# Dust Charging and Coagulation Processes in Protoplanetary Environments

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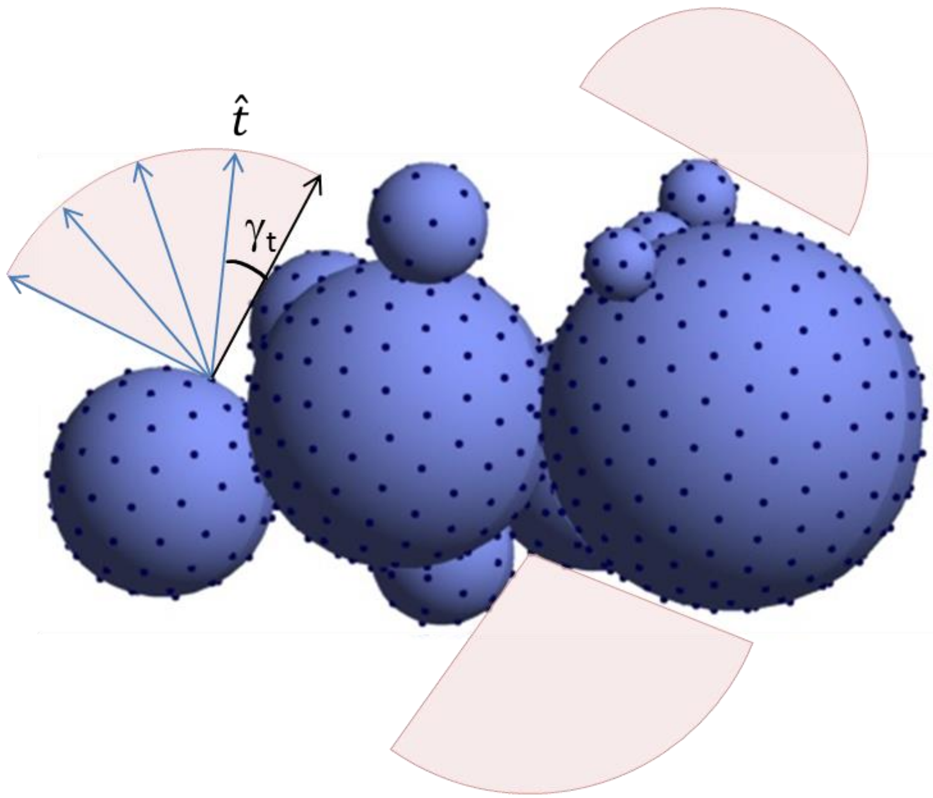
# Simulation of Dust Charging

$$J_j = n_j e_j \int_{v_{min,j}}^{\infty} \pi \left(1 - \frac{2e_j V}{m_j v_j^2}\right) f_j v_j^3 dv_j \int \cos\gamma d\Omega$$

Electron or ion current to each patch on the aggregate. Plasma species has velocity distribution $f_j(\mathbf{v})$

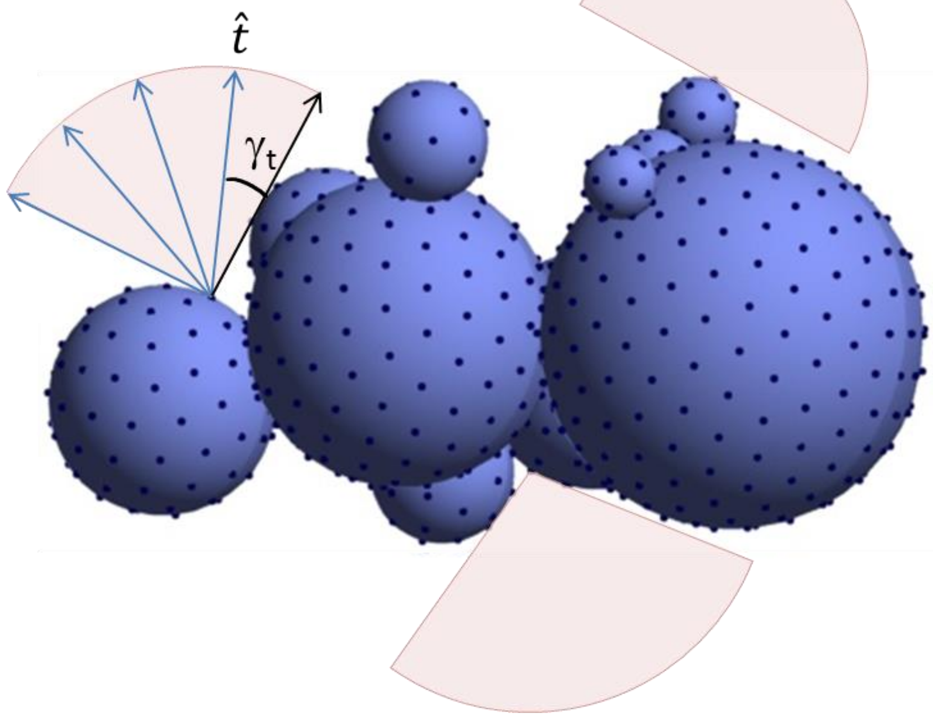


# Simulation of Dust Charging

$$J_j = n_j e_j \int_{v_{min,j}}^{\infty} \pi \left(1 - \frac{2 e_j V}{m_j v_j^2}\right) f_j v_j^3 \, dv_j \int \cos\gamma \, d\Omega$$

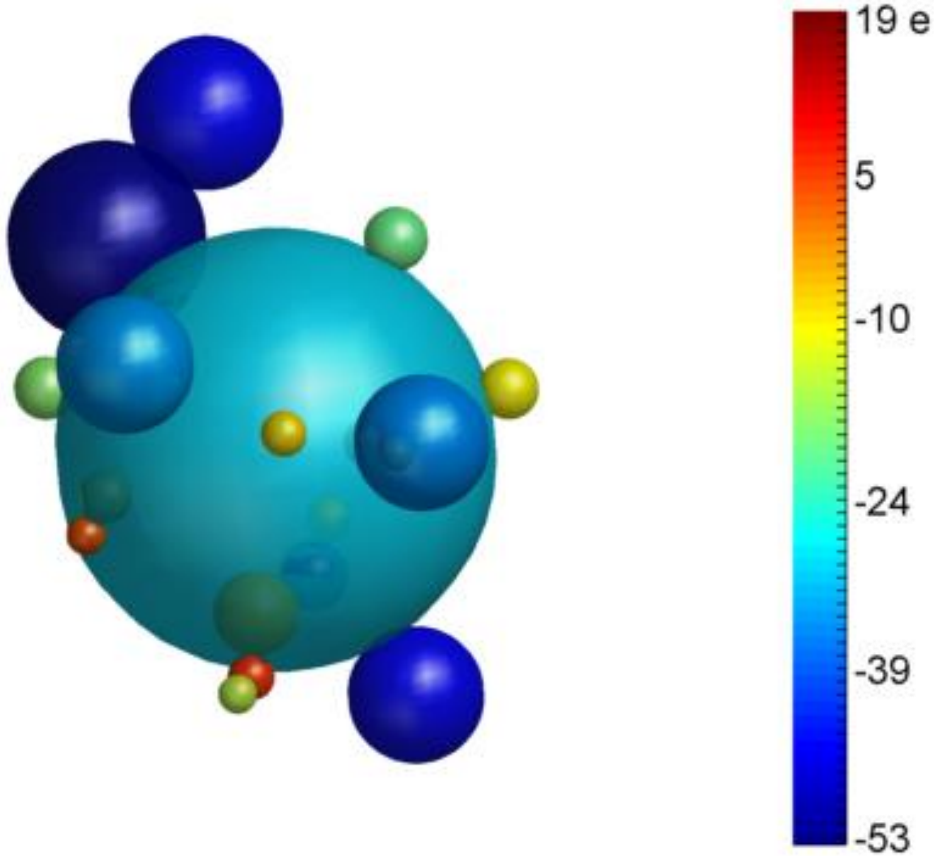
LOS_factor $= \sum_t \cos\gamma_t \Delta\Omega$

Integration over the angles is approximately numerically.



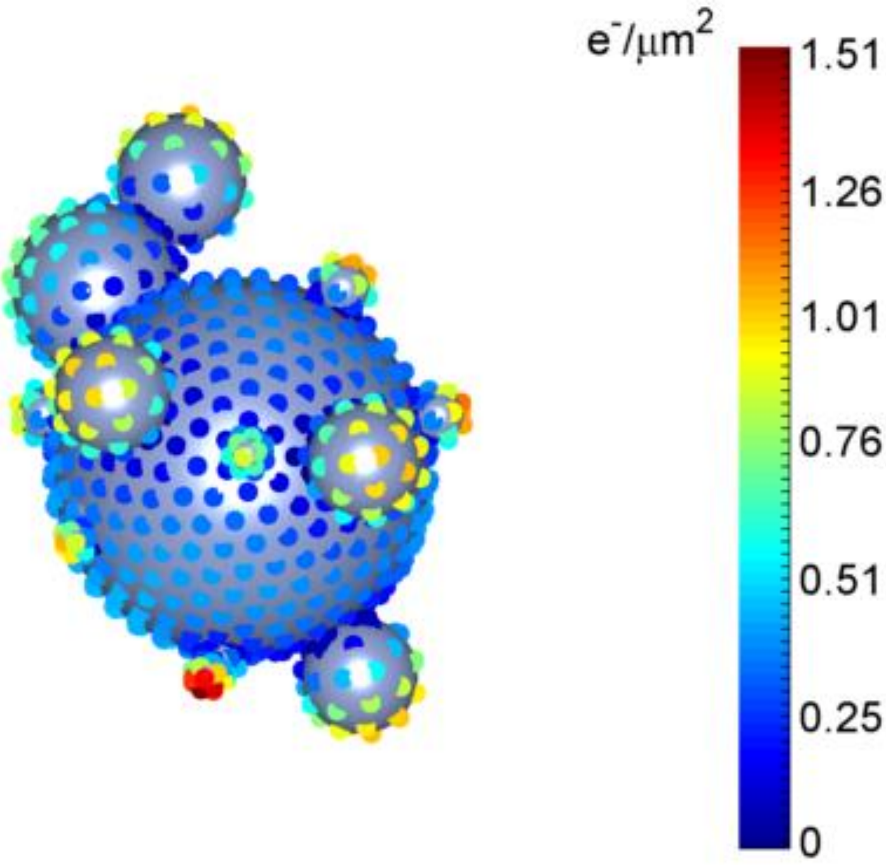
# Dust Charging

Total charge on each monomer



# Dust Charging

Charge density on each surface patch



# Formation of Planets from Dust in Protoplanetary Disks

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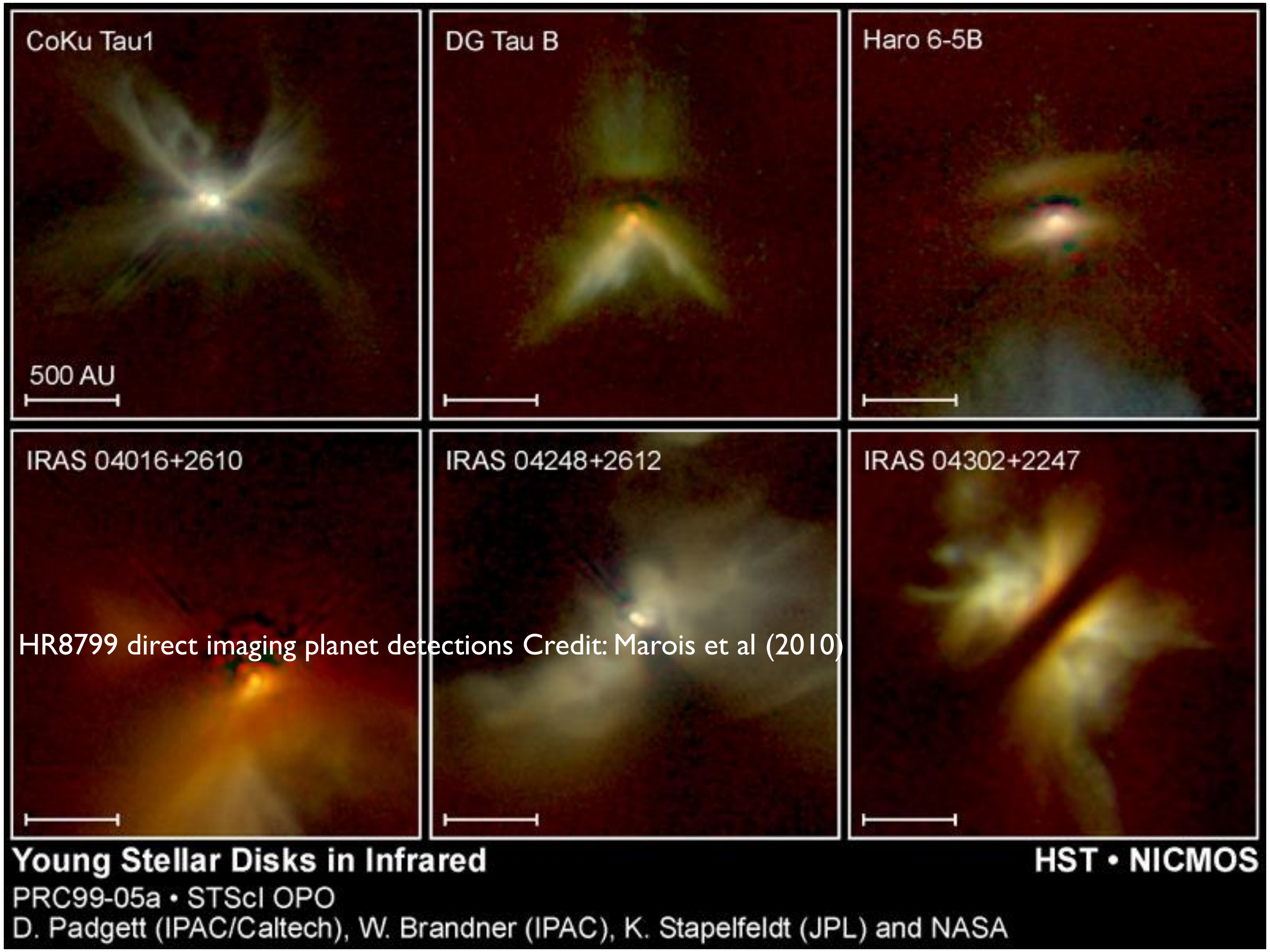
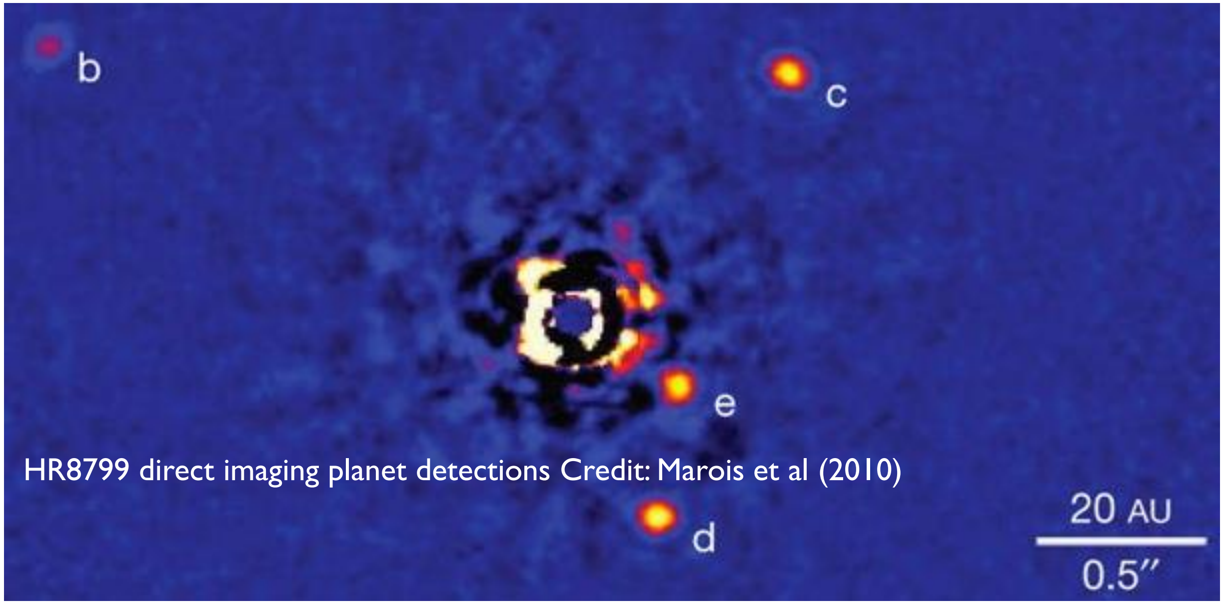



HR8799 direct imaging planet detections Credit: Marois et al (2010)

# Formation of Planets from Dust in Protoplanetary Disks



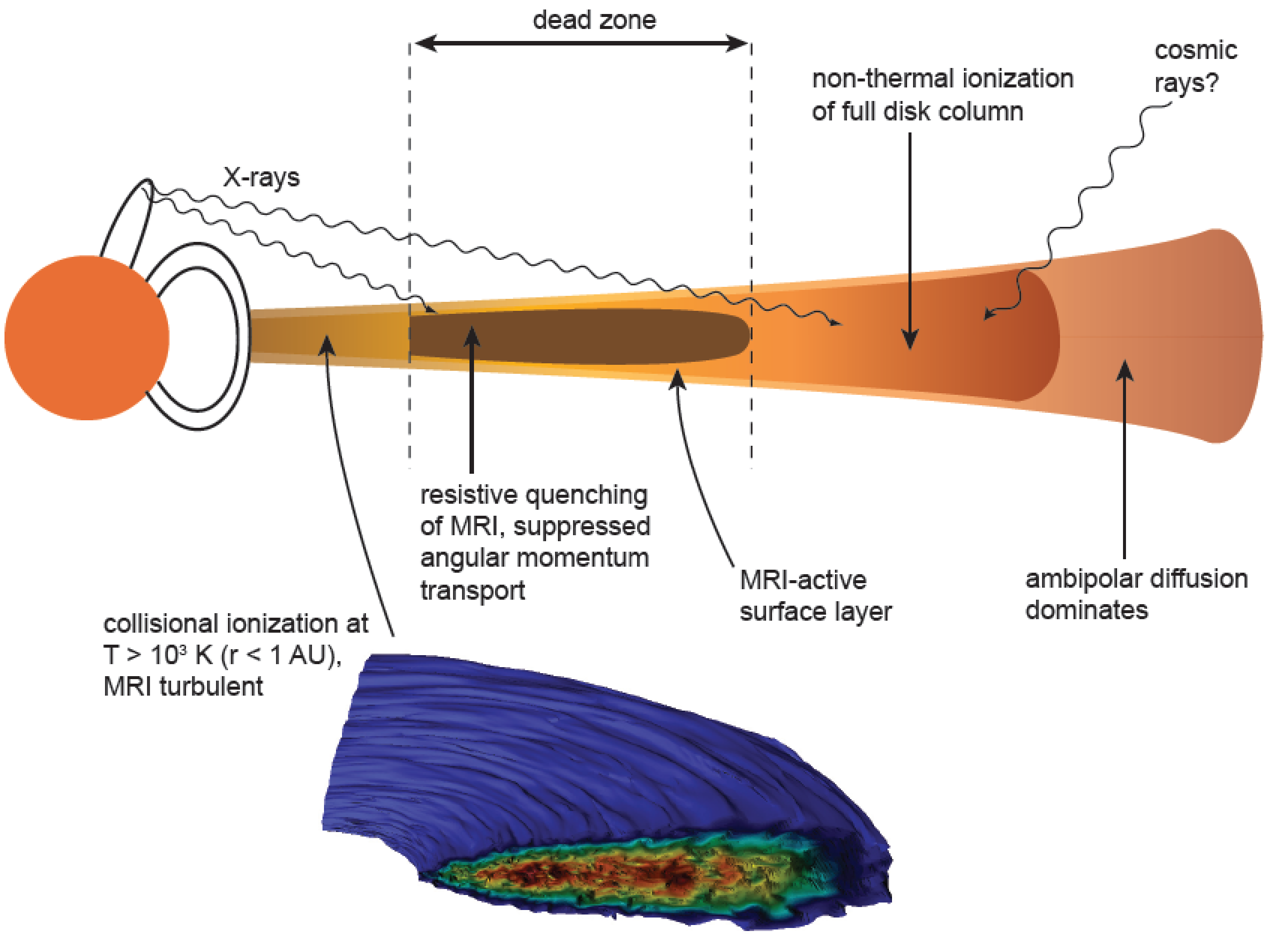
- What are the factors that determine the types and number of planets formed?




HR8799 direct imaging planet detections Credit: Marois et al (2010)

# Dust in Protoplanetary Disks

As magnetic field lines couple to plasma in the disk, the Magnetorotational Instability (MRI) causes turbulent gas flow.




Armitage, *Ann Rev Astr and Astrophys*, 2011

# PPD: Plasma Environment



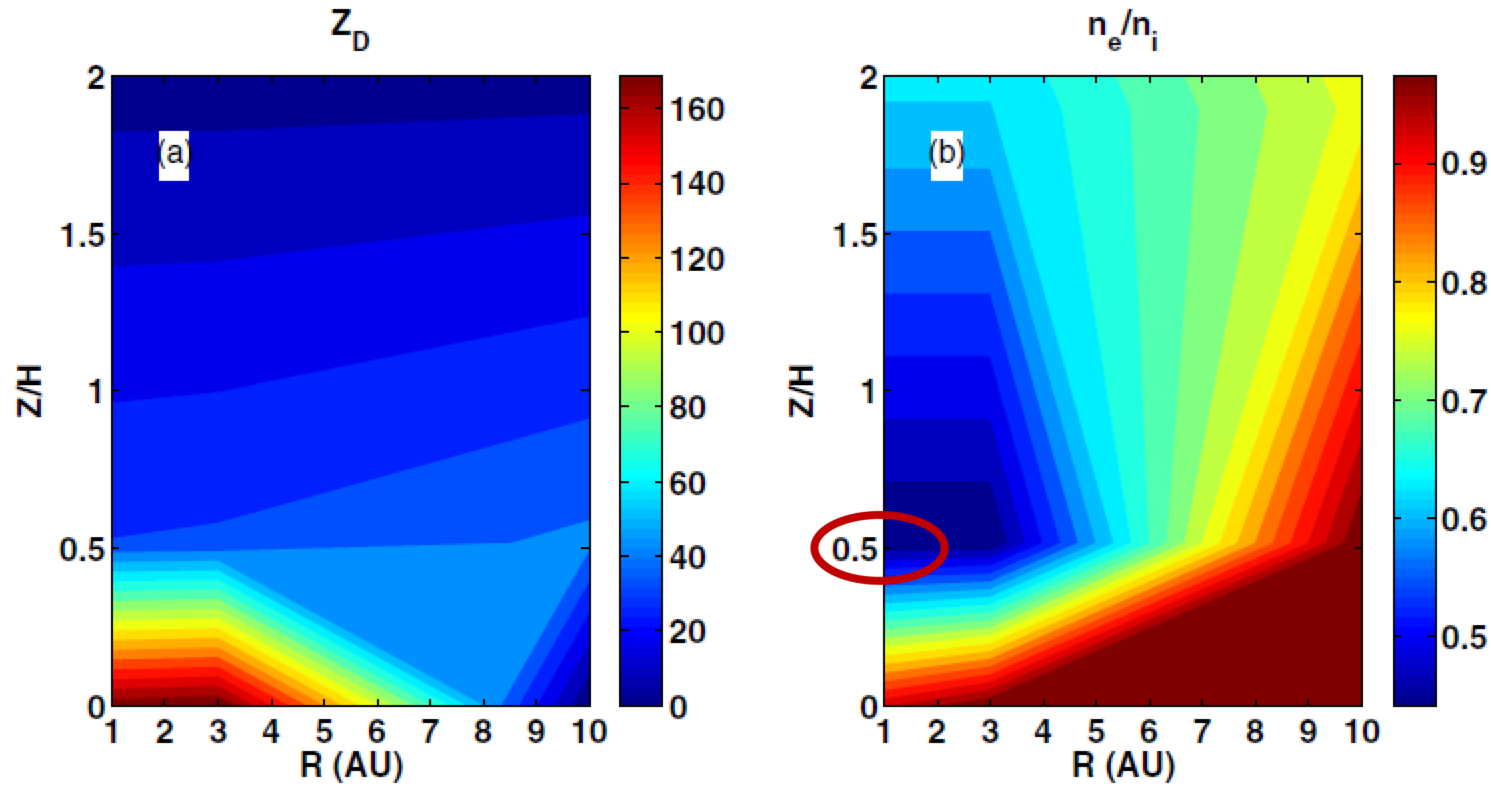
Plasma environment determined from literature profiles*:

Gas density,

Gas temperature and fractional ionization

Dust-to-gas mass density ratio of 1%

Average dust charge, $Z_D$, and ratio of electron to ion density calculated throughout the disk.




*Semenov et al. 2004, Willacy et al. 1998, Lissauer 1993, Ruden & Pollack 1991

*Matthews, Land and Hyde, ApJ, 2012*

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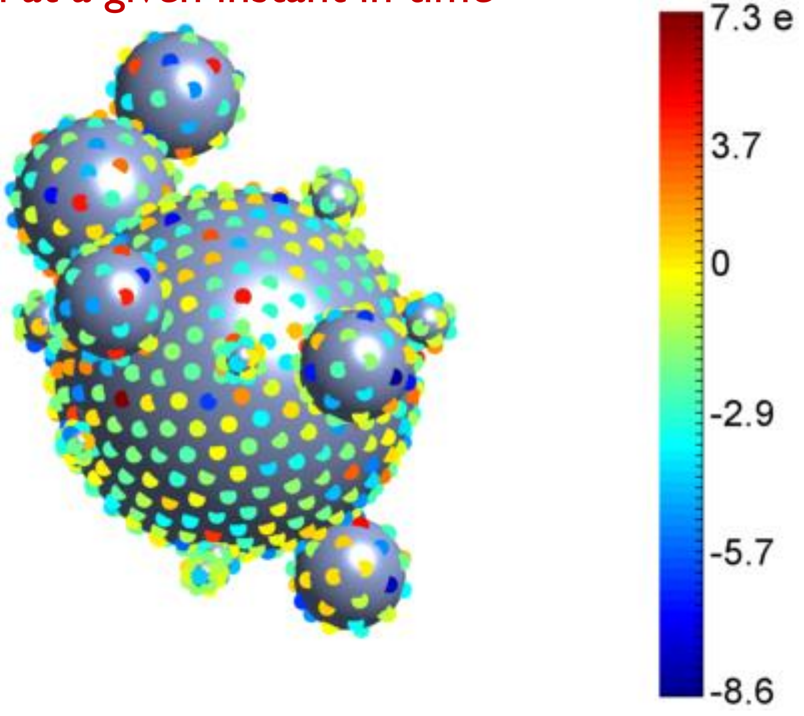
# Stochastic Charging

- Intrinsic system noise
- Ions and electrons collected at random time intervals
- Charge collected at each patch undergoes ***Markov*** process *(A random process whose future probabilities are determined by its most recent values. – Wolfram MathWorld)*

# Stochastic Charging of Aggregate

Charge on each patch at a given instant in time



# Stochastic Charging – Master Equation

$$\frac{dP(\mathbf{Z},t)}{dt} = \sum_{p=1}^{M} I_{i,p}(\mathbf{Z}-\mathbf{e}_p)\, P(\mathbf{Z}-\mathbf{e}_p,t)$$

$$+ I_{e,p}(\mathbf{Z}+\mathbf{e}_p)\, P(\mathbf{Z}+\mathbf{e}_p,t)$$

$$-\left[I_{i,p}(\mathbf{Z}) + I_{e,p}(\mathbf{Z})\right] P(\mathbf{Z},t)$$

*Probability that charge on patch p is Z(±e) at time t*

# Stochastic Charging – Master Equation

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$$\frac{dP(\mathbf{Z},t)}{dt} = \sum_{p=1}^{M} I_{i,p}(\mathbf{Z}-\mathbf{e}_p) P(\mathbf{Z}-\mathbf{e}_p, t)$$

$$+ I_{e,p}(\mathbf{Z}+\mathbf{e}_p) P(\mathbf{Z}+\mathbf{e}_p, t)$$

*Ion and electron currents to patch with given charge*

$$- \left[ I_{i,p}(\mathbf{Z}) + I_{e,p}(\mathbf{Z}) \right] P(\mathbf{Z}, t)$$

# Stochastic Charging



- Fokker-Planck Equation
  - Describes time evolution of probability distribution

$$\frac{\partial P(\mathbf{Z},t)}{\partial t} = \sum_{p=1}^{M} \frac{\partial}{\partial Z_p}\left[I_{e,p}(\mathbf{Z}) - I_{i,p}(\mathbf{Z})\right] P(\mathbf{Z},t) + \frac{1}{2}\frac{\partial^2}{\partial Z_p^2}\left[I_{e,p}(\mathbf{Z}) + I_{i,p}(\mathbf{Z})\right] P(\mathbf{Z},t)$$

- Statistically equivalent to Langevin equation

$$dZ_p(t) = \left[I_{i,p}(\mathbf{Z}) - I_{e,p}(\mathbf{Z})\right]dt + \sqrt{I_{e,p}(\mathbf{Z}) + I_{i,p}(\mathbf{Z})}\, dW_p(t)$$

*Wiener process – used to model noise in systems*

# Stochastic Charging – Implementation in Numerical Code

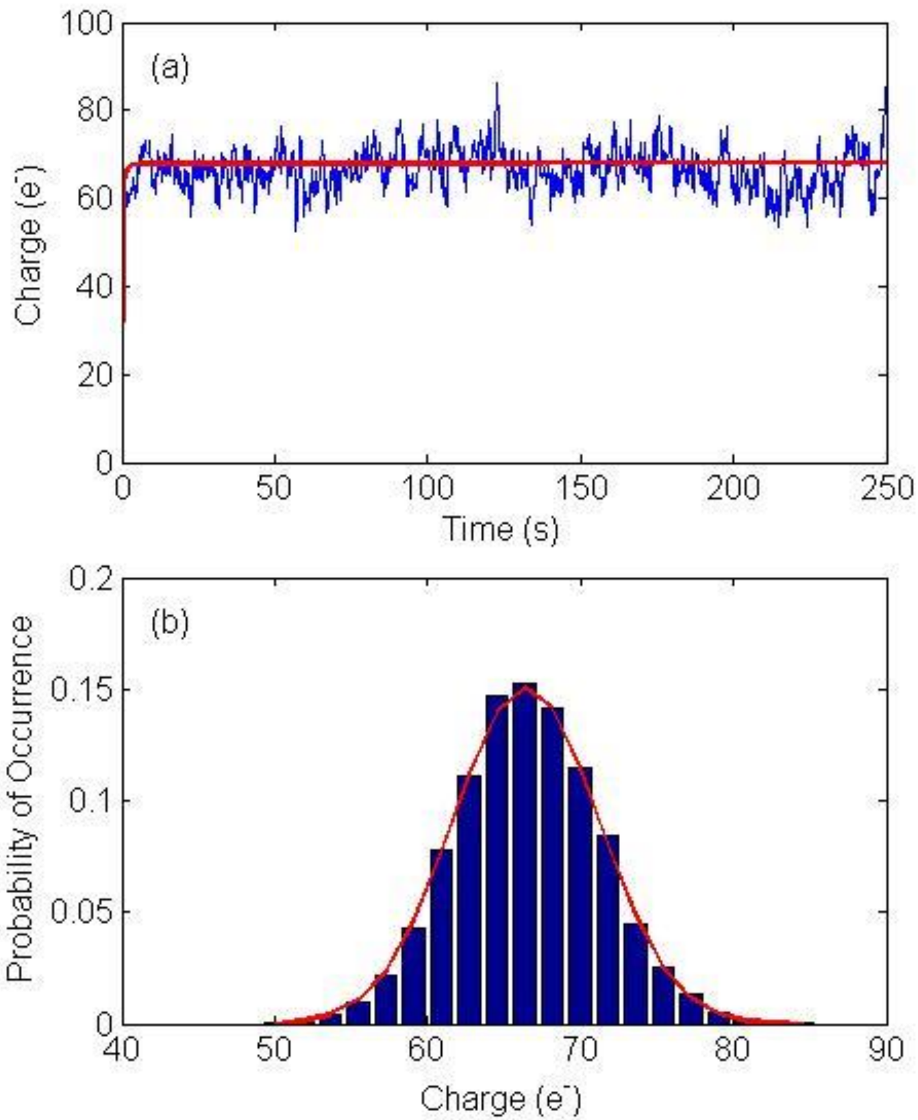


$$Z_p^{(n+1)} = Z_p^{(n)} + \left[I_{i,p}\left(\mathbf{Z}^{(n)}\right) - I_{e,p}\left(\mathbf{Z}^{(n)}\right)\right]\Delta t + \sqrt{I_{e,p}(\mathbf{Z}^{(n)}) + I_{i,p}(\mathbf{Z}^{(n)})}\sqrt{\Delta t}\,\xi_p$$

*Random number with normal distribution*

Matthews, Shotorban, and Hyde, ApJ, 2013

# Stochastic Charging of Sphere

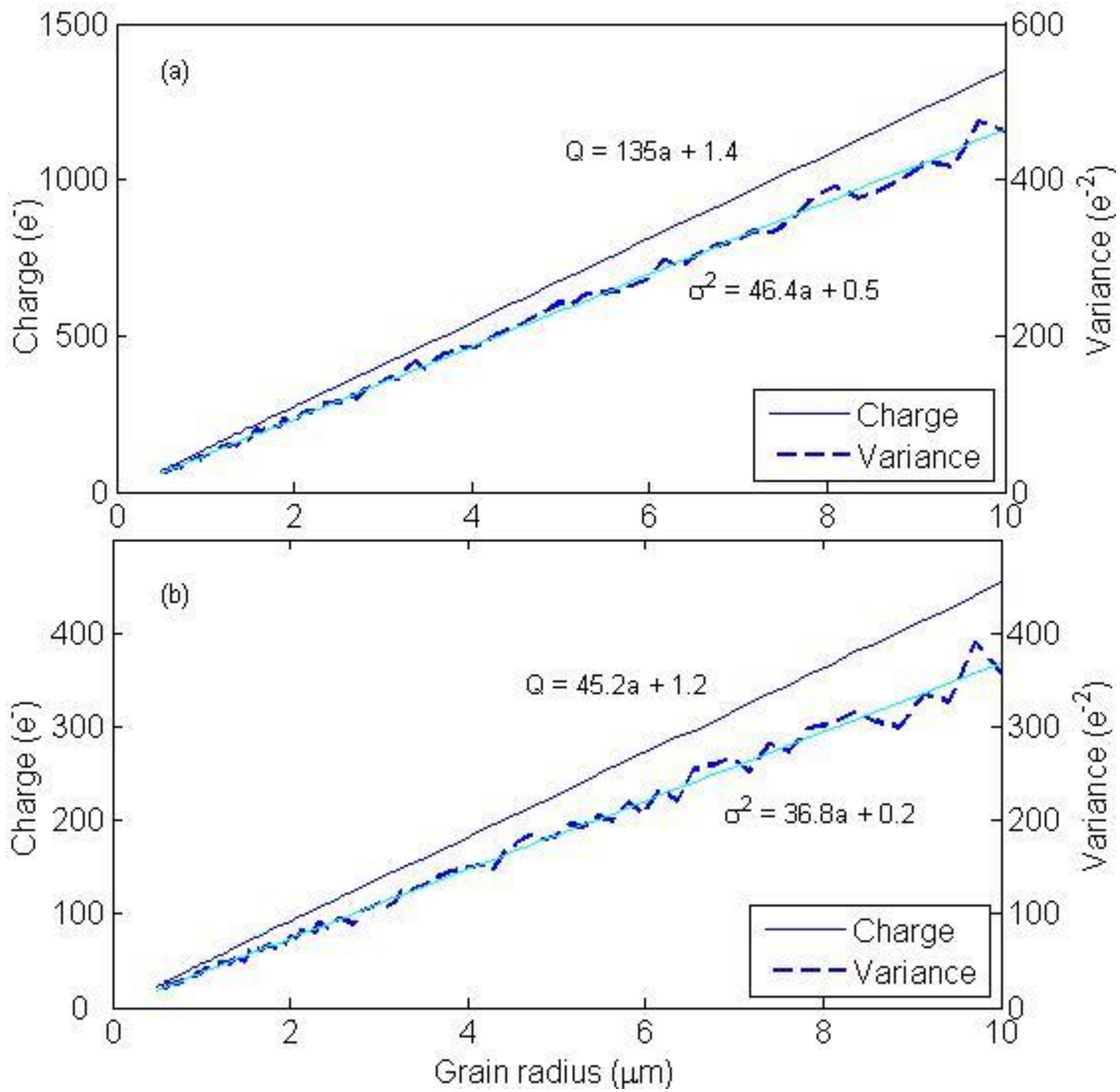


# Growth of Aggregates in PPD



- Two plasma environments: H plasma at 900K
  - $n_e = n_i$ – *large dust charges*
  - $n_e = 0.1\ n_i$ – *small dust charges*
- Compare stochastically and non-stochastically charged dust

# Stochastic Charging of Spheres – *Comparison of two plasma environments*

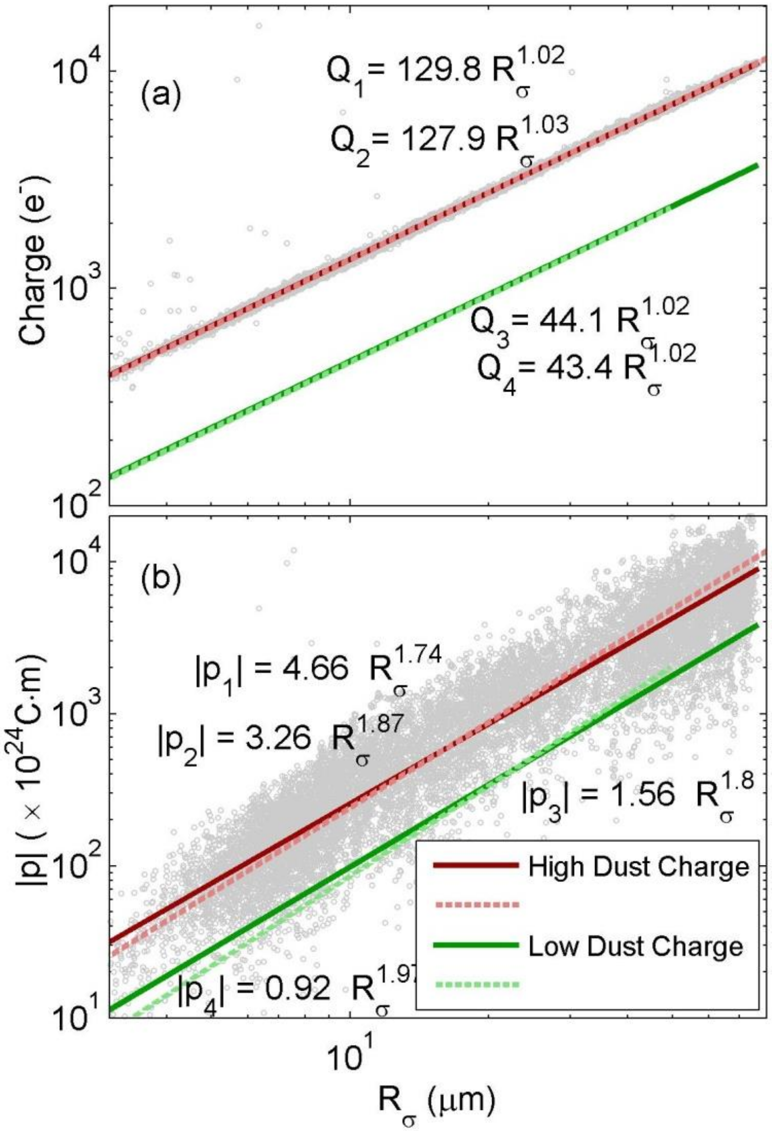



$n_e = n_i$

$n_e = 0.1\ n_i$

# Charge and dipole on aggregates

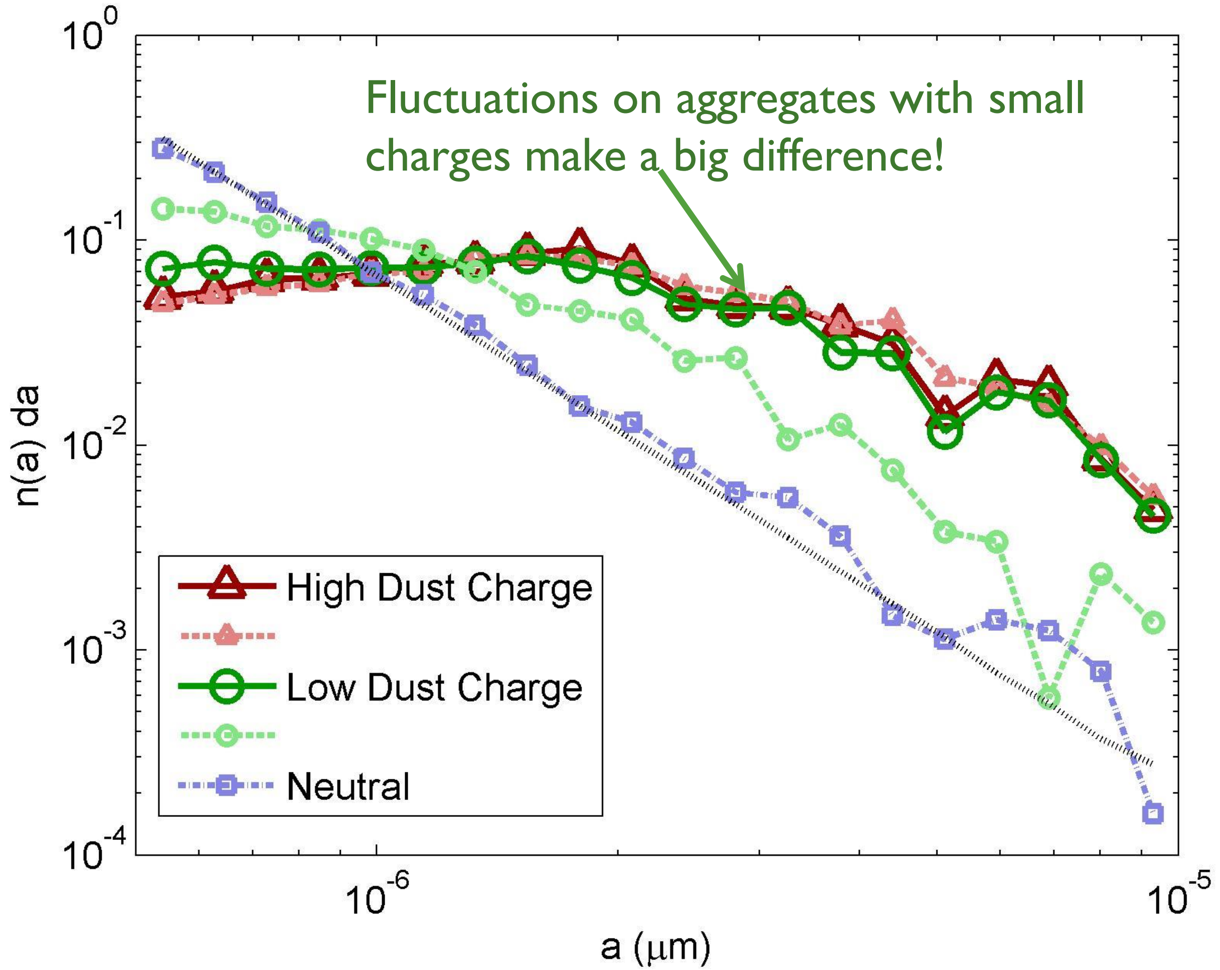





Stochastic Charging

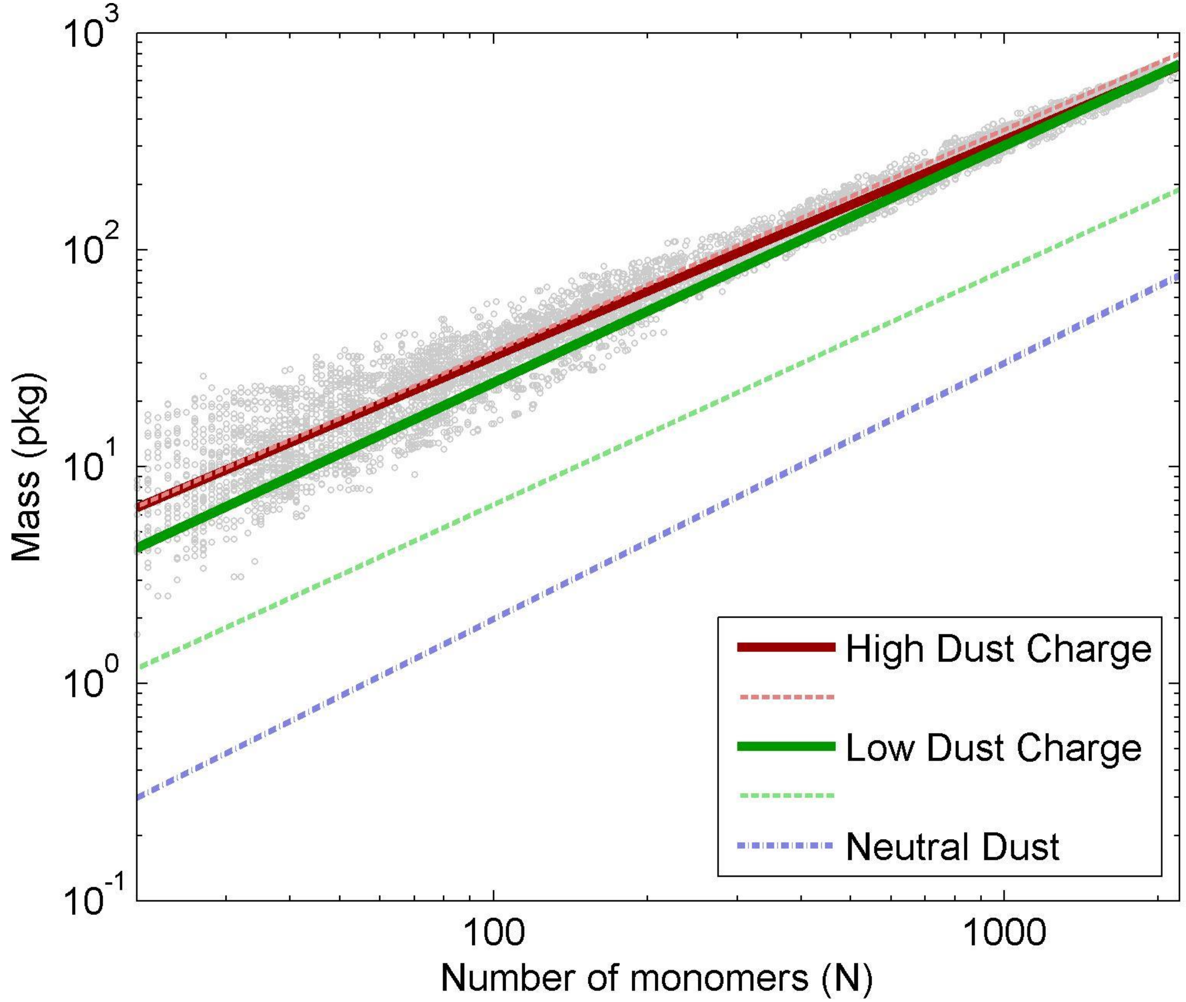
Non-Stochastic Charging

# Physical Characteristics of Aggregates – Monomer distribution



# Physical Characteristics of Aggregates – Mass

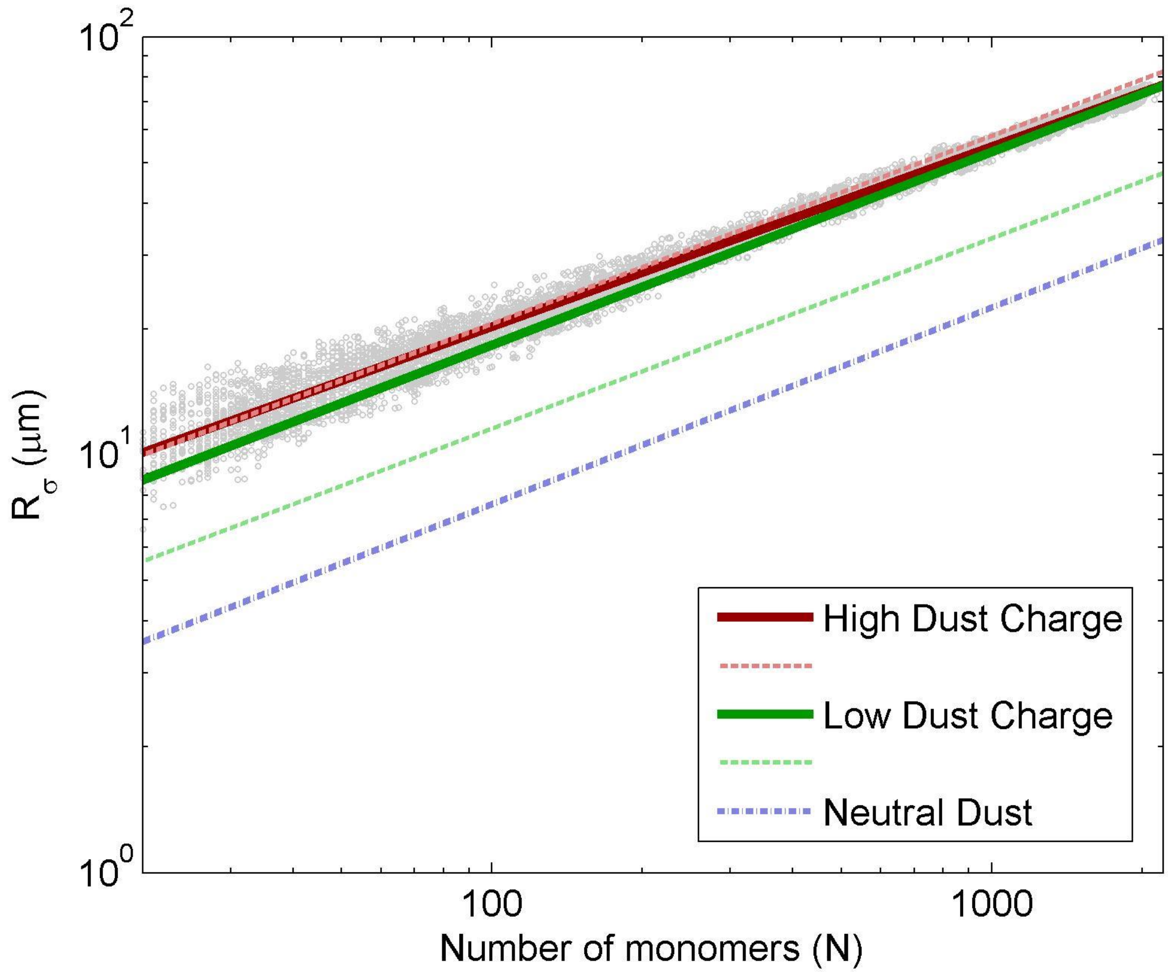
Stochastic charging leads to aggregates with greater mass for a given number of monomers



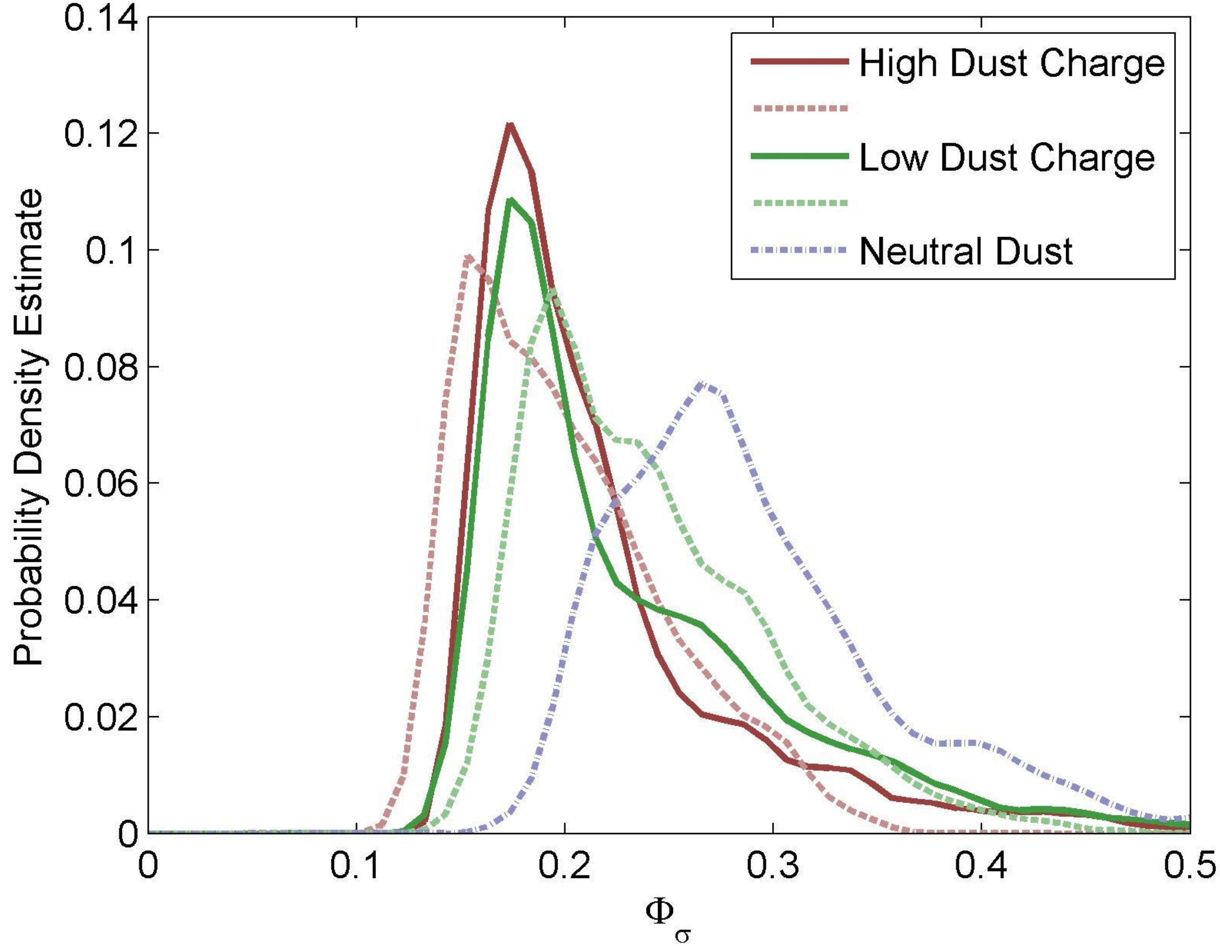
# Physical Characteristics of Aggregates – Equivalent Radius



Stochastic charging leads to aggregates with larger radius for a given number of monomers



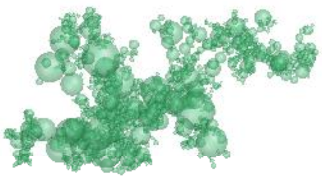
# Physical Characteristics of Aggregates – Compactness Factor



# Aggregate Morphology

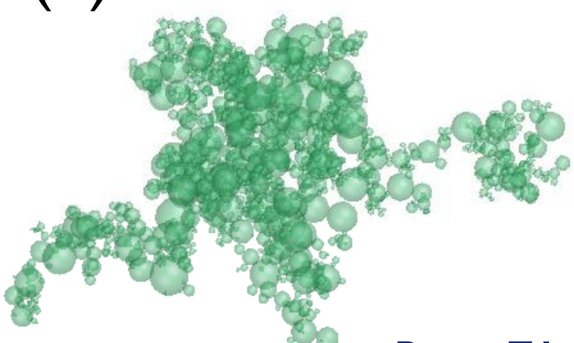
Four aggregates with N ≈ 2000 shown to scale:

(a) Low charge, non-stochastic,



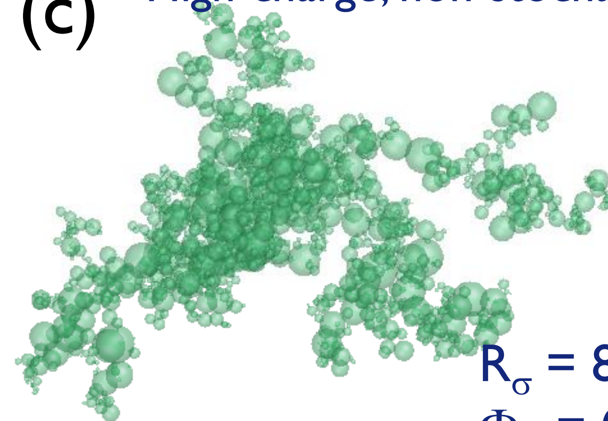
$R_\sigma = 44.4\ \mu m$
$\Phi_\sigma = 0.176$

(b) Low-charge, stochastic,



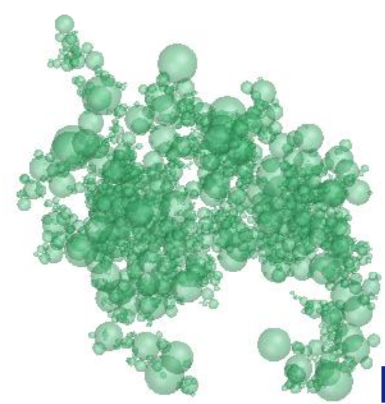
$R_\sigma = 71.5\ \mu m$
$\Phi_\sigma = 0.152$

(c) High-charge, non-stochastic



$R_\sigma = 81.2\ \mu m$
$\Phi_\sigma = 0.133$

(d) High charge, stochastic.

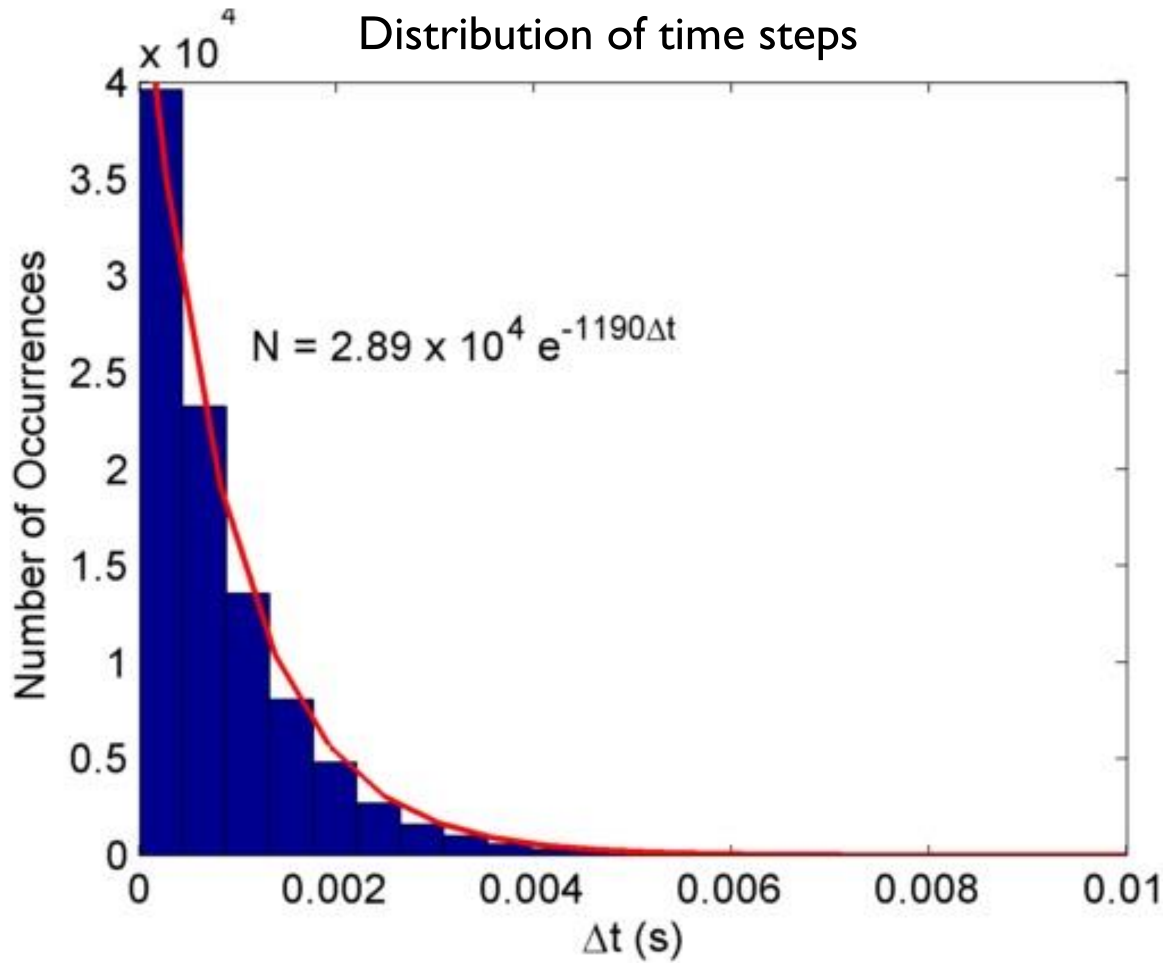


$R_\sigma = 74.7\ \mu m$
$\Phi_\sigma = 0.157$

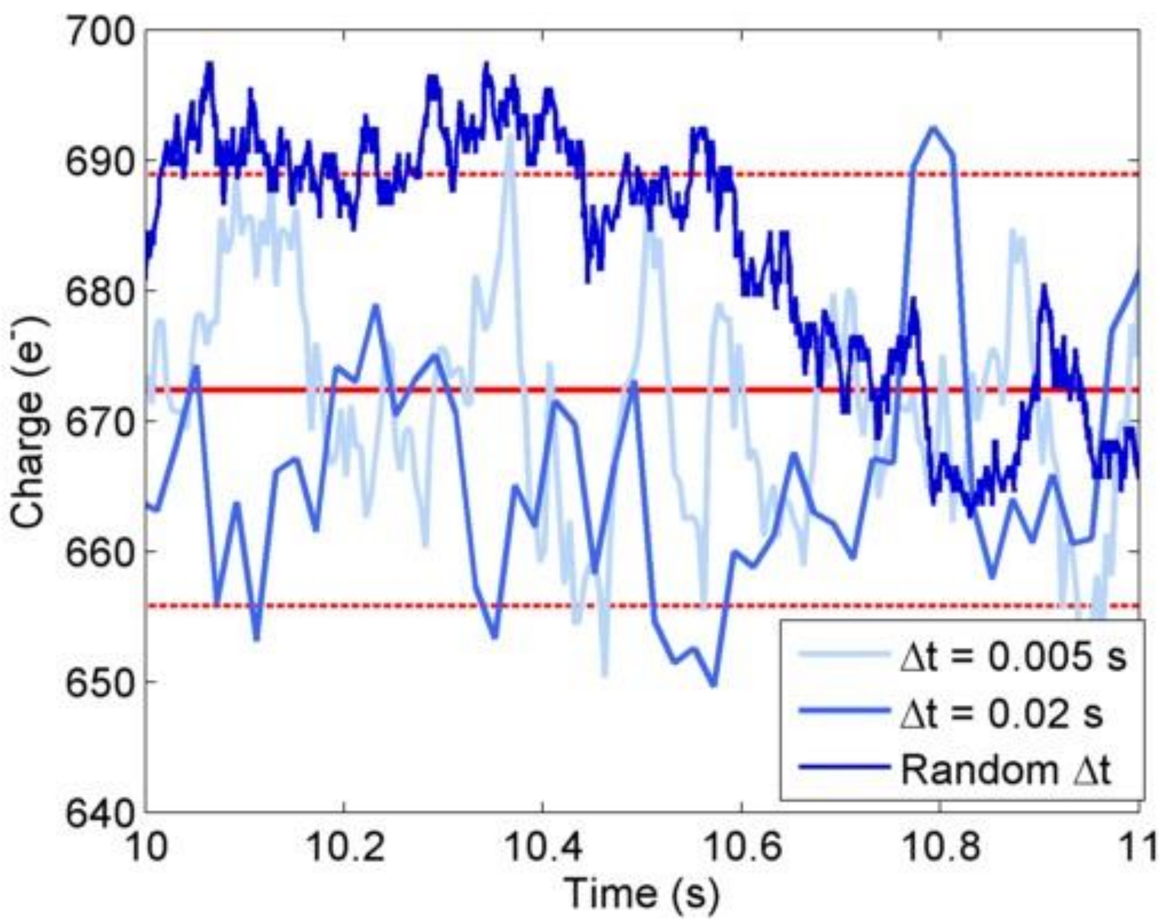
# Future Work: Discrete Stochastic Charging

- Single electrons or ions added at random time steps

Distribution of time steps



# Future work: Discrete Stochastic Charging




Comparison of charging histories

- Discrete stochastic charging with random time steps
- Continuous stochastic charging with fixed time steps

# Future work: Discrete Stochastic Charging

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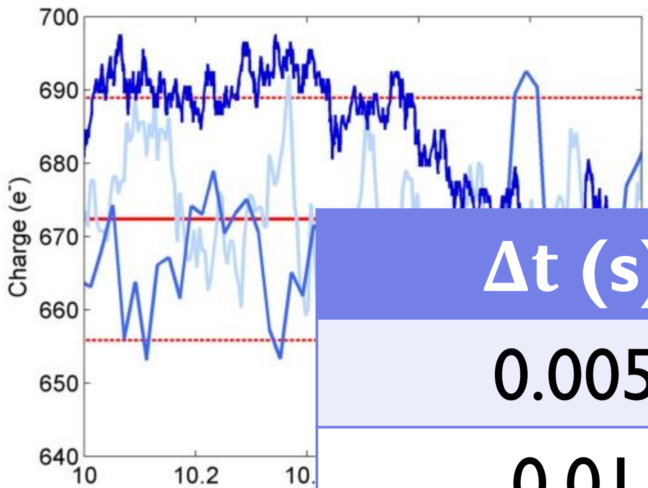


| Δt (s) | $\langle Q \rangle$ ($e^-$) | $\sigma$ ($e^-$) |
|---|---|---|
| 0.005 | 675 | 14.7 |
| 0.01 | 675 | 14.7 |
| 0.02 | 676 | 15.2 |
| 0.04 | 676 | 16.1 |
| 0.08 | 676 | 18.1 |
| **DSC: random** | **672** | **16.5** |

# Conclusion

- Effects of stochastic charging most important for small grain charges
- Dynamical timescale determines type of charging model to use

# Future Work

- Link to model of turbulence in disk
- Relative velocities, coagulation rate
- Plasma reduction due to dust densities