Simulation of a Two-Dimensional Shear Flow

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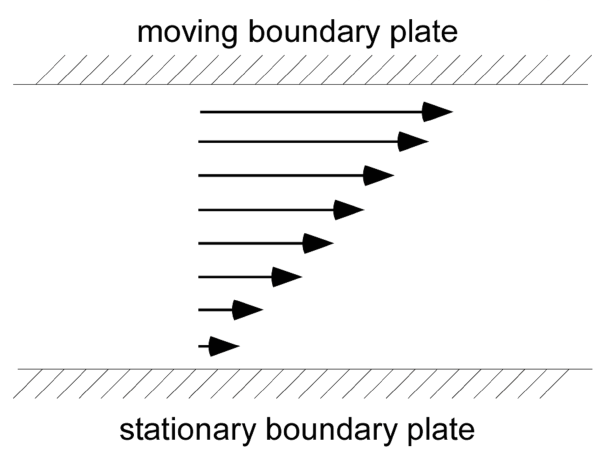
Introduction: shear flow

Shear flow: a flow in a fluid, with a gradient in its hydrodynamic flow velocity

Shear rate γ

- strength of the gradient in the flow velocity
- $\gamma = \dfrac{\partial v_x}{\partial y}$

Example:
Couette flow (uniform γ)



Shear stress P_{xy}:

- describes transfer of momentum through the fluid
- $P_{xy} = \dfrac{1}{A}\left[\sum_i m v_{i,x} v_{i,y} + \sum_i \sum_{j \neq i} r_{ij,y} F_{ij,x}\right]$ [1]

Viscosity η:

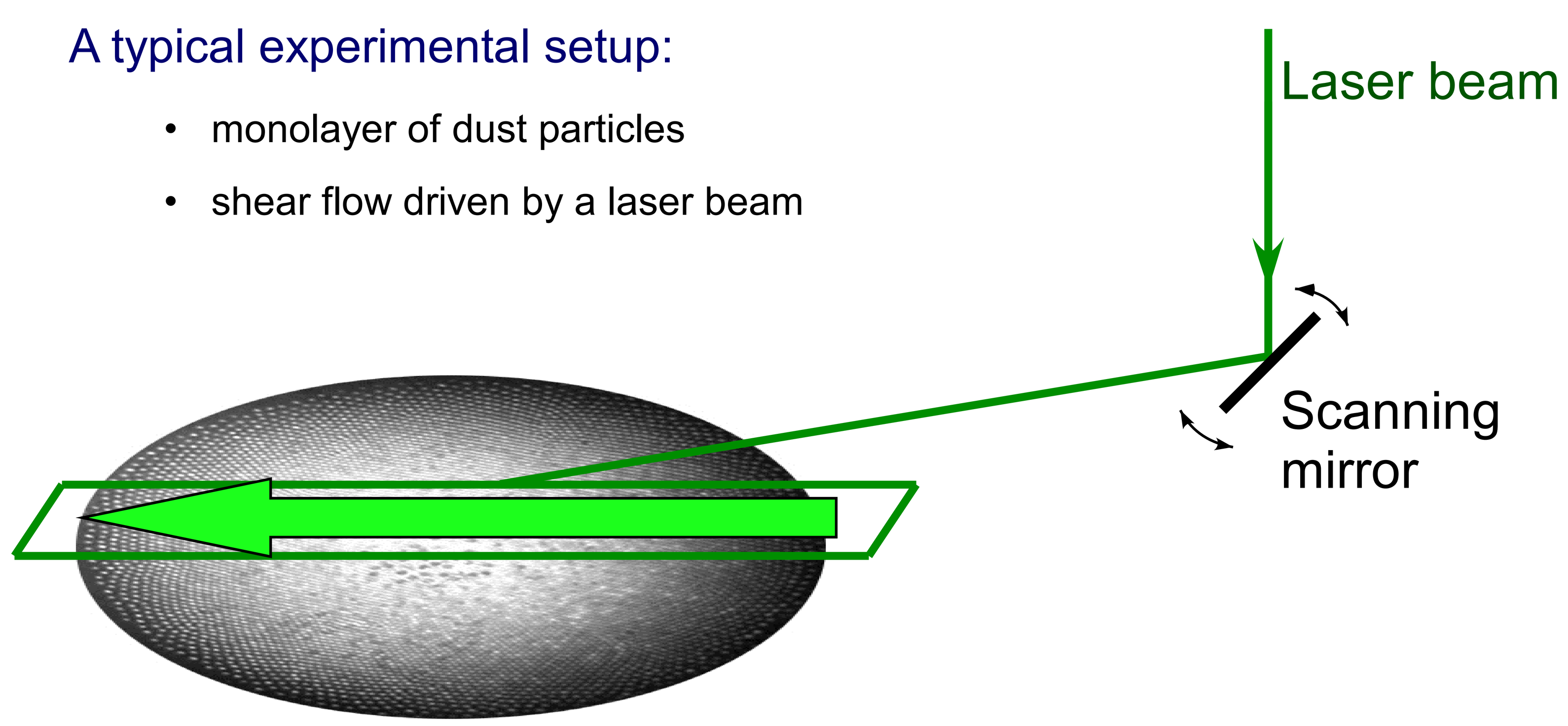
- measure of a fluid's resistance to deformation by shear stress
- $\eta = -\dfrac{\langle P_{xy}(t) \rangle}{\gamma}$

Reference:
[1] P. Hartmann, M.C. Sándor, A. Kovács, and Z. Donkó. Phys. Rev. E **84**, 016404 (2011).

Experimental motivation

A typical experimental setup:

- monolayer of dust particles
- shear flow driven by a laser beam



Treatment of experimental conditions in simulation

Four-component mixture:

Dust:
- individual particles
- interact through Yukawa potential, prescribed λ_D and *Q*
- fixed charge *Q*

Gas:
- drag force on dust is neglected
- random force on dust is neglected

Electrons and ions: contribute only to screening in Yukawa potential

Confinement:

Not simulated. Periodic boundary conditions are used instead.

Normalization:

- distance: $x' = x/a$, where a is the 2D Wigner-Seitz radius
- mass: $m' = m/m_p$, where m_p is the mass of a dust particle
- time $t' = \omega_{pd} t$, where $\omega_{pd} = \sqrt{\dfrac{Q^2}{2\pi\varepsilon_0 m_p a^3}}$

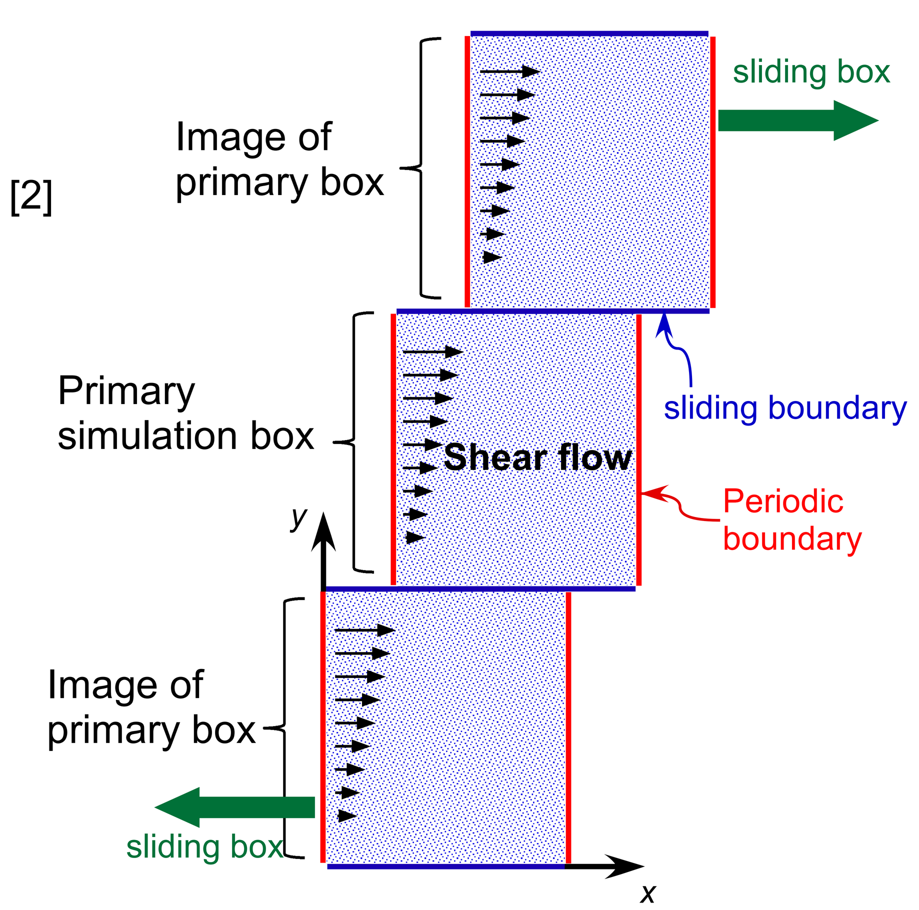
Simulation: equations and parameters

SLLOD equations of motion [2]

$$\dot{\boldsymbol{q}}_i = \frac{\boldsymbol{p}_i}{m} + \boldsymbol{i}\gamma y_i$$
$$\dot{\boldsymbol{p}}_i = \boldsymbol{F}_i - \boldsymbol{i}\gamma p_{yi} - \alpha \boldsymbol{p}_i$$

α is a thermostatting multiplier, Eq. (6.45) in Ref. [2]

Lees-Edwards boundary conditions [3]



Parameters:

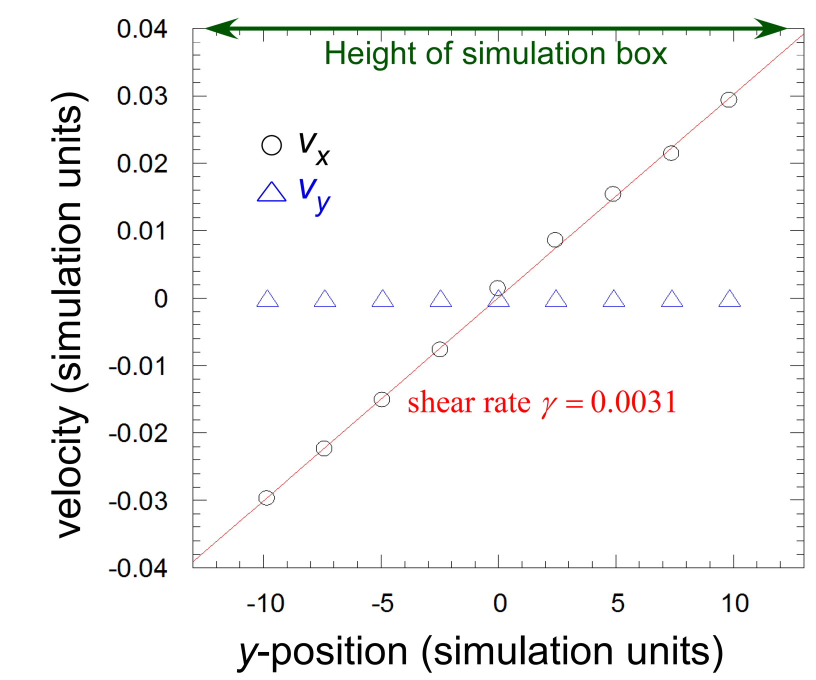
- System size $N = 256$
- Time step $\Delta t = 0.01$
- Screening parameter $\kappa = 1.00$
  where $\kappa = a/\lambda_D$
- Coupling parameter $\Gamma = 66.7$
  where $\Gamma = \dfrac{E_{potential}}{kT_{dust}}$

References:
[2] D. J. Evans and G. P. Morriss, Statistical Mechanics of Nonequilibrium Liquids, 2nd ed. (ANU E Press, Canberra, Australia, 2007), pp.138.
[3] A. W. Lees and S. F. Edwards, J. Phys. C **5**,1921 (1972).

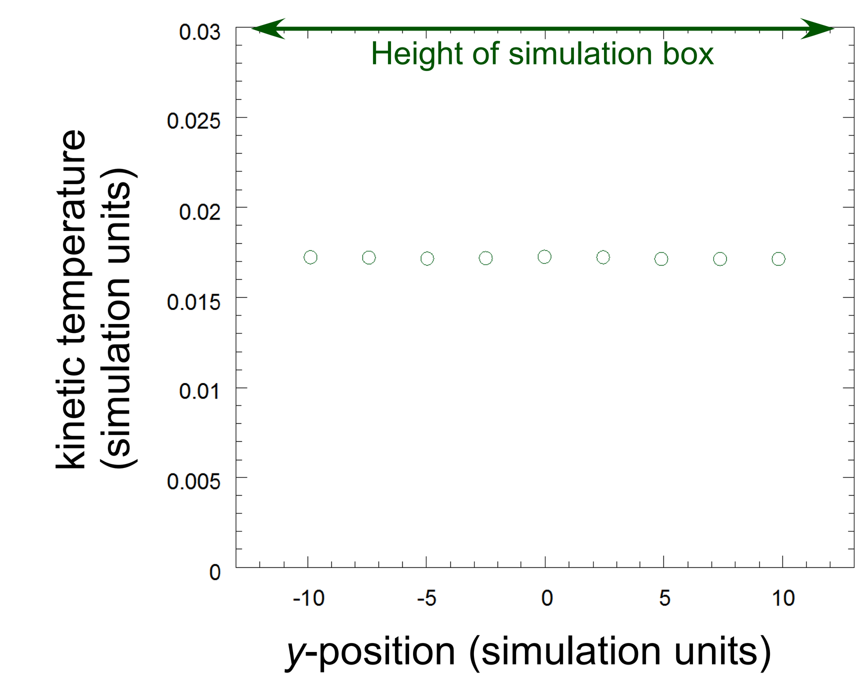
Results: velocity and temperature profiles

Simulation yields a constant shear rate and constant temperature as desired:

Velocity profile

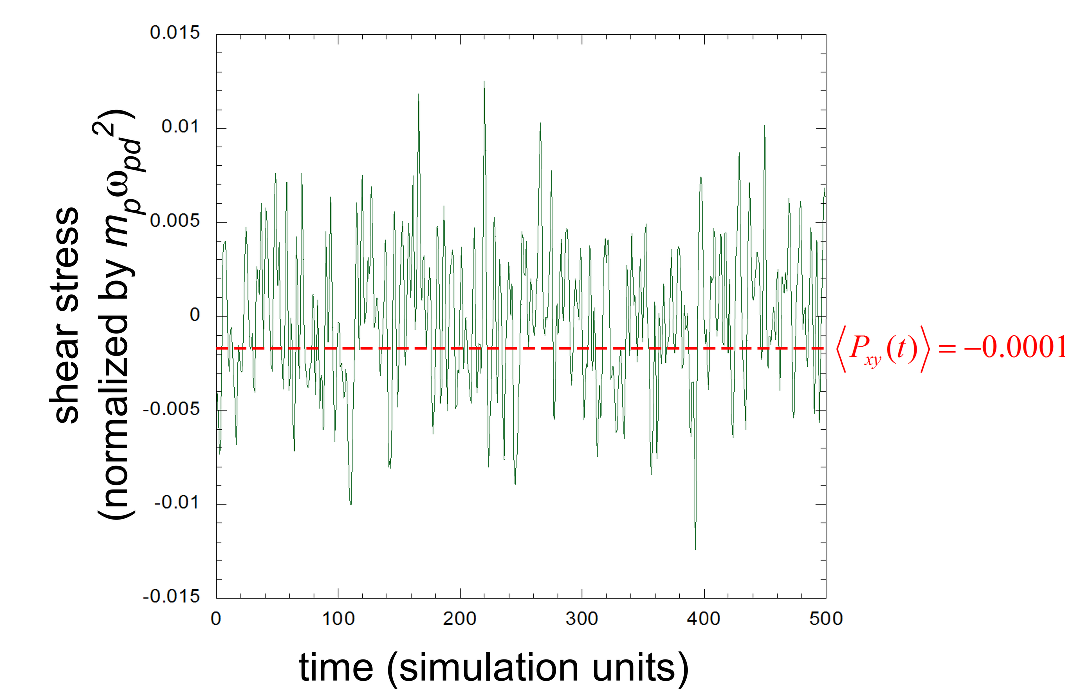


Kinetic temperature profile



Results: shear stress and viscosity

Time series of shear stress

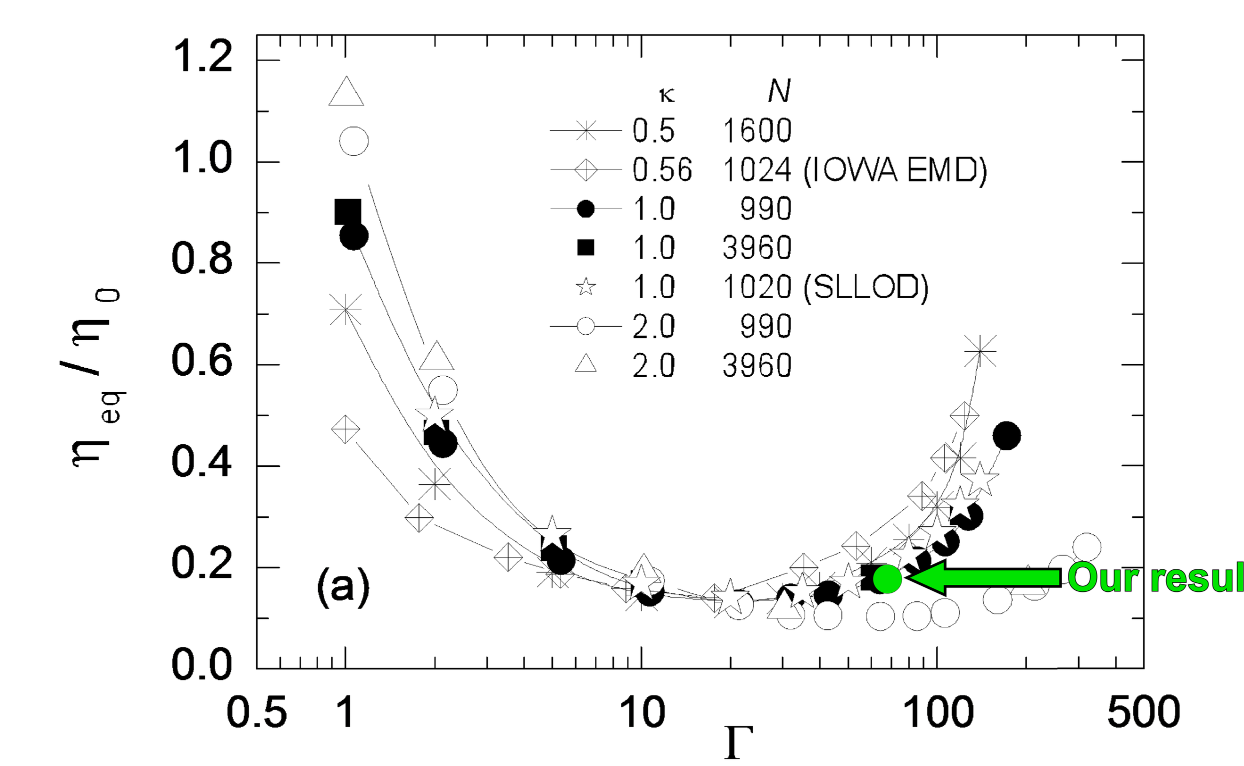


Normalized shear viscosity

$$\frac{\eta}{\eta_0} = 0.184$$

where $\eta_0 = mn\omega_{pd} a^2$

Comparison to other simulations of 2D Yukawa systems



Z. Donkó, J. Goree, P. Hartmann, and K. Kutasi, Phys. Rev. Lett. **96**,145003 (2006).