Energy Substitution in a Factor Proportions Model of the US

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June 2011

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Keywords: energy, substitution, production, factor proportions

Thanks go to Roy Ruffin, Henry Kinnucan, and Yaoqi Zhang for suggestions.

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This paper estimates energy substitution with annual US data from 1951 to 2008 in a production function based on the definition of physical work with energy and labor interacting separately with capital. Estimated substitution elasticities are applied in two factor proportions models with outputs of manufactures and services, one assuming an endogenous energy price and the other endogenous energy imports at the world price. Policy implications arise from the comparative static results.

The production function specifies output as real gross domestic product with inputs of total Btu energy, real fixed capital assets, and the labor force. The series are difference stationary leading to aggregate substitution elasticities derived from an error correction regression. Own price elasticities are weak for energy but strong for labor. Energy is a strong substitute for labor, while capital is a weak substitute for labor.

The following section introduces the production function followed by sections on the data and estimates of substitution elasticities. Sections then simulate the two general equilibrium factor proportions models. The conclusion discusses a range of policy implications including tariffs on manufactures and energy, subsidies for energy, loss of labor force through tougher immigration policy, and capital taxes.

1. The physical production function

Physical work is equal to force times distance suggesting the production function

$$Y = A(LK)^{\alpha_L}(EK)^{\alpha_E}. \tag{1}$$

Energy E and labor L interact separately with capital K to produce output Y in an analogy to physical work. Energy and labor provide the force to produce work. This functional form is the familiar log linear production function with exponents constrained to $Y = AL^{\alpha_L}E^{\alpha_E}K^{\alpha_L+\alpha_E}$. This specification proves superior to log linear and translog estimates for the present data.

This physical production function is homothetic with constant returns only if $\alpha_L + \alpha_E = ½$. The null hypothesis of constant returns is marginally rejected in the present estimates, relaxing the familiar theoretical properties of substitution elasticities. Marginal products are

$$Y_K = (\alpha_L + \alpha_E)Y/K,$$

$$Y_L = \alpha_L Y/L, \text{ and} \tag{2}$$

$$Y_E = \alpha_E Y/E.$$

First order conditions of cost minimization lead to the symmetric Hessian matrix,

$$\begin{pmatrix} 0 & Y_K & Y_L & Y_E \\ . & Y_{KK} & Y_{KL} & Y_{KE} \\ . & . & Y_{LL} & Y_{LE} \\ . & . & . & Y_{EE} \end{pmatrix} \begin{bmatrix} \partial\lambda \\ \partial K \\ \partial L \\ \partial E \end{bmatrix} = \begin{bmatrix} 0 \\ \partial r \\ \partial w \\ \partial e \end{bmatrix}, \tag{3}$$

as in Allen (1938) and Takayama (1993). Cross price elasticities such as $\varepsilon_{Ke} = (\partial K/\partial e)(e/K)$ are derived in the solution to this comparative static Hessian system. The cost minimization in (3) is given for the implicit national income maximization in the general equilibrium system (7).

The physical production function (1) in natural log form is estimated as

$$\ln Y = \alpha_0 + \alpha_L(\ln L + \ln K) + \alpha_E(\ln E + \ln K) + \varepsilon, \tag{4}$$

where ε is a white noise residual. Constant returns imply output elasticities equal factor shares, $\partial \ln Y/\partial \ln L \equiv \varepsilon_L = \alpha_L$ for labor, $\varepsilon_E = \alpha_E$ for energy, and $\varepsilon_K = \alpha_L + \alpha_E$ for capital.

2. Data and stationarity pretests

Real gross domestic product Y, real fixed capital assets K, and the labor force L are from the Department of Commerce (2010). Total Btu energy input E is from the Department of Energy (2010).

Figure 1 shows the mean weighted series. Output Y grows steadily with some irregularity and at a faster pace following the early 1980s. Capital K grows more regularly at a slightly increasing rate. The labor force L generally grows but with occasional variation. Energy input E grows at a relatively fast pace up to the energy crises during the 1970s when it declines before growing at a slower pace.

* Figure 1 *

The series are not stationary but are difference stationary as suggested by Figure 2 and shown in Table 1. Output Y is difference stationary by the Dickey-Fuller (1979) DF test with a constant and time trend. Capital K and labor L have residual correlation in Durbin-Watson DW (1951) tests but are difference stationary by augmented Dickey-Fuller ADF tests. Energy E is more volatile with ARCH(1) residual heteroskedasticity but is ADF difference stationary with six lags. Difference stationary variables suggest estimation of (4) with a difference regression or error correction regression.

* Figure 2 * Table 1 *

3. Substitution in the physical production function

Table 2 reports estimates of the physical production function (4). The regression on levels in the first column has residual correlation by the Durbin Watson DW test and heteroskedasticity by the ARCH(1) test. The error correction model ECM in the second column produces reliable statistics with standard errors from error propagation calculations. Although the series are not cointegrated according to the Engle-Granger (1987) EG test in the first column, the significant error correction coefficient is evidence the variables are cointegrated.

* Table 2 *

Derived output elasticities from the ECM include the error correction effect. Output elasticities are 0.48 (0.12) for energy, 0.20 (0.12) for labor, and 0.68 (0.17) for capital with standard errors derived by error propagation. The sum of these coefficients 1.26 (0.24) marginally rejects constant returns. Labor is overpaid and energy vastly underpaid relative to productivities. The error correction regression has no residual correlation or heteroskedasticity in the residual. The white noise residual suggests separate treatment of technology is unnecessary, while log linear and translog regressions produce highly autocorrelated residuals. Regressions excluding energy also display strong autocorrelation suggesting misspecification relative to the present production function.

* Figure 3 *

Marginal products and related second order terms evaluated at sample means lead to the Hessian matrix in (3),

$$\begin{pmatrix} 0 & .5982 & .0448 & .4278 \\ . & -.0046 & .0003 & .0004 \\ . & . & -.0004 & .0002 \\ . & . & . & -.0030 \end{pmatrix}. \tag{5}$$

Invert (5) to derive symmetric partial derivatives of inputs with respect to input prices as elements of the inverse matrix,

$$\begin{pmatrix} \partial K/\partial r & \partial L/\partial r & \partial E/\partial r \\ \partial K/\partial w & \partial L/\partial w & \partial E/\partial w \\ \partial K/\partial e & \partial L/\partial e & \partial E/\partial e \end{pmatrix} = \begin{pmatrix} -49.3 & 87.3 & 59.9 \\ . & -2381 & 127.5 \\ . & . & -97.0 \end{pmatrix}. \tag{6}$$

Cross price elasticities are evaluated at sample means and the derived marginal products in (2). Thompson (2006) summarizes the related literature on applied energy substitution.

Table 3 reports these derived substitution elasticities σ along with Cobb-Douglas σ_{CD} for comparison. As an example, the cross price elasticity of energy input with respect to the wage σ_{Ew} =

$(\partial E/\partial w)w/E = (\partial E/\partial w)Y_L/\mu_E = (127.5 \times 0.0448)/72.7 = 0.08$ is twice the Cobb-Douglas elasticity. Linear homogeneity is relaxed implying row sums of elasticities are not zero as with Cobb-Douglas.

* Table 3 *

Cross price elasticities are weak with respect to the wage but own labor substitution is elastic, not good news for labor. In contrast, the moderate cross price substitution between capital and energy has about the same order of magnitude as those own elasticities. The physical production function has stronger own labor substitution and much weaker own energy substitution than Cobb-Douglas, and stronger substitution of energy for capital. Rising capital costs encourage more energy efficient capital than revealed by Cobb-Douglas.

4. Factor shares, industry shares, and factor intensities

Factor shares of industry payments and industry shares of factor employment are primary building blocks of comparative static factor proportions models. Factor intensities anticipate comparative static properties.

Table 4 presents the factor payment matrix for 2008. Row sums equal factor income, and column sums sector income. Assuming an input has the same competitive price across sectors leads to the industry shares λ_{ij} of factor distribution across sectors in Table 5. As an example 16% = \$787/\$4894 of labor is employed in manufactures. The service sector employs the majority of capital and labor producing 85% of aggregate output. Manufactures appear energy intensive.

* Table 4 * Table 5 *

Factor payment shares θ_{ij} in Table 5 are based on sector incomes. As an example the labor share of income in services is 45% = \$787/\$1759. Labor receives about the same share of revenue in services and manufactures. Energy receives a much larger share of revenue in manufactures, and capital a somewhat smaller share. Comparing industry shares, manufactures appears energy intensive and services capital intensive.

Table 6 reports factor intensities derived from industry shares. For instance, the ratio of industry shares between capital and labor is $\lambda_{Kj}/\lambda_{Lj} = (a_{Kj}x_j/K)/(a_{Lj}x_j/L) = (a_{Kj}/a_{Lj})(K/L)$. Services are capital intensive relative to labor and especially relative to energy. Manufactures are energy intensive relative to both capital and labor. Theoretical misconceptions arise excluding energy input. Labor is the middle factor, intensive in manufactures relative to capital but intensive in services relative to energy.

* Table 6 *

5. Factor proportions model simulations

Theoretical foundations for the present simulations are Heckscher (1919) and Ohlin (1924) formalized by Samuelson (1953), Jones (1965), and Chipman (1966) and reviewed by Jones and Neary (1984). The comparative static analysis is clearly developed by Jones and Scheinkman (1977) and Chang (1979). The present three factor model of Jones and Easton (1983) and Thompson (1983, 1985) is the simplest to allow complements, an issue in the energy economics literature.

Behavioral assumptions are full employment, cost minimization, and competitive pricing. The full employment condition is $Ax = v$ where A is the matrix of cost minimizing unit inputs, x the output vector, and v the input vector. Differentiate and introduce substitution elasticities to derive the first equation in the system (7). Competitive pricing in each industry is stated $A^T w = p$ where w is the factor price vector and p the price vector. Differentiate and use the cost minimizing envelope condition to find the second equation in the system (7).

The comparative static model in elasticity form where the prime ′ represents percentage change is

$$\begin{pmatrix} \sigma & \lambda \\ \theta^T & 0 \end{pmatrix} \begin{pmatrix} w' \\ x' \end{pmatrix} = \begin{pmatrix} v' \\ p' \end{pmatrix}. \tag{7}$$

The σ matrix is the estimated substitution elasticities in Table 3. The λ matrix of industry shares is from Table 6, and the θ matrix of factor shares from Table 7. Factor prices w and outputs x adjust endogenously to changes in factor endowments v and prices p.

Table 7 is the inverse of the system matrix (7) in the array of general equilibrium comparative static elasticities. Cobb-Douglas results are included for comparison. The upper left quadrant presents elasticities of factor prices with respect to changes in factor endowment. Input prices are insensitive to energy input E. Capital K and energy E are weak enemies, an increase in the availability of one lowering the return to the other. The relatively strong negative effect of capital K on the energy price e suggests new capital is more energy efficient. Capital and energy are moderate technical substitutes in the estimate, but general equilibrium complements in the general equilibrium illustrating the importance of factor intensity relative to substitution as in Thompson (1995). Elasticities in this quadrant of comparative static properties are similar to Cobb-Douglas.

* Table 7 *

The lower left quadrant in Table 7 shows the Rybczynski (1955) effects of factor endowment changes on outputs. Increases in energy E or capital K have elastic output effects favoring their intensive sector. Outputs fall for the other sector, especially for manufactures when capital increases. Inelastic effects of labor on outputs favor manufactures. These endowment/output effects are similar to Cobb-Douglas.

The production frontier effects in the lower right quadrant of Table 7 are elastic, especially for manufactures. Thompson and Toledo (2007) show these production frontier elasticities rapidly diminish with disaggregation. The falling manufactures price due to import competition would substantially reduce manufactures output, but at the industrial level there would be winners and losers. Production frontier effects for changes in the price of services price are about twice as strong with the physical production function as with Cobb-Douglas.

Factor intensities account for the elastic Stolper-Samuelson (1941) effects of changing prices on factor prices in the upper right quadrant of Table 7. These effects differ considerably from Cobb-Douglas, especially for the effects of manufactures prices on the capital return and wage. A falling manufactures price raises the wage, strongly lowers capital return, and especially lowers the energy price. This strong effects on the energy price is due to energy intensive manufactures. With Cobb-Douglas production in contrast, a falling manufactures price lowers the wage and raises the capital return. A rising price of services also strongly lowers the energy price while raising the capital return and wage. The inelastic effects on the capital return imply ambiguous effects on the real capital return.

6. The model with imported energy

This section assumes energy is imported from the global market at an exogenous world price. The comparative static system becomes

$$\begin{pmatrix} \sigma_{KK} & \sigma_{KL} & 0 & \lambda_{KM} & \lambda_{KS} \\ \sigma_{LK} & \sigma_{LL} & 0 & \lambda_{LM} & \lambda_{LS} \\ \sigma_{EK} & \sigma_{EL} & -1 & \lambda_{EM} & \lambda_{ES} \\ \theta_{KM} & \theta_{LM} & 0 & 0 & 0 \\ \theta_{KS} & \theta_{LS} & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} r' \\ w' \\ E' \\ x_M' \\ x_S' \end{pmatrix} = \begin{pmatrix} K' - \sigma_{KE}e' \\ L' - \sigma_{LE}e' \\ -\sigma_{EE}e' \\ p_M' - \theta_{EM}e' \\ p_S' - \theta_{ES}e' \end{pmatrix}, \tag{8}$$

as in Thompson (1983). Partial derivative comparative static effects are found multiplying the inverse A^{-1} of this system matrix by the vector for each exogenous change. For K′, L′, p_M′, and p_S′ the inverse isolates the effects while for the energy price e′ the inverse is multiplied by $(-\sigma_{KE}, -\sigma_{LE}, -\sigma_{EE}, -\theta_{EM}, -\theta_{ES})^T$.

Comparative static elasticities are in Table 8. Cobb-Douglas presents very similar comparative static results that differ only in sizes of the effects of (e^*, p_M, p_S) on (E, x_M, x_S). Factor price equalization FPE holds in the upper left corner relating endowments and prices of domestic inputs. The implication of FPE is that similar freely trading economies would have the same wages and capital returns given endowments within a common production cone.

* Table 8 *

Stolper-Samuelson elasticities in the upper right corner and Rybczynski elasticities in the lower left corner are consistent with capital intensive services and labor intensive manufactures. The elastic Stolper-Samuelson effects are opposite and much larger than in the model with domestic energy. Aside from international energy, manufactures is labor intensive and services capital intensive. The production frontier effects in the lower right corner are exaggerated due to the highly aggregated model. One fundamental lesson is that manufactures output appears especially sensitive to price changes facing an exogenous international price of energy.

Energy imports E in the middle row of Table 8 responds strongly to changes in exogenous variables with positive links to labor and the manufactures price. The high degree of sensitivity is due to the high level of aggregation. Increased labor endowment raises energy demand and manufactures output, increasing energy imports. A falling manufactures price and rising services price would reduce energy imports.

An increase in the international energy price e* reduces energy imports with an elastic effect, implying reduced import spending. Manufactures output falls in favor of services. The wage falls with a nearly elastic effect while the capital return rises moderately. Tariffs on energy imports would then raise the trade balance but lower wages and manufactures relative to services output.

Total energy input E might include domestic energy input E_{dom} implying imports E_{imp} equal E – E_{dom}. At the perfectly inelastic E_{dom} the domestic price would be e_{dom} but the lower international energy price e* implies imports. Comparative static properties would be the same as in Table 8.

7. Conclusion

Model simulations address a wide range of policy issues including tariffs on manufactures, rising prices of services due to free trade agreements, capital taxes, the effect of immigration policy on the labor force, domestic energy subsidies, and tariffs on imported energy.

Tariffs on manufactures raise energy demand, increasing either the price or import of energy. The wage falls in the model with domestic energy but rises in the international energy model. The capital return moves opposite to the wage with an ambiguous real effect in the domestic energy model.

A higher price of services raises the wage in the domestic energy model. In the international energy model, the wage falls along with energy imports as the economy specializes away from manufactures. A higher price of services raises the capital return with an ambiguous real effect in the domestic energy model.

Taxes on capital lower the wage and services output. With international energy, capital taxes have no effect on the wage as output adjustments relieve labor market pressure. Capital taxes raise the price or imports of energy.

Tightened immigration policy that lowers the labor force raises the wage in the domestic energy model, and lowers the capital return. With international energy, there are no effects on the wage or capital return due to factor price equalization. Energy imports and both outputs fall as both sectors lose labor.

A subsidy for domestic energy strongly favors manufactures output, raises the wage, and lowers the capital return. The effect on the real wage depends on the burden of the subsidy.

A tariff on international energy lowers imports with an elastic effect implying reduced import spending inclusive of the tax, the Metzler (1949) paradox. Production shifts strongly away from manufactures due to the higher price of intensive energy. The wage falls and the capital return rises.

The present estimates of energy substitution and the factor proportions simulations reflect the critical role of energy input for the US economy since the middle of the 20th century. A basic lesson of the present paper is that excluding energy leads to theoretical misconceptions and empirical misspecification. Energy input should be included in the models of macroeconomics, economic growth, and international production and trade.

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Table 1. Stationarity Analysis

τ_{DF} -3.80	DF	ADF	ADF_6
lnY	-2.83		
ϕ 6.73	6.01		
DW 1.65	1.68		
ARCH F	0.18		
lnK	-1.38	-1.72	
ϕ	1.01	4.40	
DW	1.18*	1.96	
ARCH F	0.91	0.48	
lnL	-2.16	-3.10	
ϕ	3.09	5.03	
DW	1.38*	1.98	
ARCH F	-1.61	-0.57	
lnE	-1.32	-1.01	-1.13
ϕ	6.05	3.35	2.20
DW	1.68	2.00	1.80
ARCH F	4.06*	2.64*	-0.20

Table 2. Physical Production Function

	Levels	ECM
constant	-1.28***	0.001
	(0.05)	(0.006)
K+L	0.49***	0.14
	(0.06)	(0.12)
K+E	0.15**	0.46***
	(0.06)	(0.12)
EC		-0.13***
residual		(0.07)
R^2	.996	.425
DW	0.22*	1.59*
ARCH F	7.78*	-0.03
AIC	-139	-236
EG	-1.74	

Table 3. Substitution Elasticities

σ σ_{CD}	∂lnK	∂lnL	∂lnE
∂lnr	-0.52 -0.49	0.60 0.45	0.49 0.04
∂lnw	0.07 0.51	-1.23 -0.55	0.08 0.04
∂lne	0.44 0.51	0.63 0.45	-0.57 -0.96

Table 4. US Factor Payment Matrix, 2008, $bil

	M	S	Factor
K	$730	$5379	$6109
L	$787	$4107	$4894
E	$242	$178	$420
Sector	$1759	$9664	$11423

Table 5. Industry Shares and Factor Shares, 2008

λ_{ij} θ_{ij}	M	S
K	0.12 0.42	0.88 0.56
L	0.16 0.45	0.84 0.42
E	0.58 0.14	0.42 0.02

Table 6. Factor Intensity Ratios, 2008

Θ	M	S
K/L	0.8	1.1
K/E	0.2	2.1
L/E	0.3	2.0

Table 7. Comparative Static Factor Proportions Model, 2008

Physical CD	∂K	∂L	∂E		∂p_M	∂p_S
∂r	-0.36 -0.38	0.39 0.41	-0.04 0.05		0.79 -0.47	0.86 1.47
∂w	0.49 0.52	-0.54 -0.57	0.05 0.05		-1.46 0.28	1.56 0.72
∂e	-0.52 -0.54	0.57 0.60	-0.05 -0.05		9.62 7.80	-7.67 -6.80
∂x_M	-1.58 -1.65	0.72 0.79	1.86 1.86		14.9 15.3	-14.1 -15.3
∂x_S	1.10 0.91	0.18 0.39	-0.28 -0.30		-5.95 -2.83	5.64 2.83

Table 8. Comparative Statics with an Exogenous Energy Price, 2008

	∂K	∂L		∂e^*		∂p_M	∂p_S
∂r	0	0		0.69		-5.87	6.17
∂w	0	0		-0.95		7.68	-5.73
∂E	-10.1	11.1		-19.6		188	-150
∂x_M	-20.4	21.4		-29.6		365	-294
∂x_S	3.90	-2.90		4.52		-58.2	47.3

Figure 1. Output and Inputs

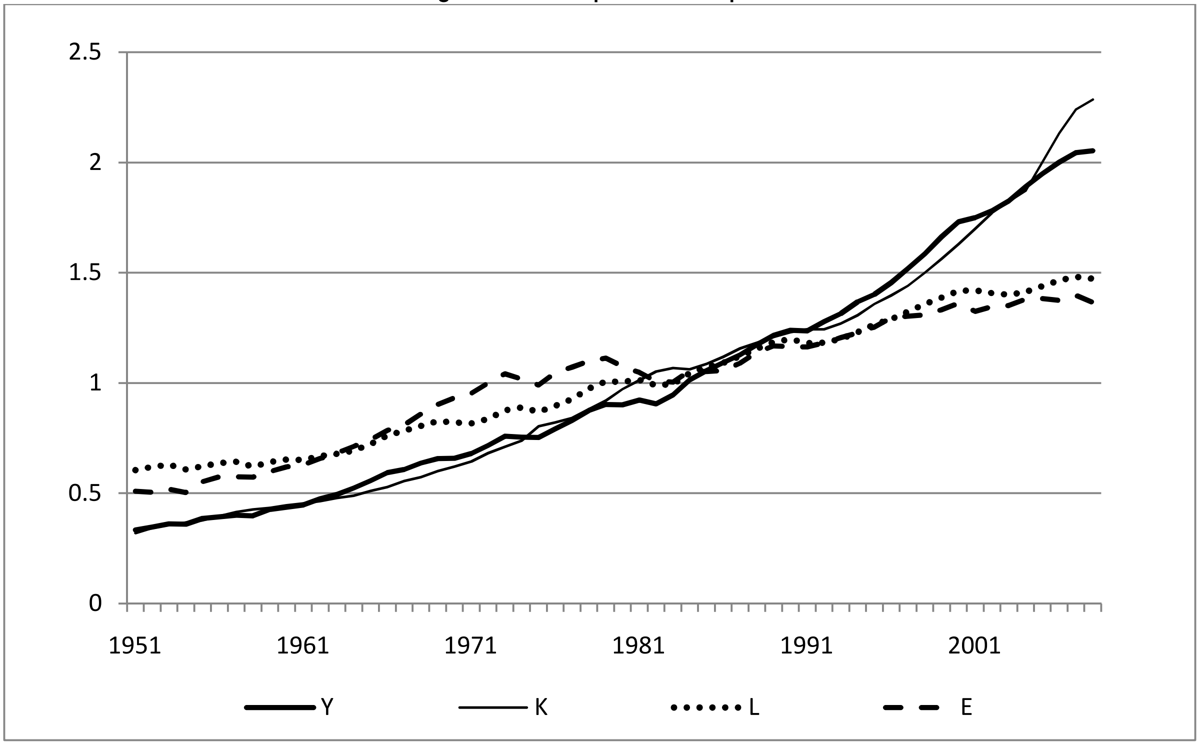

Figure 2. Differences in Output and Inputs