma = 0.25$ but slightly negative labor growth $\lambda = -0.05$. Both outputs fall with income as capital growth does not offset the declining depletion and labor force. Consumption of both goods falls even though the level of trade could increase with a sharper reduction in import competing production.

* Figure 4 *

Figure 5 illustrates the effects of a depletion tax of $t_N = 10\%$ imposed at $t = 4$. Depletion N_t falls as the tax raises the input price to $(1 + t_N)n_t$ along a higher optimal path. Resource intensive output x_{1t} and the capital return r_t both fall before resuming negative trends. Labor intensive output x_{2t} increases as labor and capital are released from export production.

* Figure 5 *

Figure 6 shows the effects of a 10% import tariff at t = 4 with saving $\sigma = 0.25$ and labor growth $\lambda = 0.01$ at the original levels in Figure 2. The tariff reinforces the underlying paths of the wage w_t and capital return r_t in (9). Depletion N_t and export production x_{1t} both fall to lower paths as import competing x_{2t} jumps to a higher trend.

* Figure 6 *

Figure 7 shows adjustments to an export subsidy of 5% at t = 4 based on (11), (12), and (13). Export production x_{1t} jumps with depletion N_t as both overshoot their new downward trends. Production of import competing x_{2t} similarly overshoots its lower upward trend as does the wage w_t. Effects of the change in the export price at t = 4 weaken the following period and reverse at t = 6 before resuming new trends at t = 7. Effects of the rising resource price n_t outweigh the higher export price but only after adjustment over three time periods. The capital return r_t rises temporarily before resuming its decline, implying the negligible adjustment in the resource price n_t. An export subsidy of 10% collapses the import competing sector.

* Figure 7 *

Reaction to an export tax is similar to the import tariff in Figure 6. Differences in factor price and depletion paths, however, relax the familiar Lerner symmetry theorem.

7. Alternate assumptions on depletion

A constant depletion rate implies the same fraction α of the resource stock S_t is depleted each period according to $N_t = \alpha S_t$. The condition $(N_t/S_t)' = 0$ implies $N_t' = -\alpha N_t$ added to (3) with the endogenous n_t'. The resulting system is similar to the static model with three factors and two goods. A higher depletion rate α implies higher N_t', n_t', and r_t' but lower w_t'. A higher saving rate σ raises capital growth and w_t' but lowers r_t' and n_t' as capital replaces the resource in export production. Higher labor growth λ would lower w_t' but raise r_t' and n_t'. Other model properties depend on substitution. An import tariff and an export subsidy have expected output effects but their factor price effects depend on the state of the economy.

A tragedy of the commons implies the resource is priced at marginal physical extraction cost E_t. Assuming E_t is constant $n_t' = 0$ eliminates $r_t n_t$ from the exogenous vector of (3). The constant resource price implies w_t and r_t are constant as well. Production of the export evolves according to $x_{1t}' = (a_{L2}\sigma Y_t - a_{K2}\lambda L_t)a_{KL}^{-1}$ as in the first term of (7). Higher capital growth and lower labor growth both favor increased export production x_{1t}. Import competing output $x_{2t}' = (a_{K1}\lambda L_t - a_{L2}\sigma Y_t)a_{KL}^{-1}$ would decrease with higher capital growth or lower labor growth. Depletion rises according to $N_r' = \Sigma_j a_{Nr} x_j' = (a_{NL}\sigma Y_t + a_{NK}\lambda L_t)a_{KL}^{-1} > 0$. Income rises with gains from competition, $Y_t' = w_t L_t' + r_t K_t' + n_t N_t' = [(w_t a_{KL} + a_{NK})\lambda L_t + (r_t a_{KL} + a_{NL})\sigma Y_t]a_{KL}^{-1} > 0$. Rising marginal extraction cost would lead to more complex properties similar to the optimal depletion model.

A myopic monopolistic resource owner maximizes immediate profit disregarding the asset value of the resource stock, setting marginal revenue R_t equal to marginal extraction cost E_t. Total resource revenue $n_t N_t$ implies $R_t = (n_t N_t)'/N_t' = n_t + N_t n_t'/N_t' = E_t$ and $n_t' = N_t'(E_t - n_t)/N_t$. For viability, the resource price n_t would have to be greater than E_t implying opposite signs for n_t' and N_t'. The resource owner suffers a falling income share.

8. Conclusion

Optimal depletion and a rising resource price imply declining depletion in general equilibrium, consistent with the assumption in partial equilibrium resource economics. Depletion diminishes regardless of growth in labor and capital, substitution, and adjusting outputs. Assuming resource intensive exports and labor intensive import competing production, the wage rises and capital return falls regardless of capital deepening. Trade falls with the declining export production. Income may rise, however, due to investment, labor growth, and increased production of the import competing good.

Regarding policy issues, a depletion tax and an import tariff amplifies trends. An export subsidy for leads to overshooting new trends as the rising resource price ultimately outweighs the increased export price.

The present model can be modified in various ways. A backstop resource can be included. A renewable resource would introduce its cost to the general equilibrium. Optimal saving can be included. Labor growth can be made endogenous. Utility maximization would lead to intertemporal trade levels for a small open economy or to evolving terms of trade between two large economies. Finally, other production functions can be simulated.

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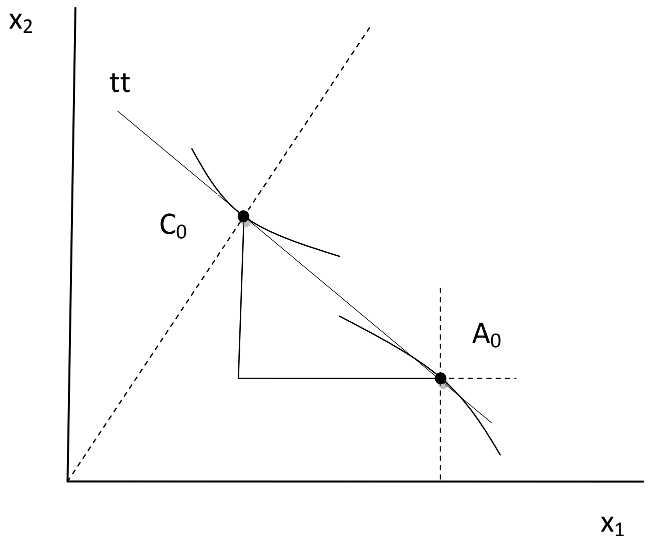

Figure 1. Production and Trade

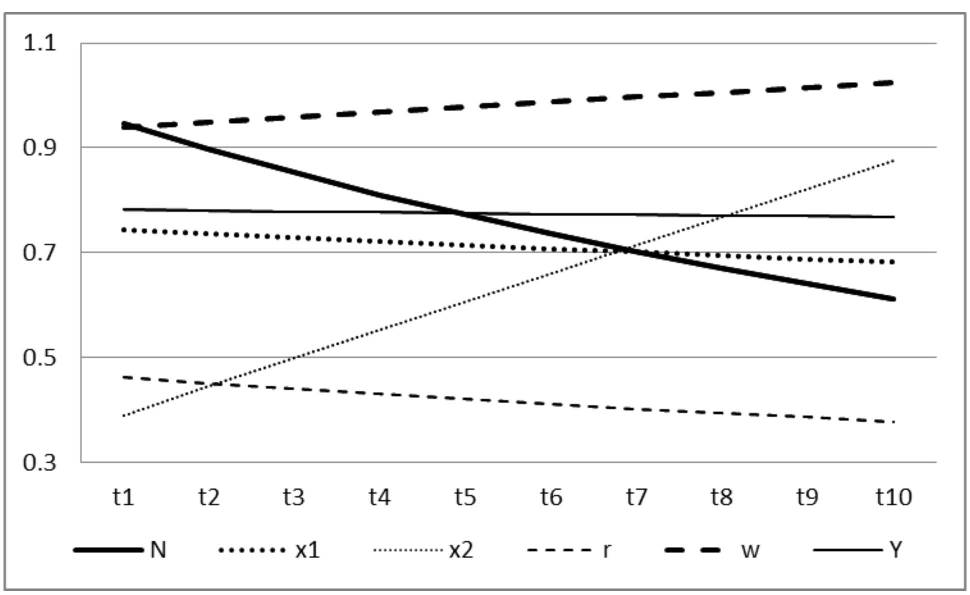

Figure 2. Cobb-Douglas production

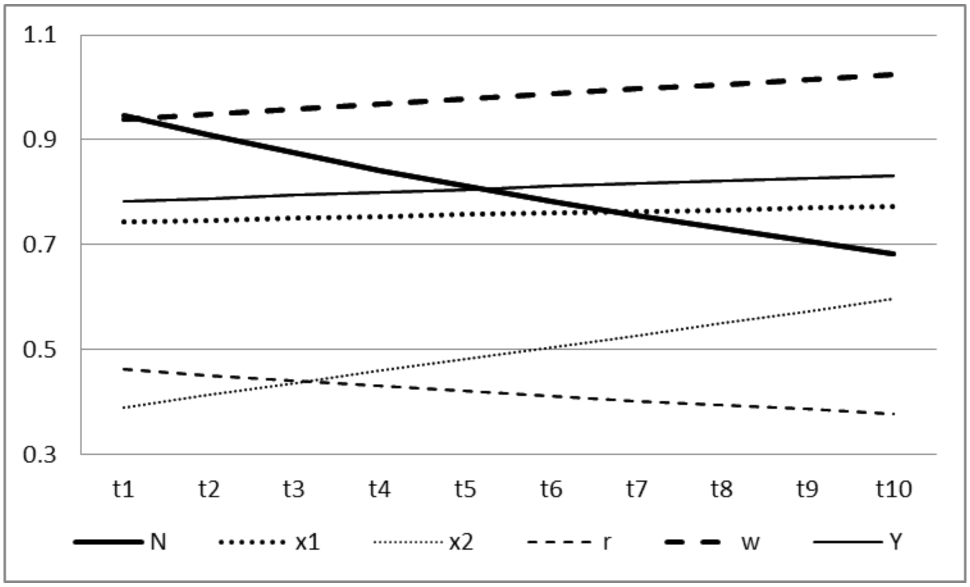

Figure 3. Higher saving and lower labor growth

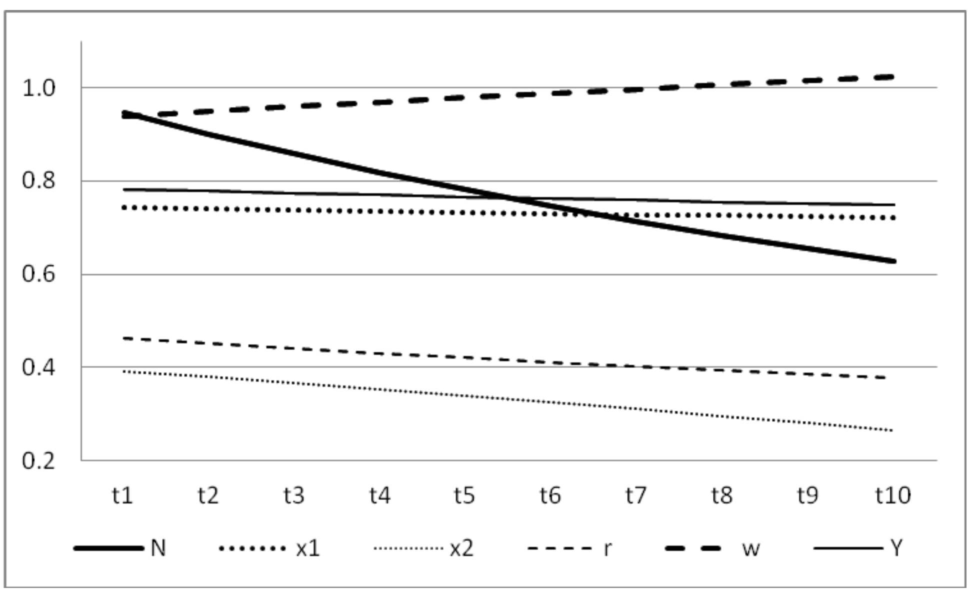

Figure 4. Negative labor growth

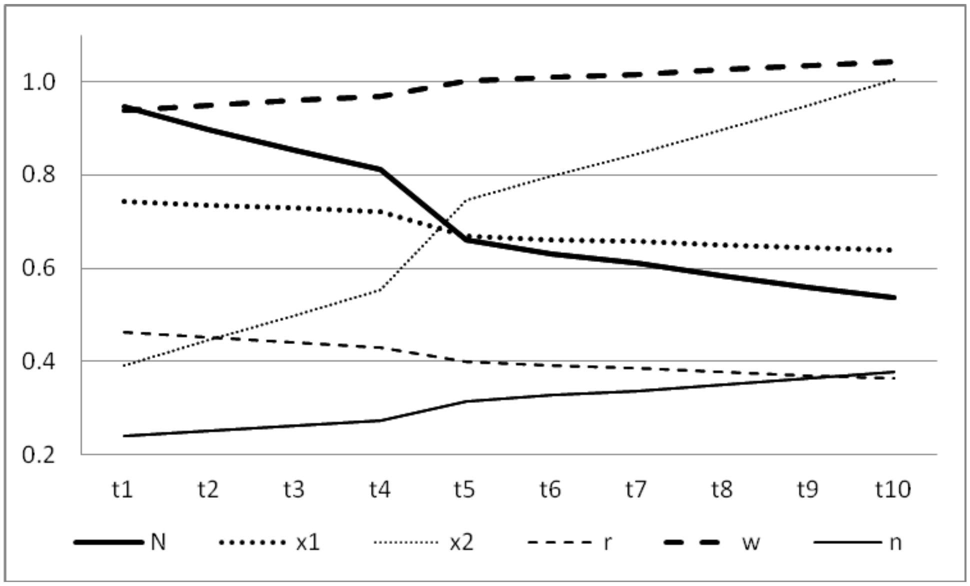

Figure 5. 10% depletion tax at t = 4

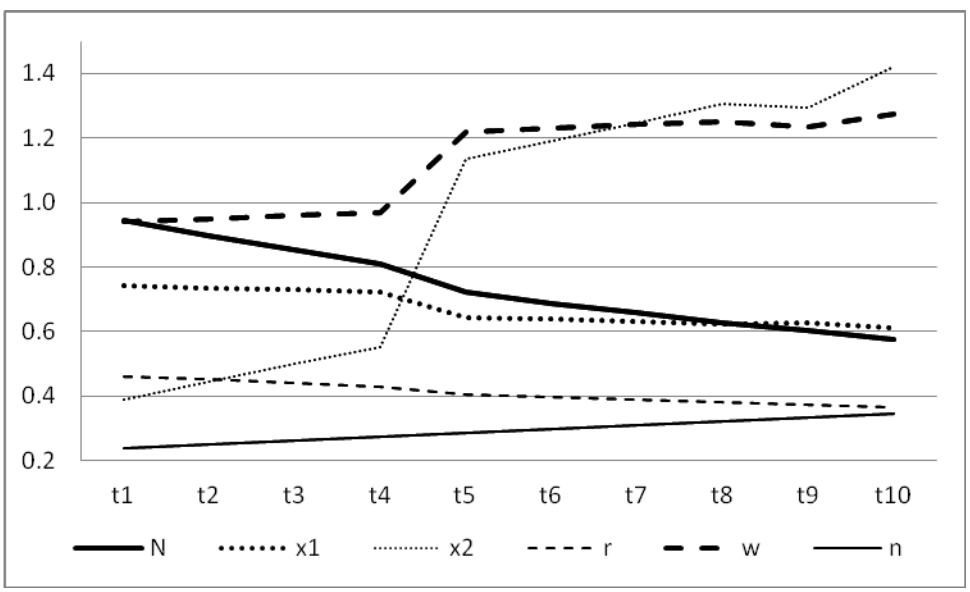

Figure 6. 10% import tariff at t = 4

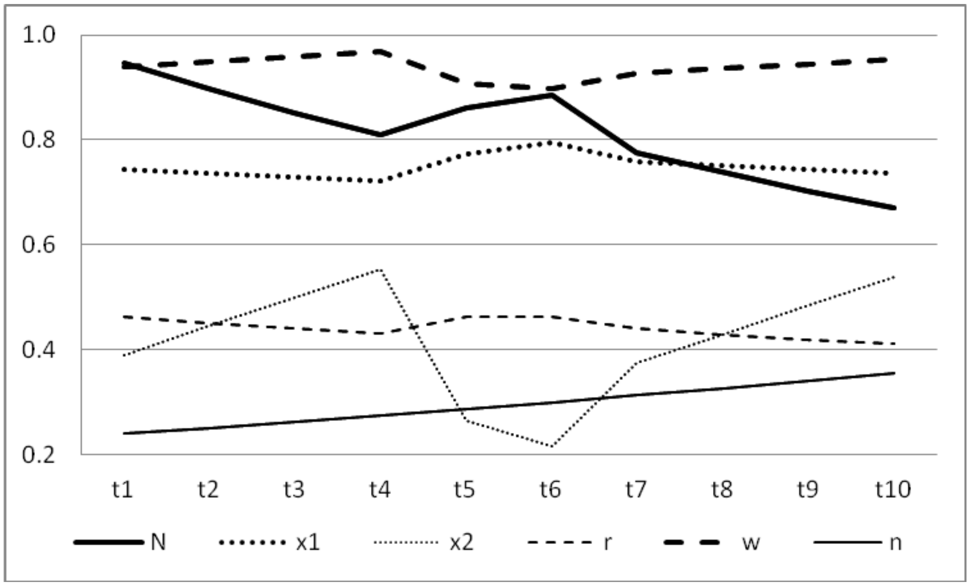

Figure 7. 5% export subsidy at t = 4