The Effect of Charge Limits on Particle Charge Distributions in Nanodusty Plasmas

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Acknowledgments
National Science Foundation
Department of Energy Plasma Science Center
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Outline

- Introduction
- Particle charge limits
- Particle charge distributions
- Analytical model for charge distributions with charge limits
- The particle-charge-limited regime
- Conclusions

Motivation

The amount of charge a dust particle can hold is limited, but this has never been taken into account in previous models of charge distributions in dusty plasmas.

Introduction

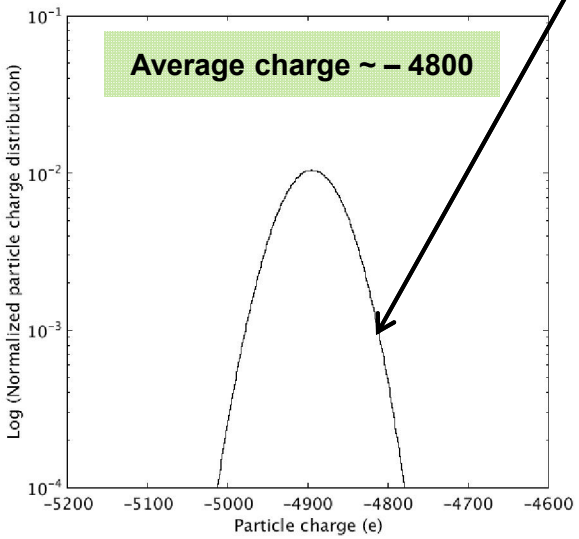
- Particle charge fluctuates due to discrete charging events

⟹ Causes particles to exhibit a **charge distribution**

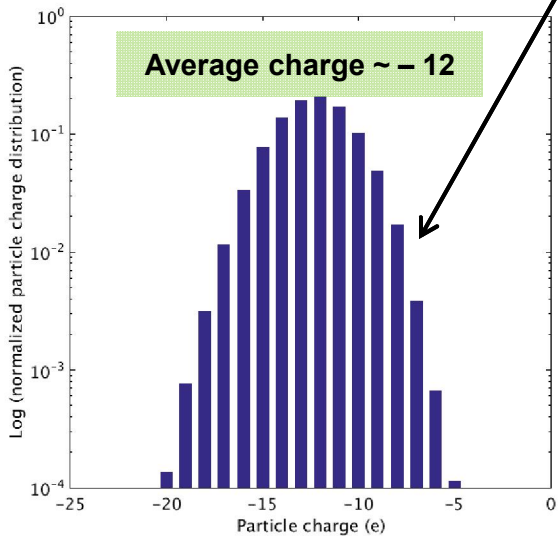
Can be treated as continuous

Discrete nature of charge becomes important

Diameter = 2 μm



Diameter = 5 nm



Monte Carlo simulations assuming OML charging

Based on Cui and Goree, IEEE Trans. Plasma Sci. (1994)

Particle charge limits

Maximum number of electrons that can coexist on a given particle

- **Electron field emission**
  Draine and Sutin, Astrophysical Journal,1987

$$q_{\lim} = 1 + 0.7\frac{R^2}{1nm^2}$$

- **Effective electron affinity**
  Based on Boufendi, Stoffels, and Stoffels,1999

Bulk electron affinity

$$q_{\lim} = A_{\infty}\frac{4\pi\varepsilon_0 R}{e^2} + \frac{3}{8}$$

- **Rayleigh limit**
  Rayleigh, Phil. Mag.,1882

$$q_{\lim} = 8\pi\sqrt{\varepsilon_0\gamma}R^{3/2}$$

Surface tension

Particle charge limits

Maximum number of electrons that can coexist on a given particle

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  Draine and Sutin, Astrophysical Journal,1987

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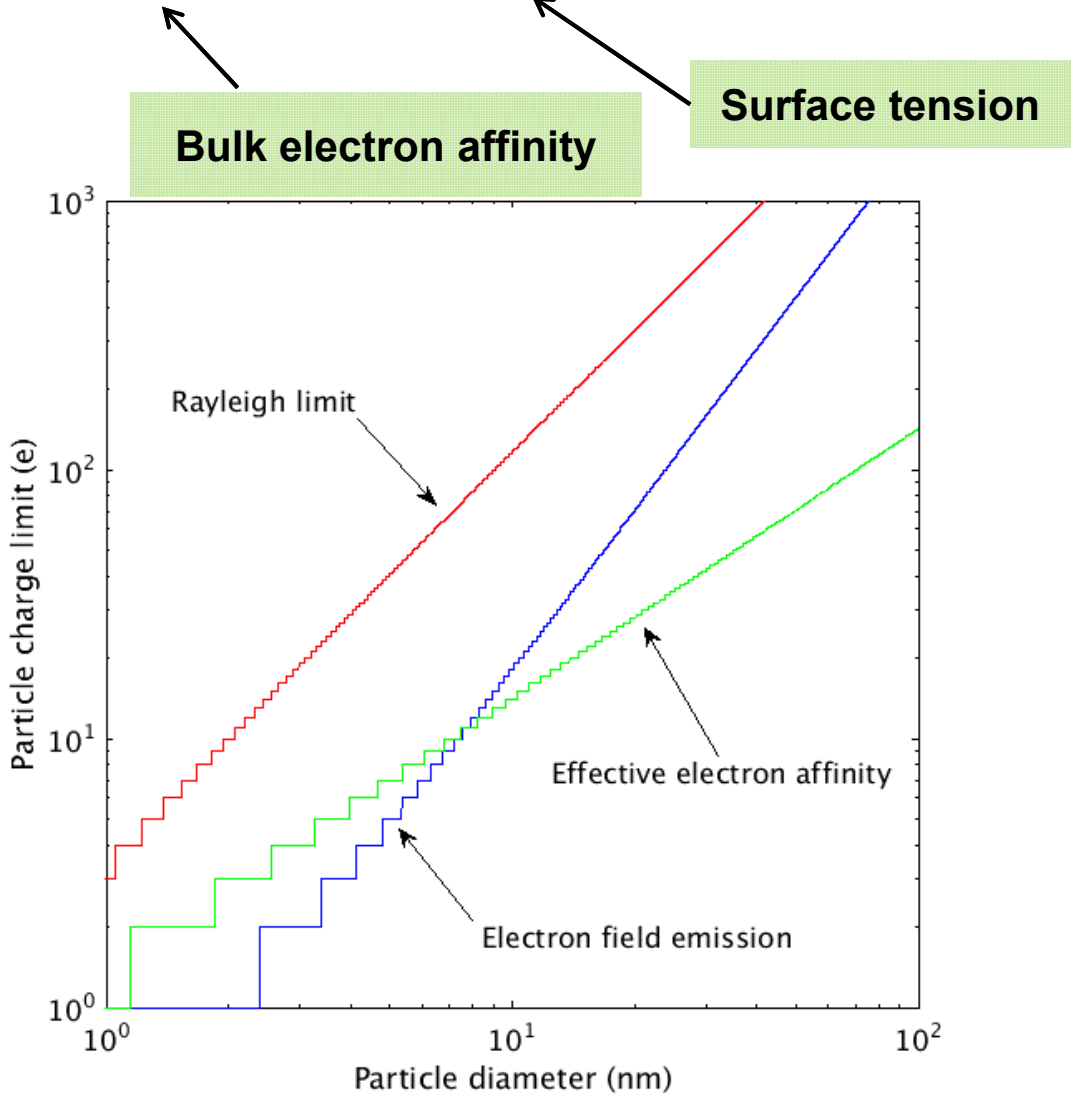
Size dependent

Particle charge limits

- Solid silicon particles (A_∞ = 4.05 eV and $\gamma > 10^{-1}$ N/m)



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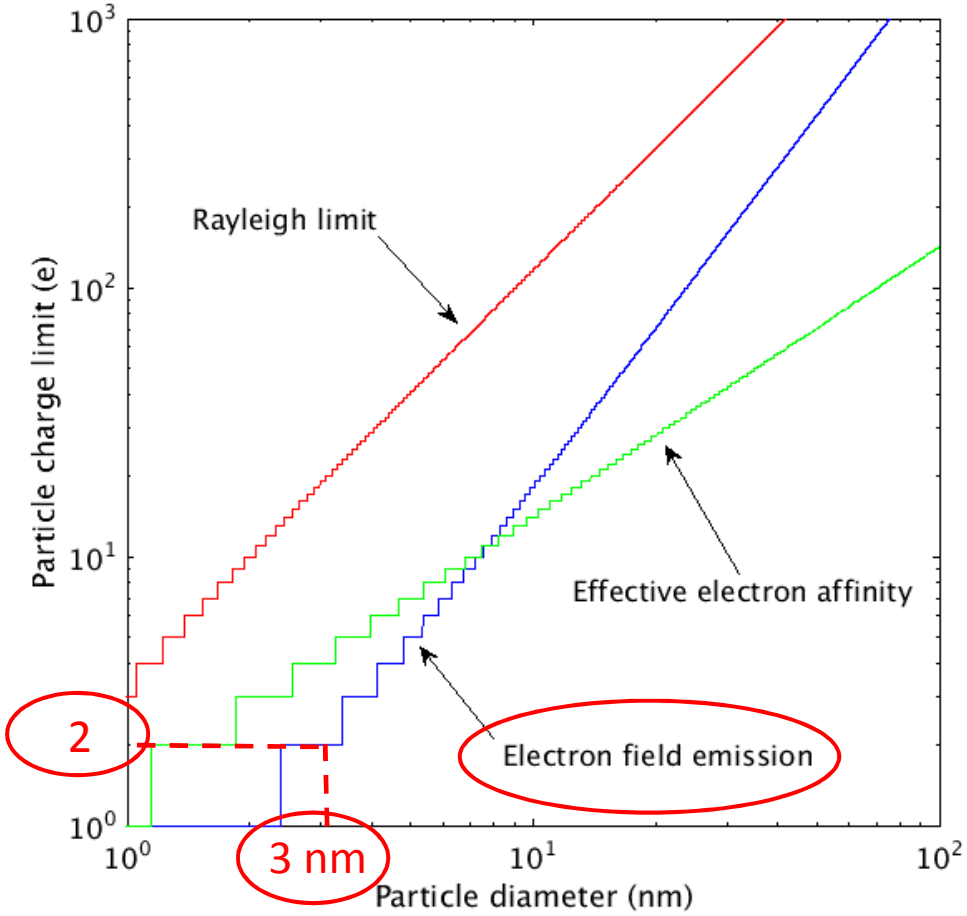
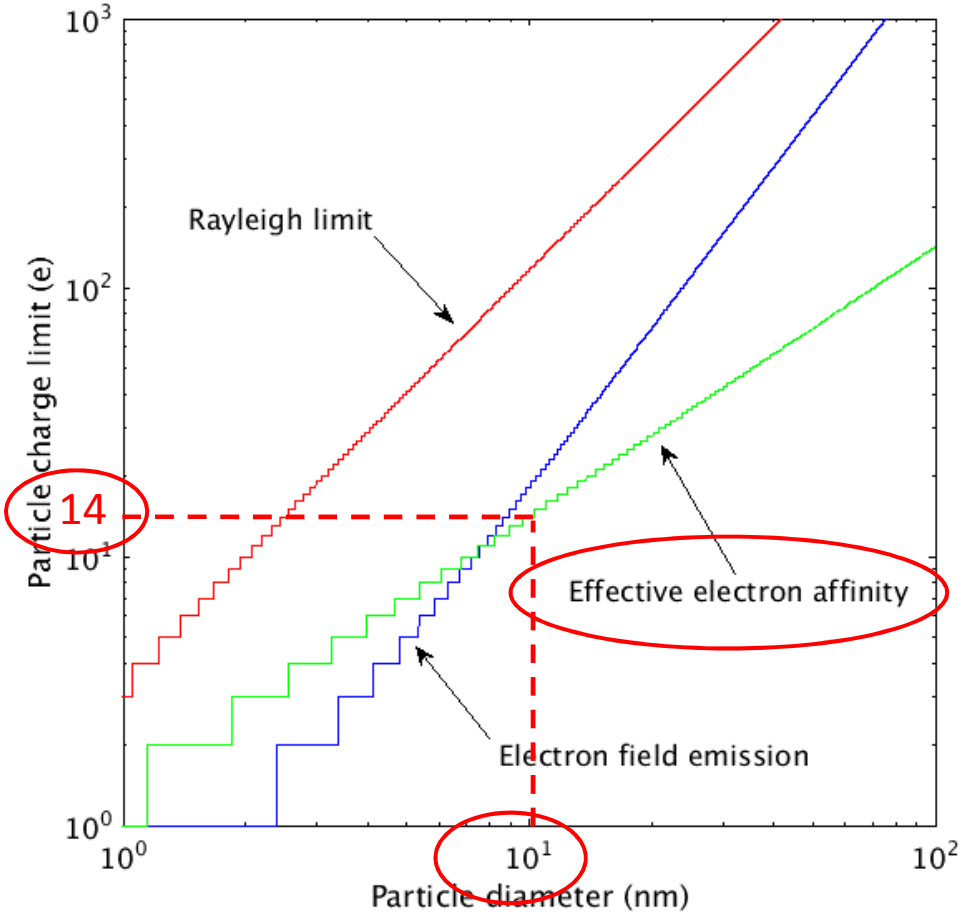


Particle charge limits

- Solid silicon particles (A_∞ = 4.05 eV and $\gamma > 10^{-1}$ N/m)
- Rayleigh limit is unimportant for **solid particles**



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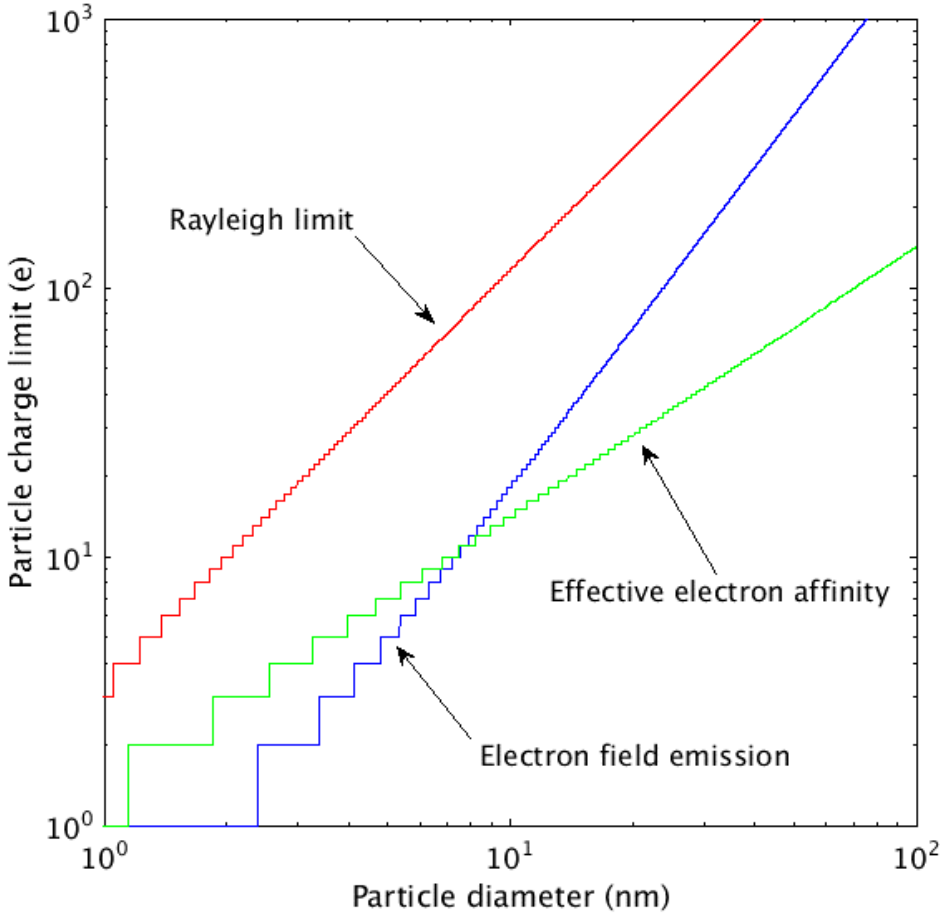


What are effects of charge limits on particle charge distributions?

Particle charge distribution

- Analytical expression for steady-state particle charge distribution from Matsoukas and Russell (1995)

Average charge

Probability distribution for charge

$$n(q) = \frac{1}{\sigma\sqrt{2\pi}} \times \exp\left[-\frac{1}{2}\left(\frac{q-\bar{q}}{\sigma}\right)^2\right]$$

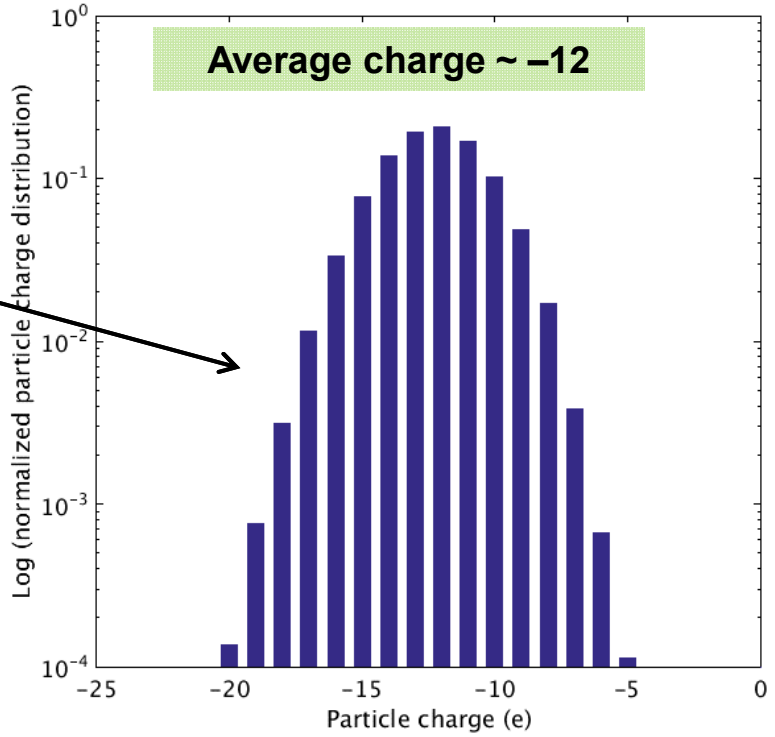
Standard deviation

- **Does not take charge limits into account**

Particle charge distribution

- **Solid silicon nanoparticles**

Charge distribution without charge limits



Simulation parameters

diameter = 5 nm

$n_e/n_i = 1$

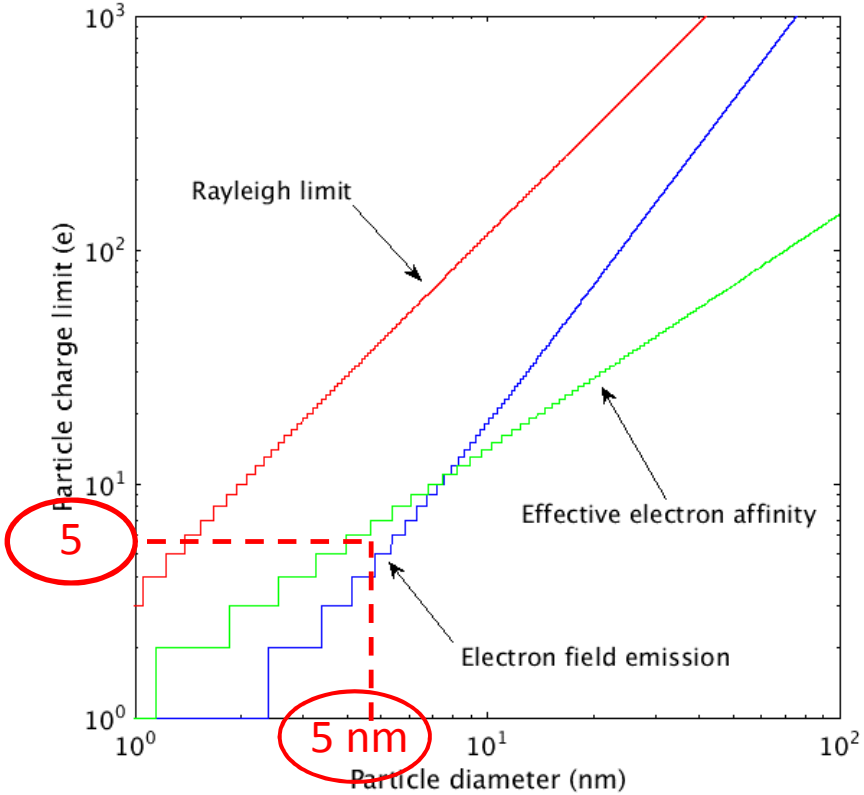
T_e= 3 eV

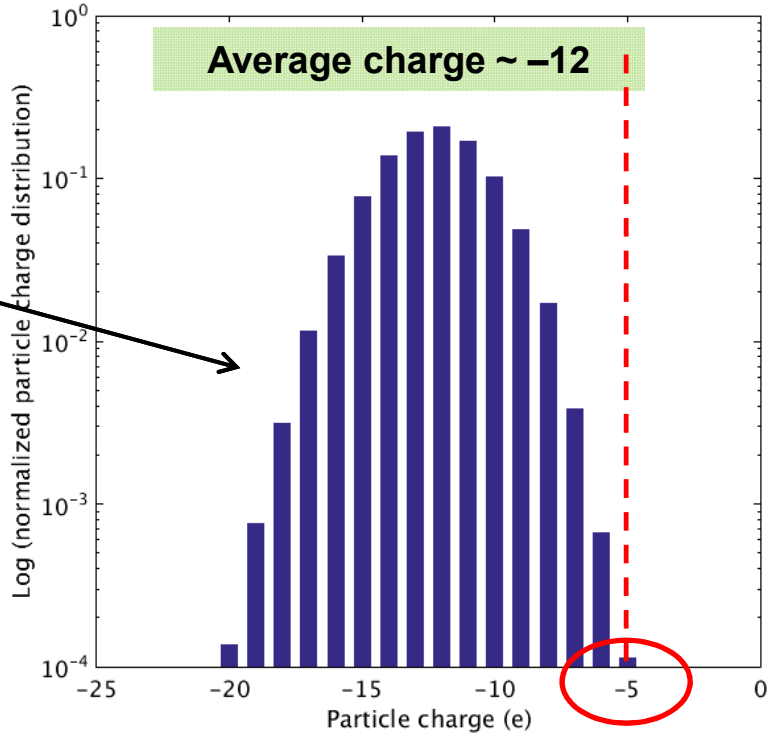
T_i = 300 K

Particle charge distribution

- **Solid silicon nanoparticles**

Charge distribution without charge limits





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Analytical expression

- Modifies distribution w/o charge limits by introducing correction factor
- Derivation: manuscript in preparation

Accounts for integer nature of charge

$$q'_{\lim} = -q_{\lim} - 0.5$$

Gaussian distribution w/o charge limits

$$n^*(q) = \left[\frac{2}{1 + erf\left(\frac{\bar{q} - q'_{\lim}}{\sqrt{2}\sigma} \right)} \times H\left(q - q'_{\lim}\right) \right] \times n(q)$$

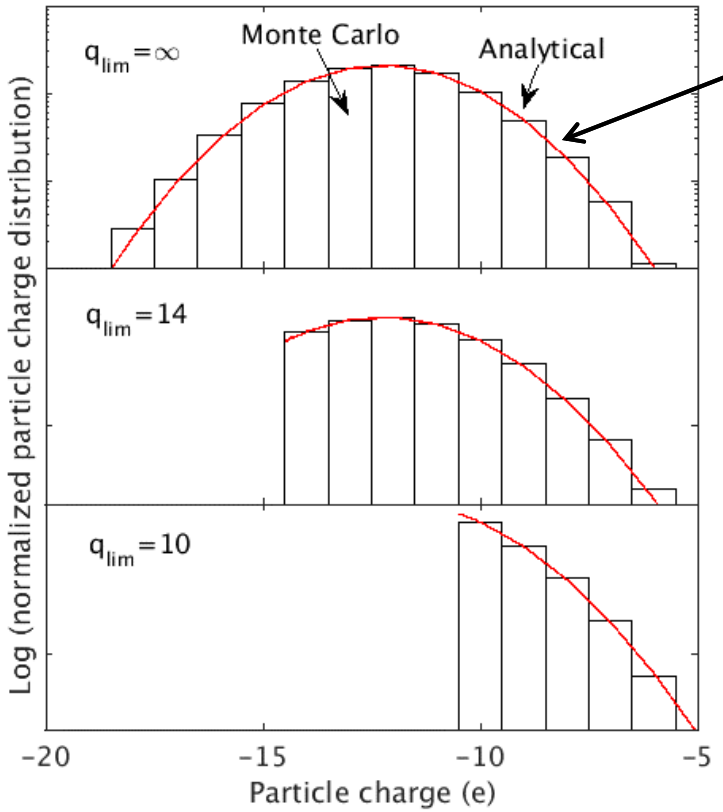
Charge distribution with charge limit

Heaviside step function

$$n^*(q) \to n(q) \quad \textbf{as} \quad q'_{\lim} \to \infty$$

Analytical expression

- Charge limit treated as free parameter
- Excellent agreement between new analytical expression & Monte Carlo simulations



Gaussian distribution

Simulation parameters
diameter = 5 nm
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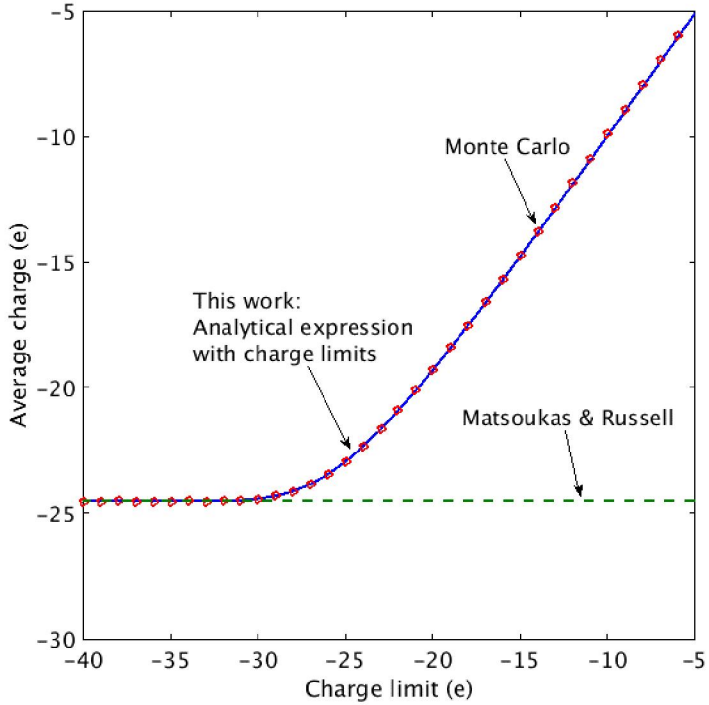
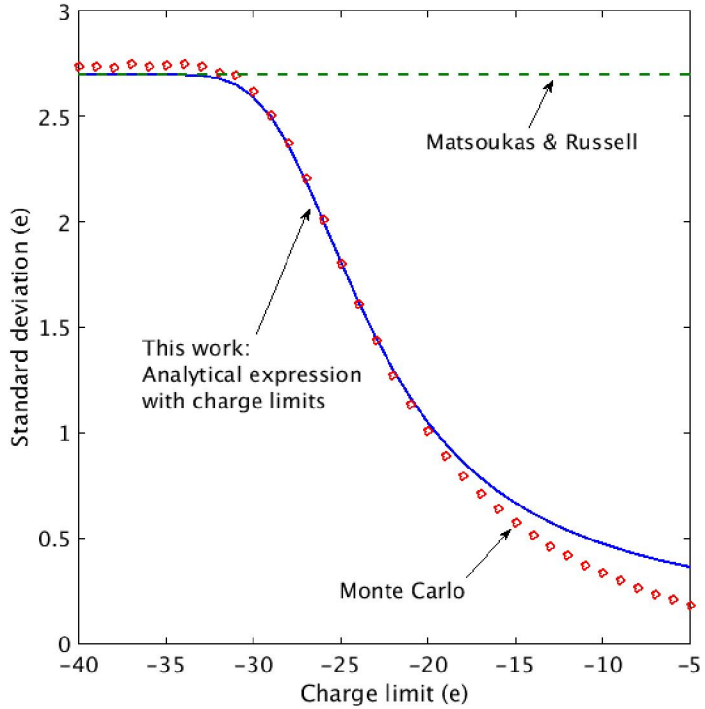
Average charge & standard deviation

- Charge limit treated as free parameter
- Excellent agreement for average charge
- Discrepancies in standard deviation at very small charge limit

Average charge



Standard deviation



Average charge & standard deviation

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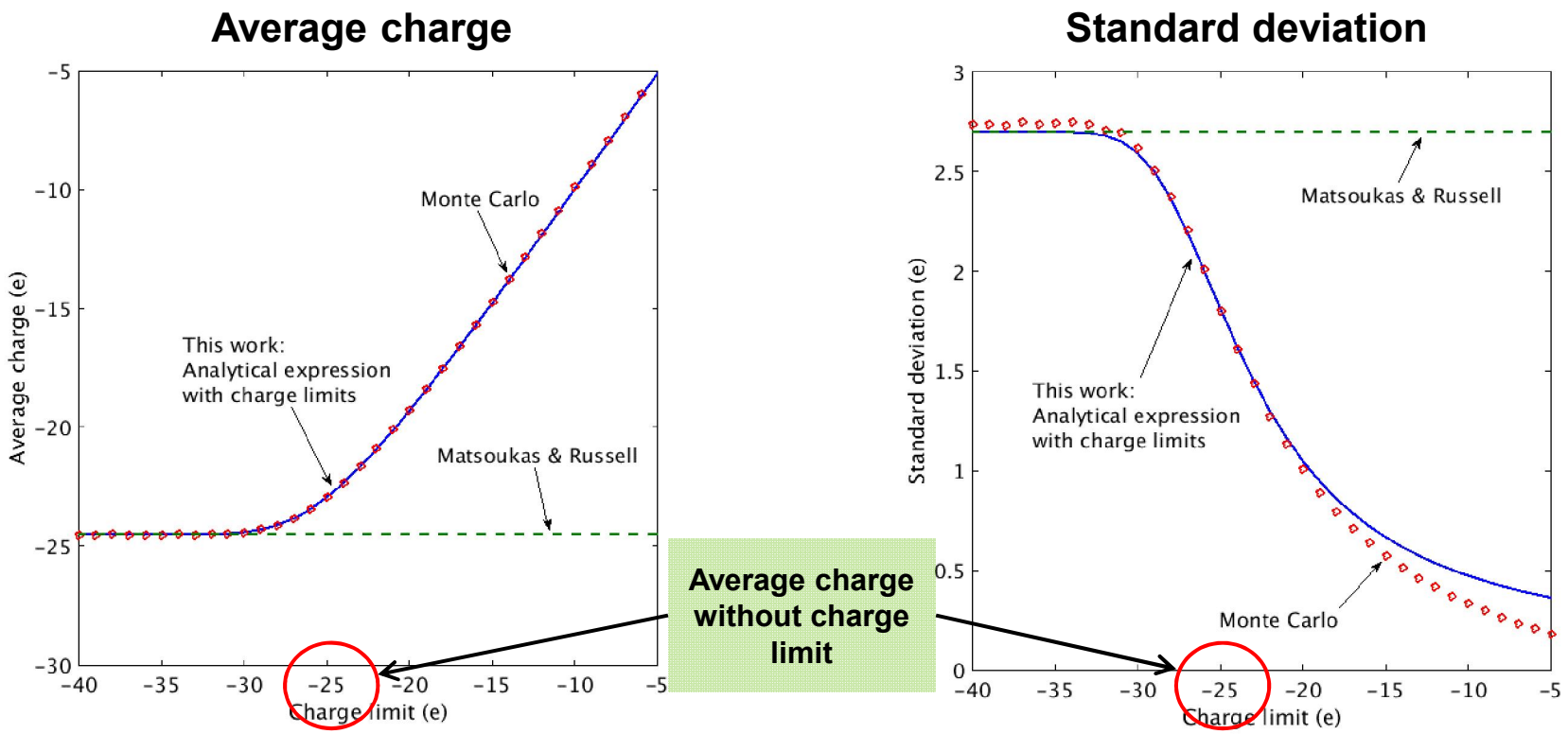

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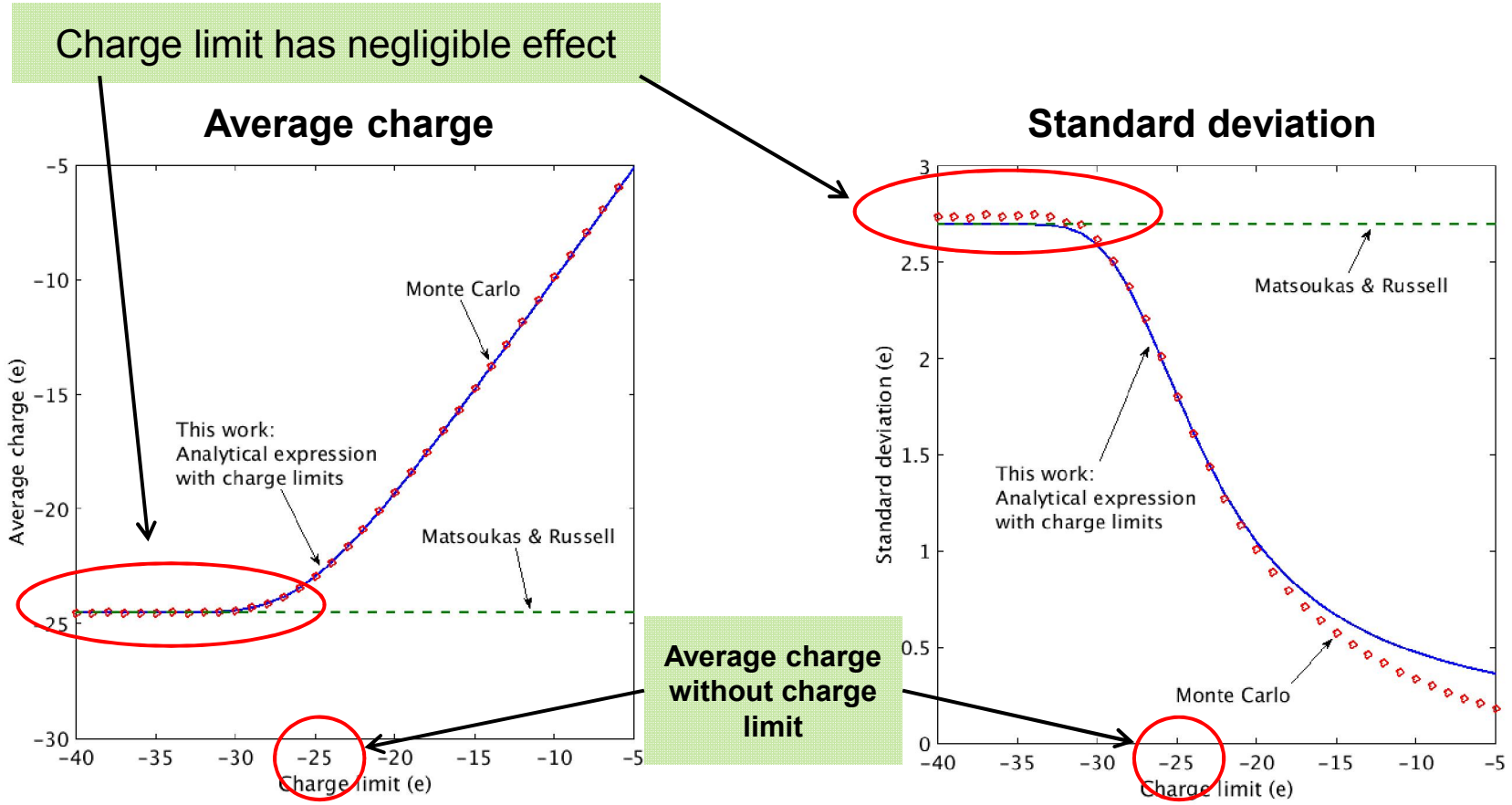

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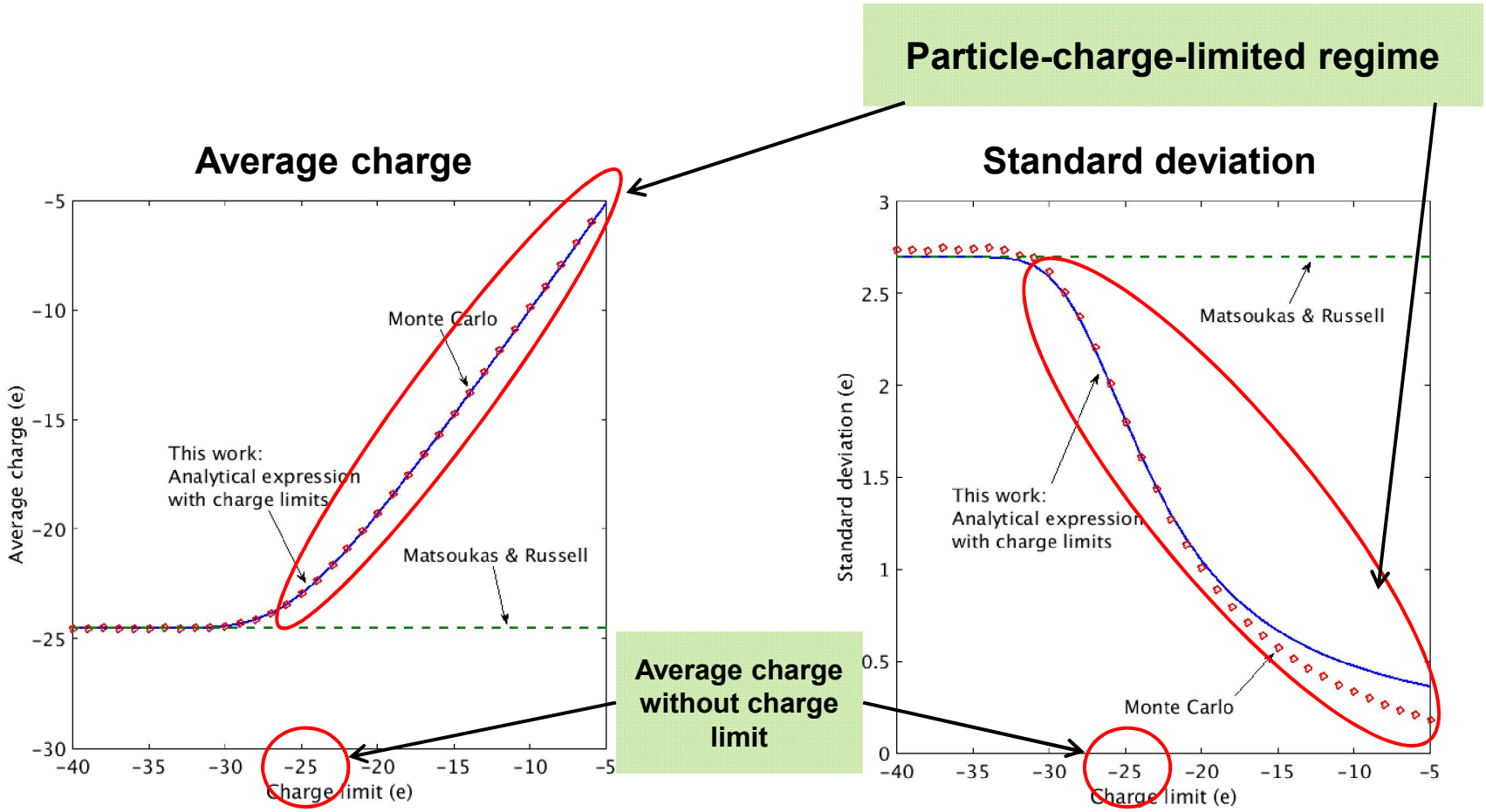
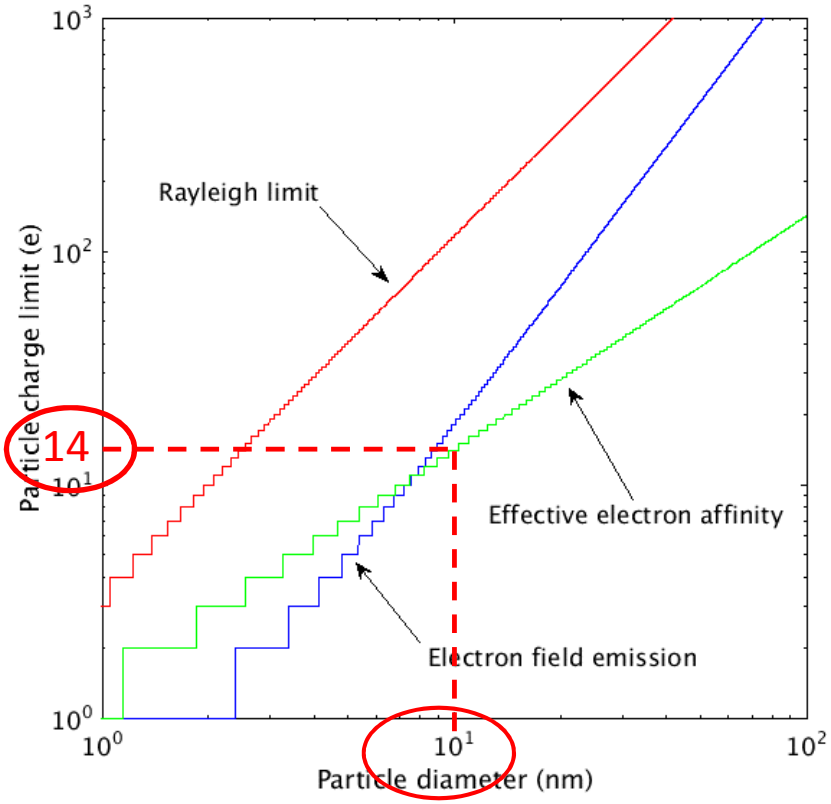


Electron-to-ion density ratio

- Dust particles deplete electrons
- Silicon nanoparticles of 10-nm diameter
- Charge limit = 14



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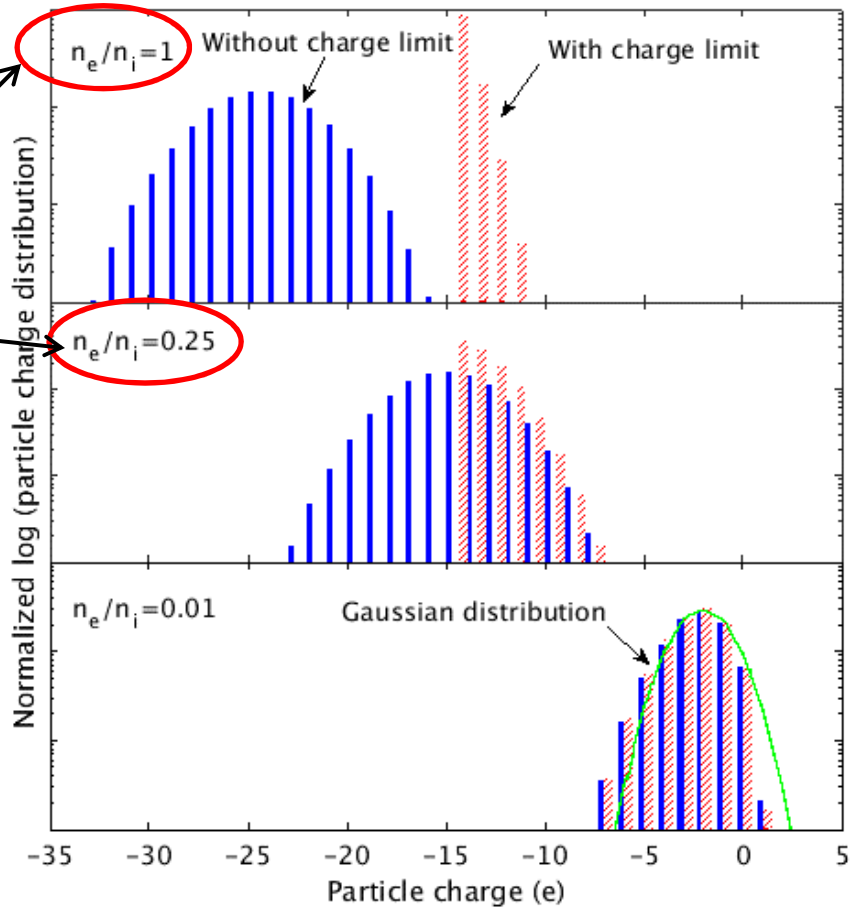
Particle-charge-limited regime

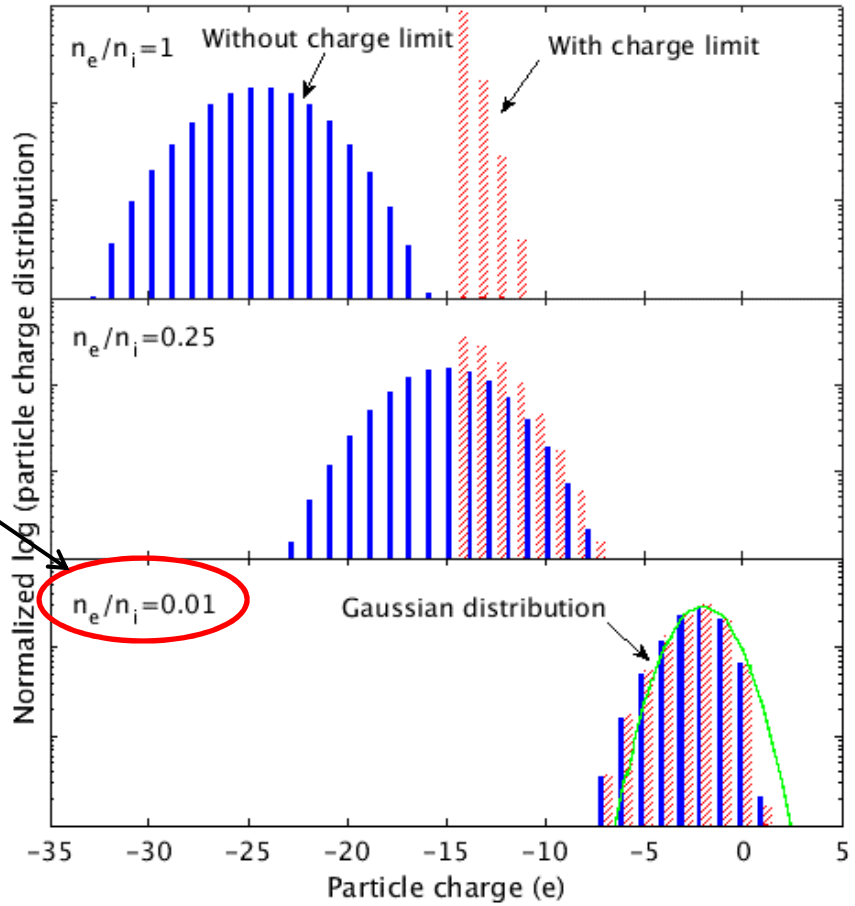


Electron-to-ion density ratio

- Dust particles deplete electrons
- Silicon nanoparticles of 10-nm diameter
- Charge limit = 14

Negligible effect of charge limits

- Strong electron depletion
- Deviation from Gaussian
- Assumption that particles are isolated is not valid at small n_e/n_i

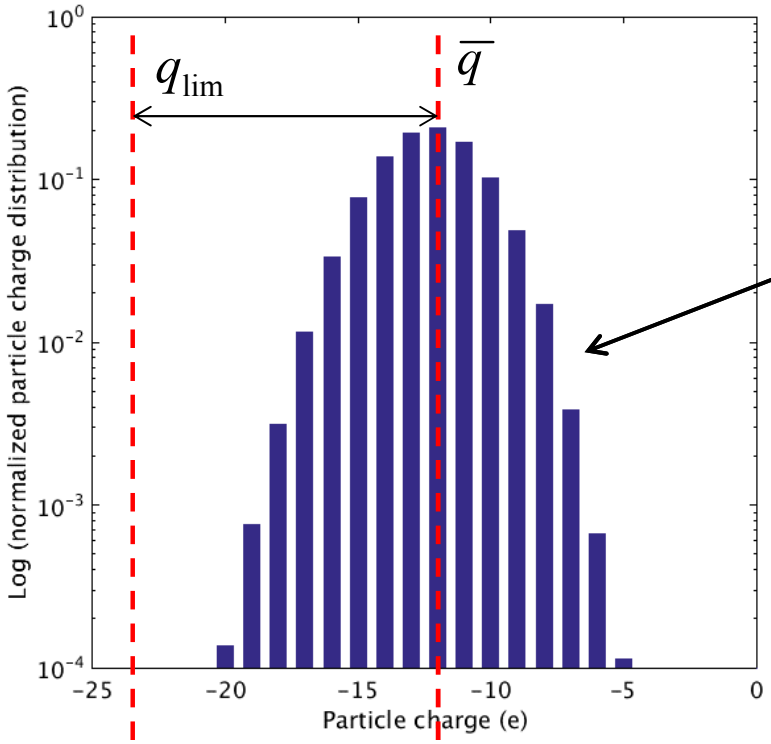


Criterion for particle-charge-limited regime

- Depends on plasma parameters and charge limit

Number of standard deviations

$$q_{\text{lim}} > \left|\bar{q} - r\sigma\right|$$



Charge distribution w/o charge limits

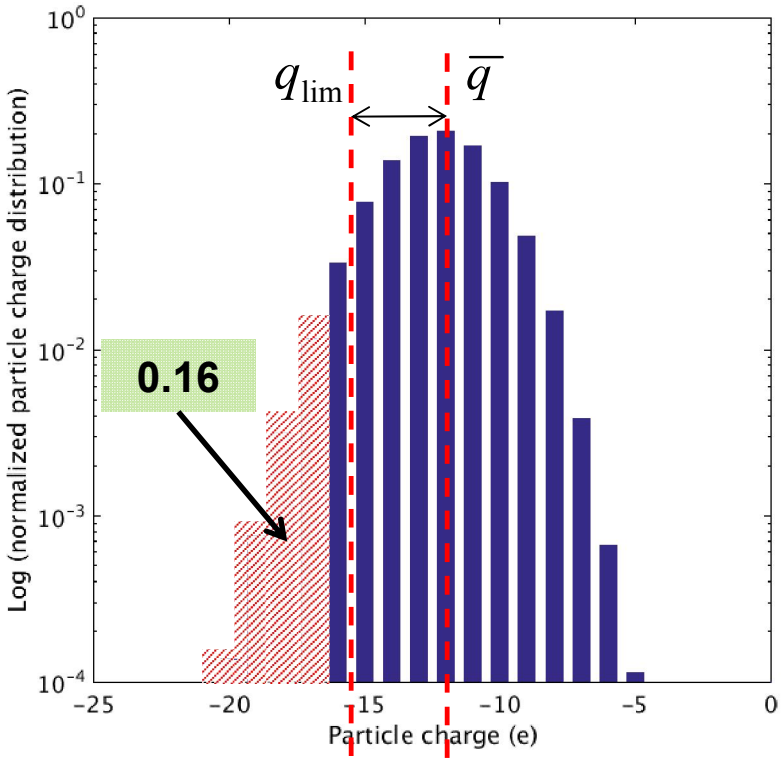
Criterion for particle-charge-limited regime

- Depends on plasma parameters and charge limit

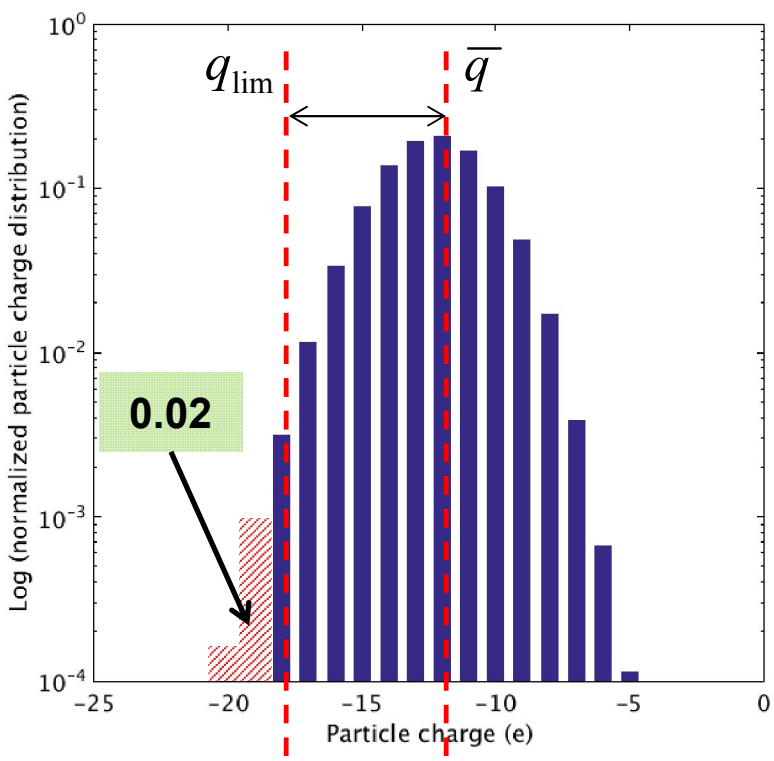
Number of standard deviations

$$q_{\text{lim}} > |\bar{q} - r\sigma|$$

***r* = 1**



***r* = 2**



Conclusions

- Developed new analytical expression for stationary charge distributions accounting for particle charge limits
- Excellent agreement with Monte Carlo charging model
- Developed criterion for whether one is in the particle-charge-limited regime