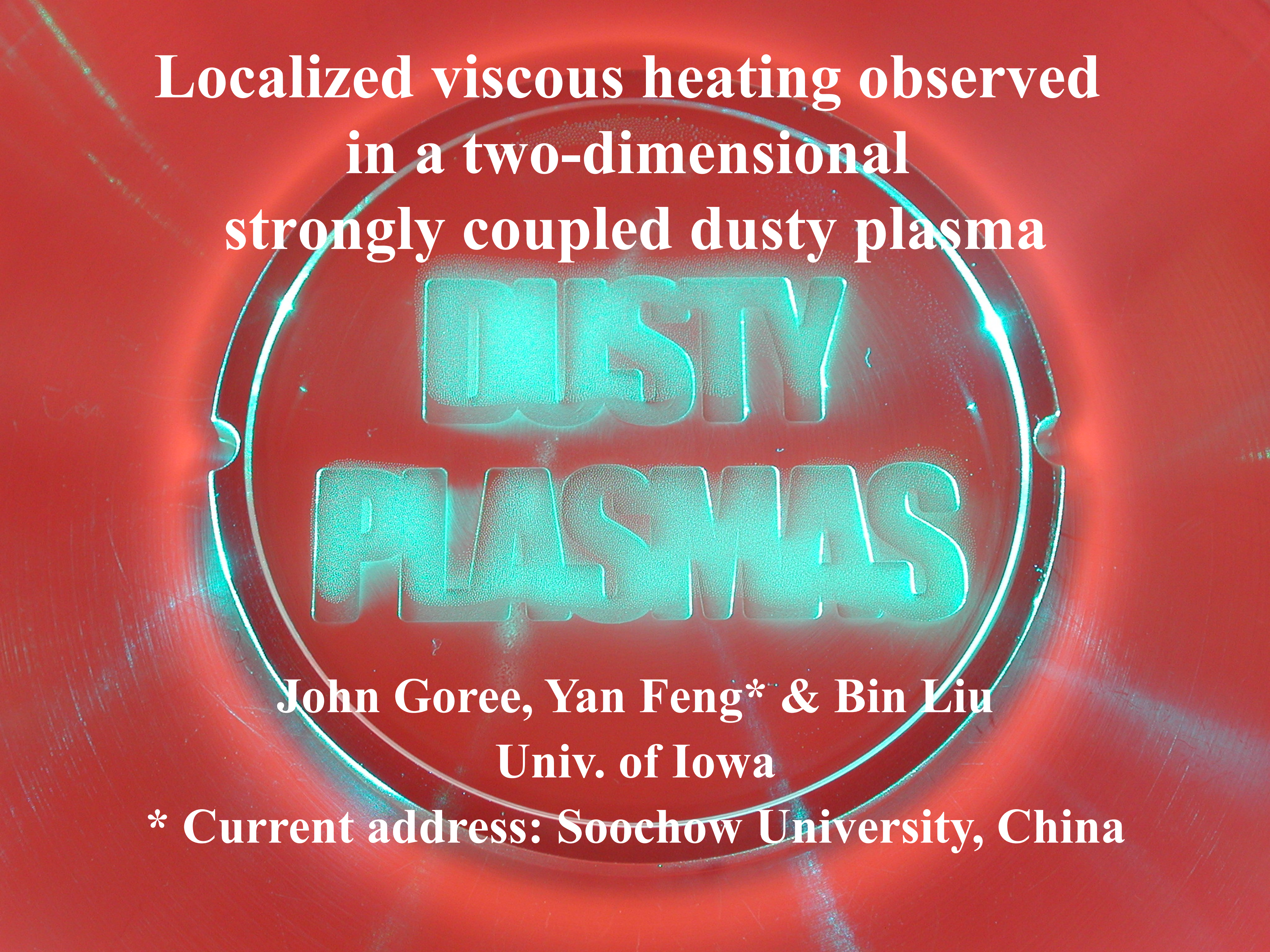
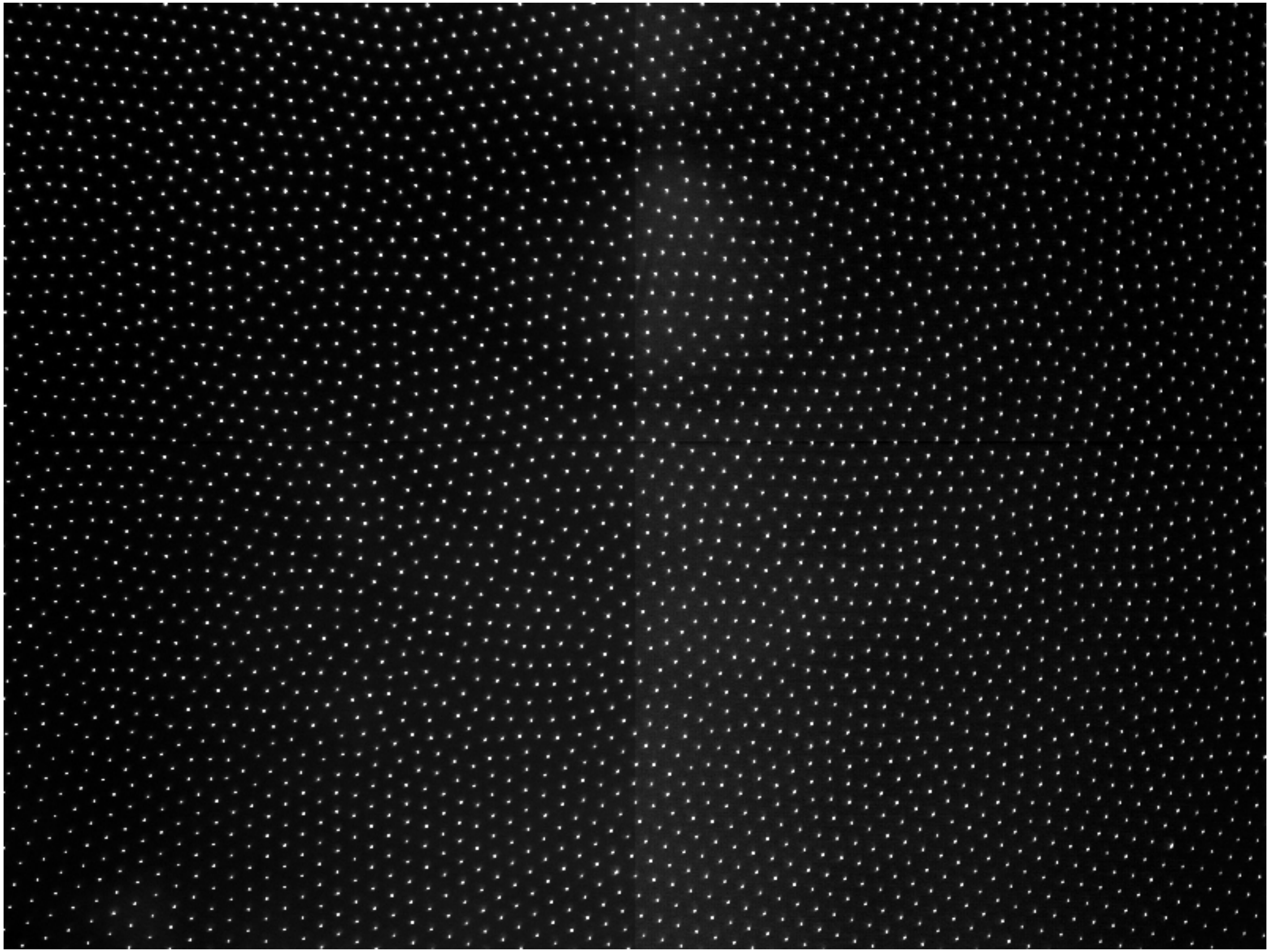


Localized viscous heating observed in a two-dimensional strongly coupled dusty plasma

John Goree, Yan Feng* & Bin Liu
Univ. of Iowa
* Current address: Soochow University, China

Shear flow in 2D dusty plasma

Shear flow in dusty plasma



Fluids, in general

Transport coefficients

μ = viscosity

κ = thermal conductivity

These coefficients are material properties.

Viscous heating: it's where a lot of energy is lost

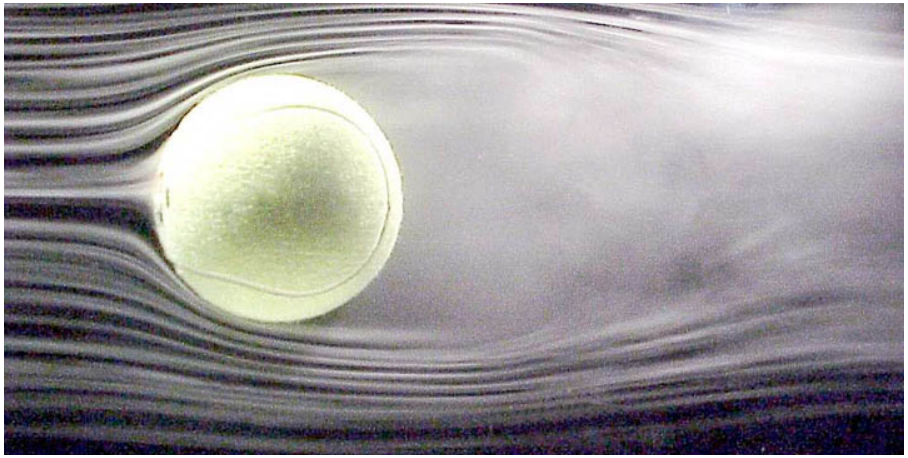


image: NASA

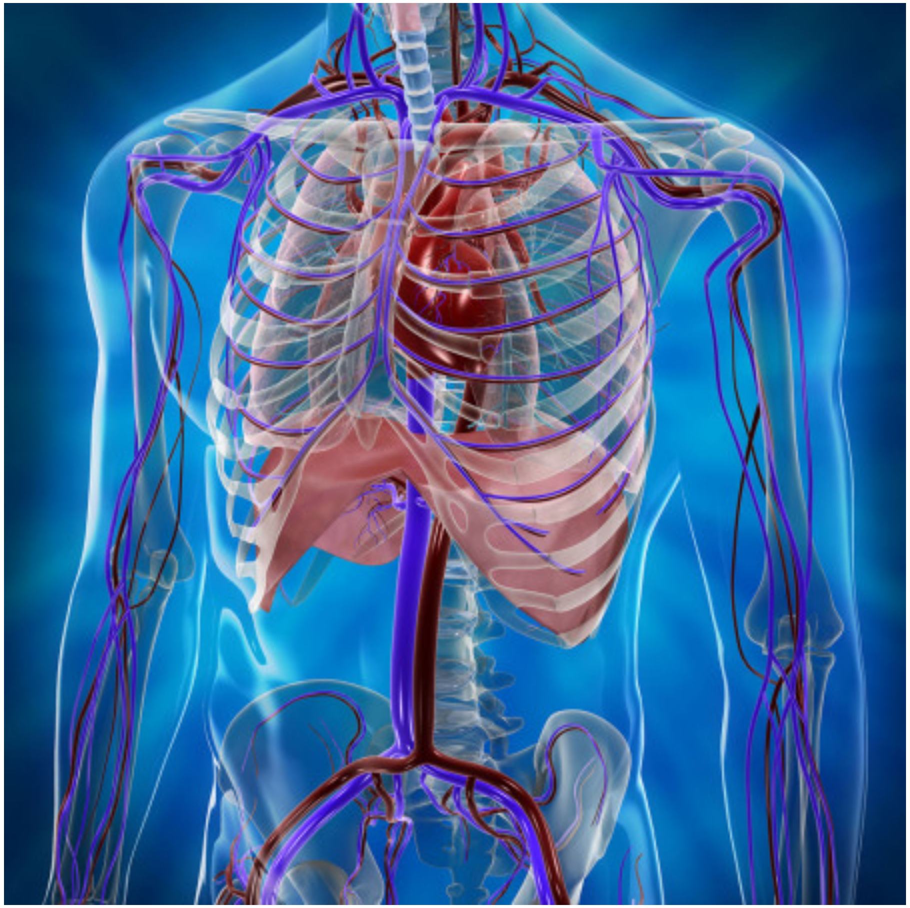
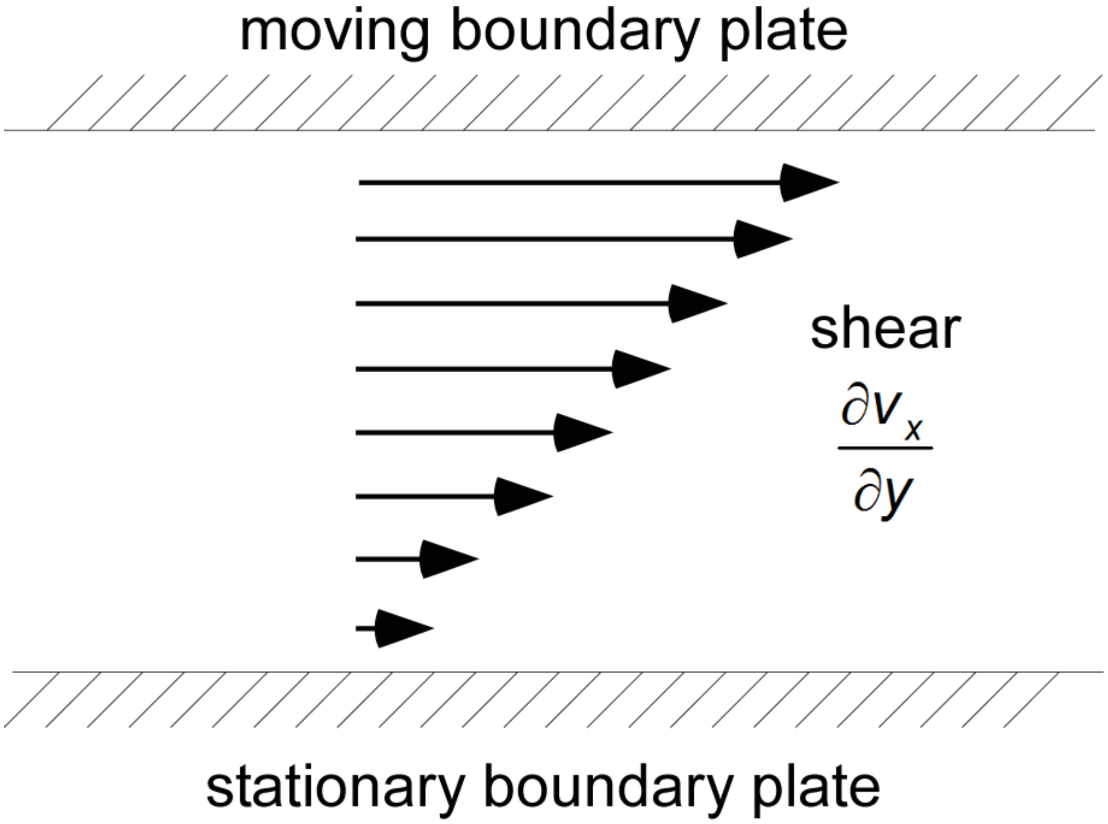


image: wordpress.com

Viscous heating happens in a sheared flow

Couette flow
between
two flat plates



Viscous heating happens in a sheared flow

"shear"

= transverse gradient of a flow velocity

= $\partial v_x / \partial y$

Heat generated $\propto (\partial v_x / \partial y)^2$

Hydrodynamic equations for Couette flow

momentum

$$\frac{\eta}{\rho}\nabla^2\mathbf{v} = 0$$

energy

$$\frac{\kappa}{\rho}\nabla^2\mathrm{T} + \frac{\eta}{\rho}\left(\frac{\partial v_x}{\partial y}\right)^2 = 0$$

thermal
conduction

viscous
heating

μ = viscosity
κ = thermal conductivity

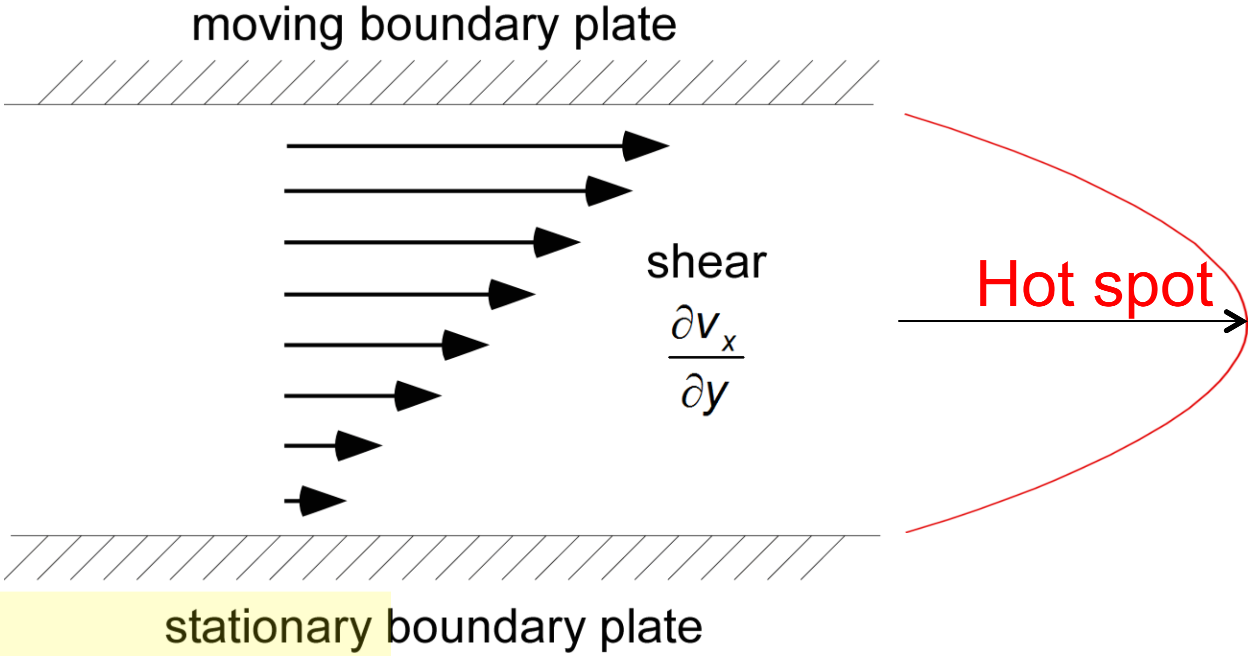
Solution of the hydrodynamic equations for Couette Flow

$$T \propto \frac{\eta}{\kappa}\left(\frac{\partial v_x}{\partial y}\right)^2$$

Solutions:

$v_x \propto$ y

$T \propto$ y^2 due to viscous heating



viscous heating:

- should cause **hot-spots** in a flow
- but they aren't observed

Why aren’t the hot spots observed?

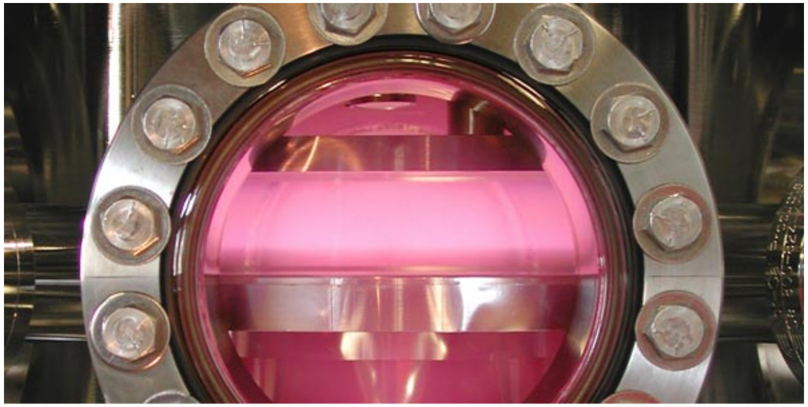
For most substances:

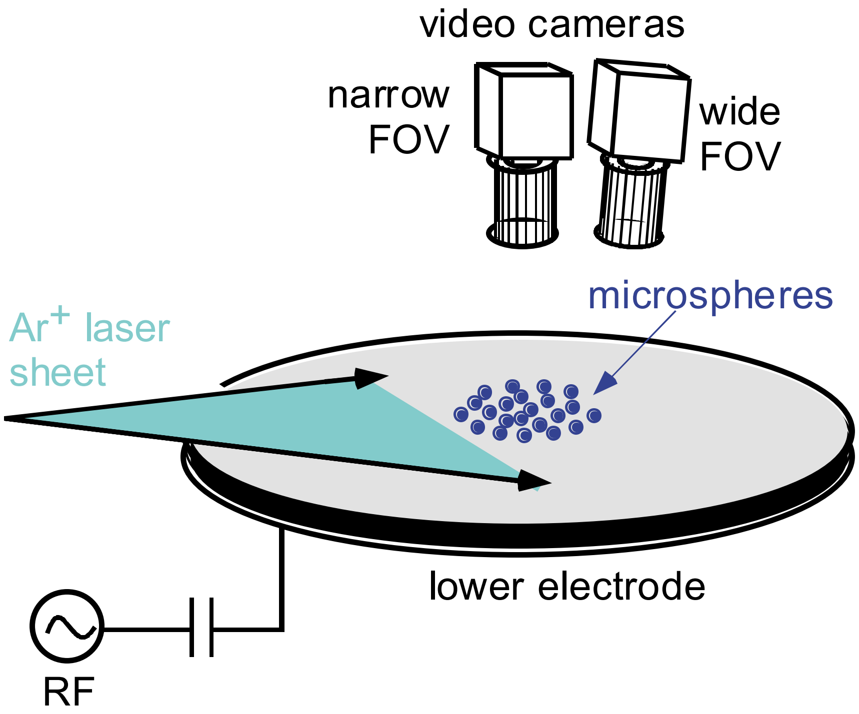
thermal conductivity κ is so large that $\nabla^2 T$ is tiny.

To observe a hot spot requires a combination of:

- Small κ thermal conductivity
- Large η viscosity
- Large $\partial v_x/\partial y$ shear
- Instrumentation for in-situ temp measurement

Experimental setup





Argon RF glow discharge:

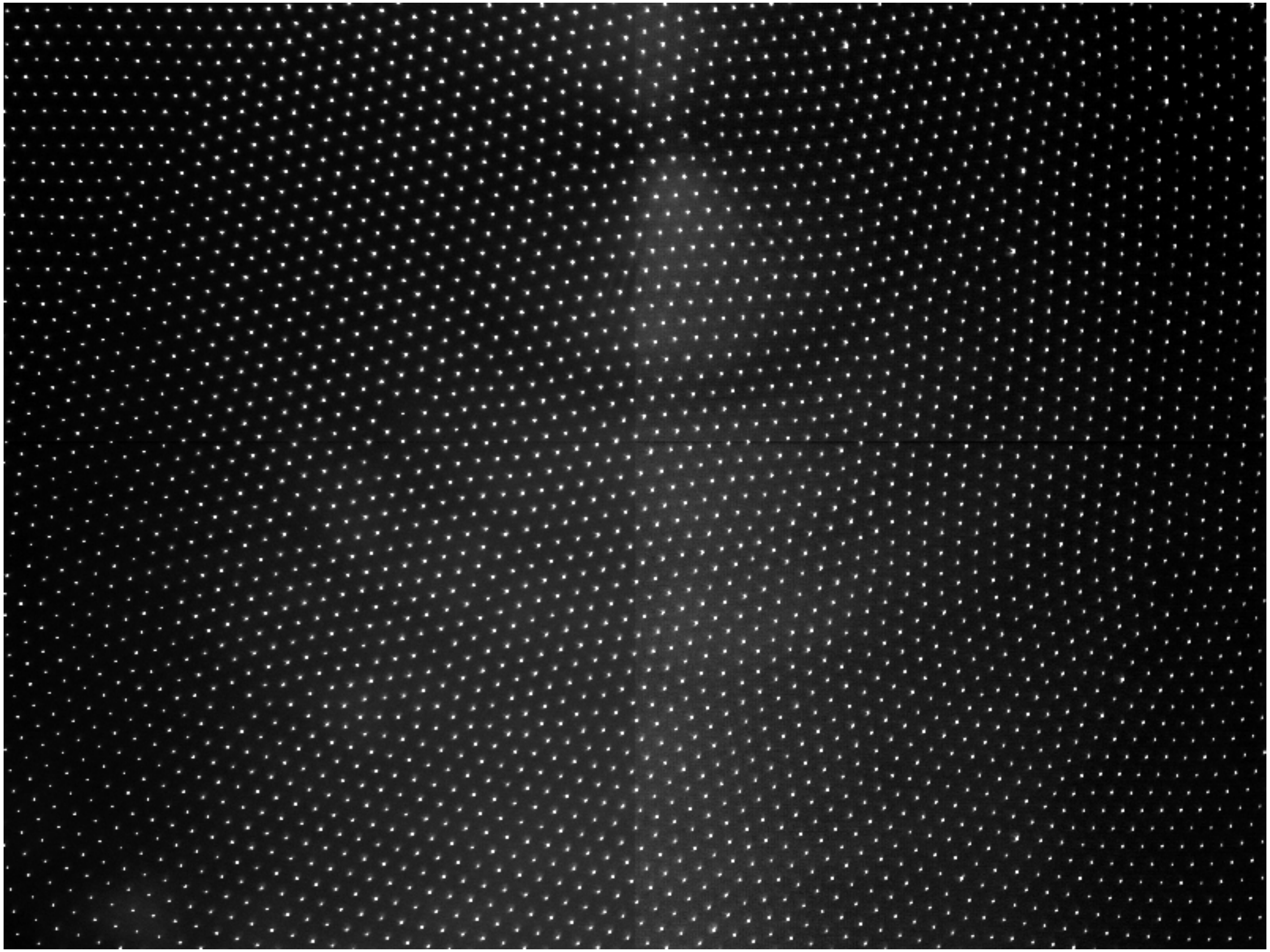
gas	15.5 mTorr Argon
RF	low power, 13.6 MHz
λ_D	0.53 mm

MF Polymer microspheres:

diameter	8.1 μm
number	>11000 particles
interparticle dist.	0.50 mm
charge	-9700 *e*

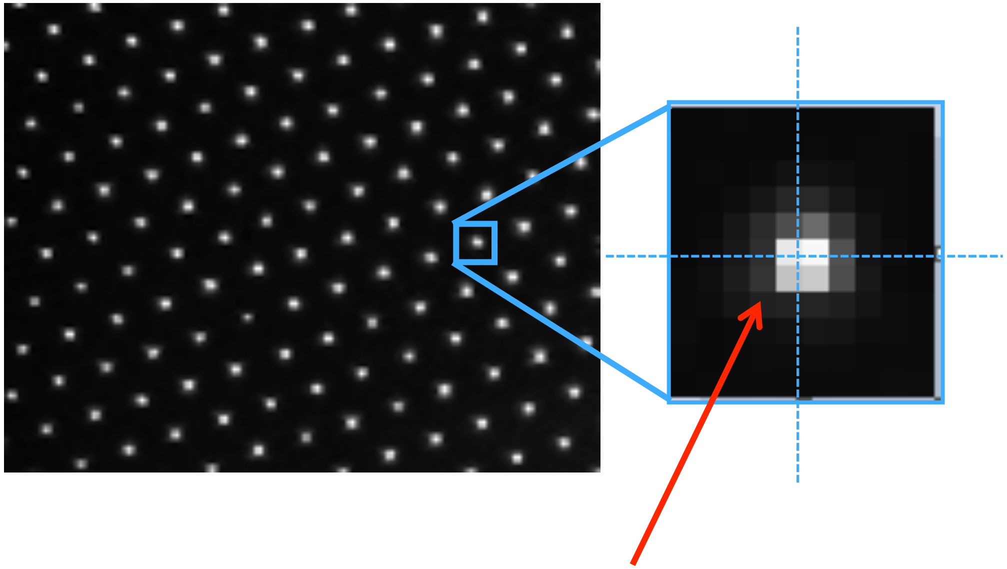
Narrow field of view

Top view



Random motion in a crystalline phase

Measurements of particles



Intensity in pixel i is I_i

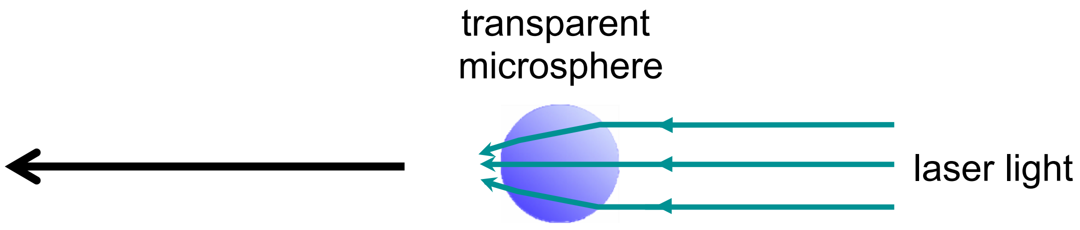
- Measure particle *x*,*y* positions

$$X = \frac{\sum (X_i I_i)}{\sum I_i}$$

- Track particles from frame to frame

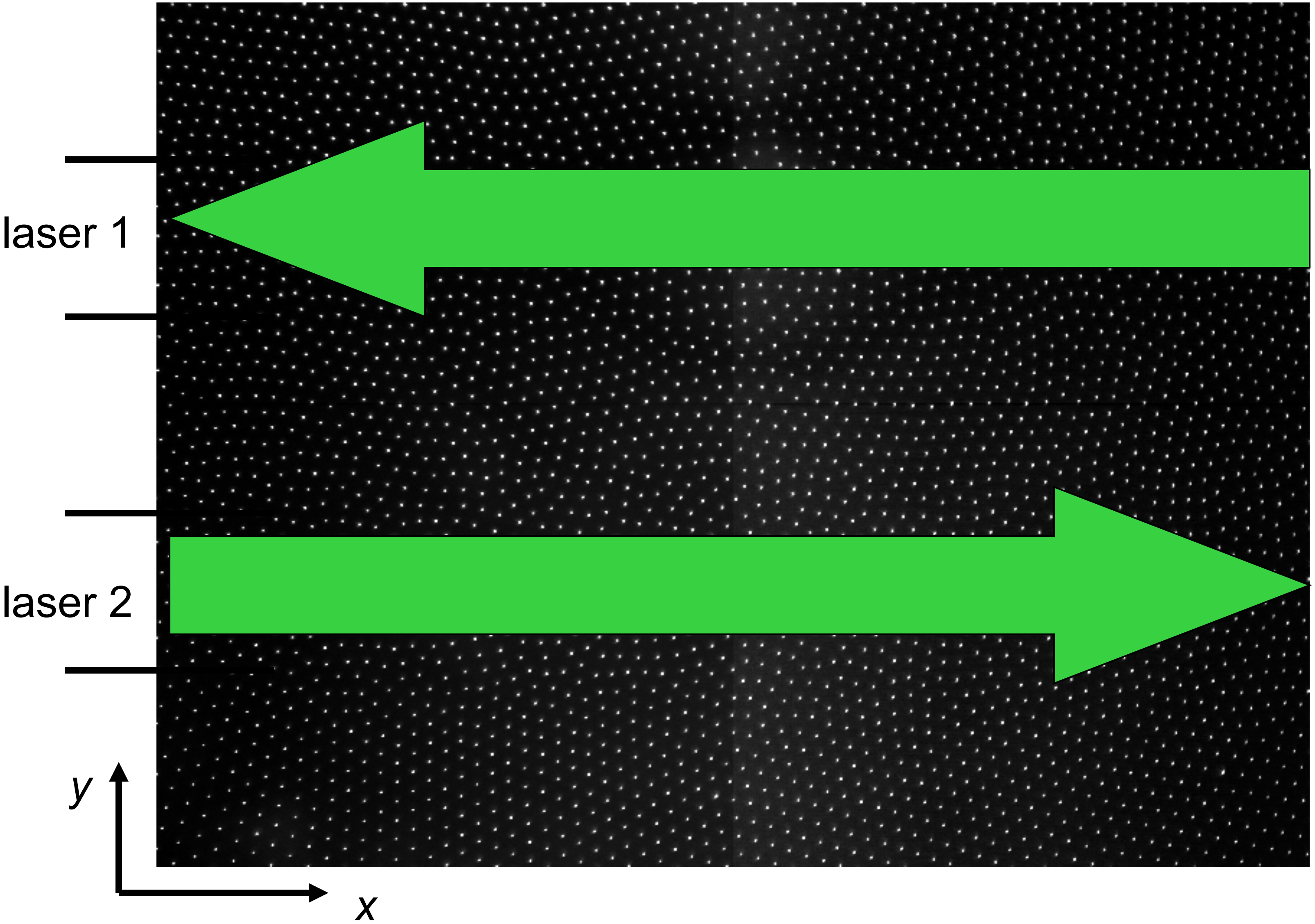
Yields time series for
positions & velocities of particles

Manipulating particles using Radiation Pressure Force

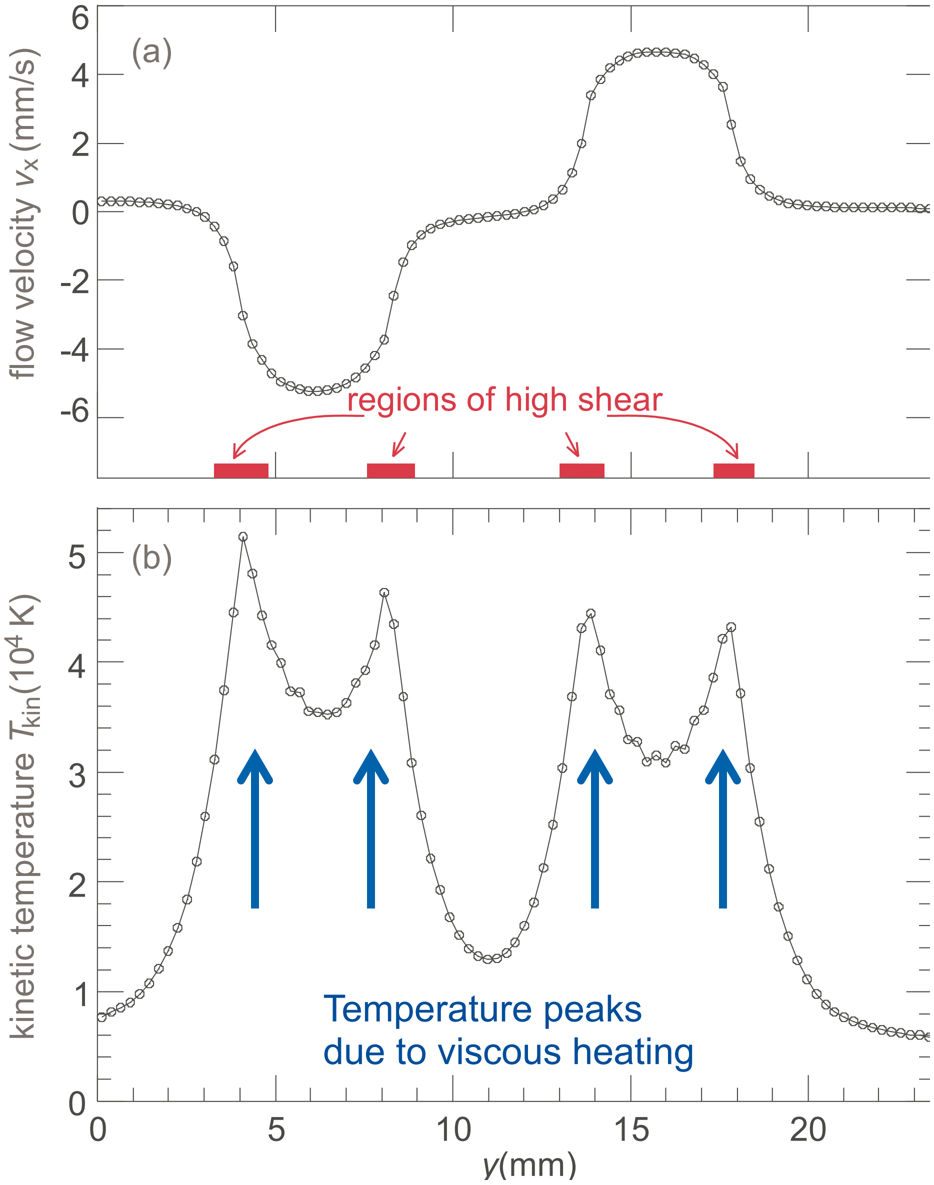


$$F \propto I_{laser}$$

Movie of the laser-driven shear flow



Results: profiles of flow velocity & temperature



flow velocity profile v_x

temperature profile T

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Confirm the conclusion

To confirm that the peaks are due to viscous heating:

- Calculate the Brinkmann number

$$\mathrm{Br} \equiv \frac{\eta}{\kappa}\frac{\Delta v_x^{\ 2}}{\Delta T}$$

- Confirm that it is of order unity

We require:

$$\left.\begin{array}{l}\Delta v_x \\ \Delta T\end{array}\right\}\ \text{Flow parameters}$$

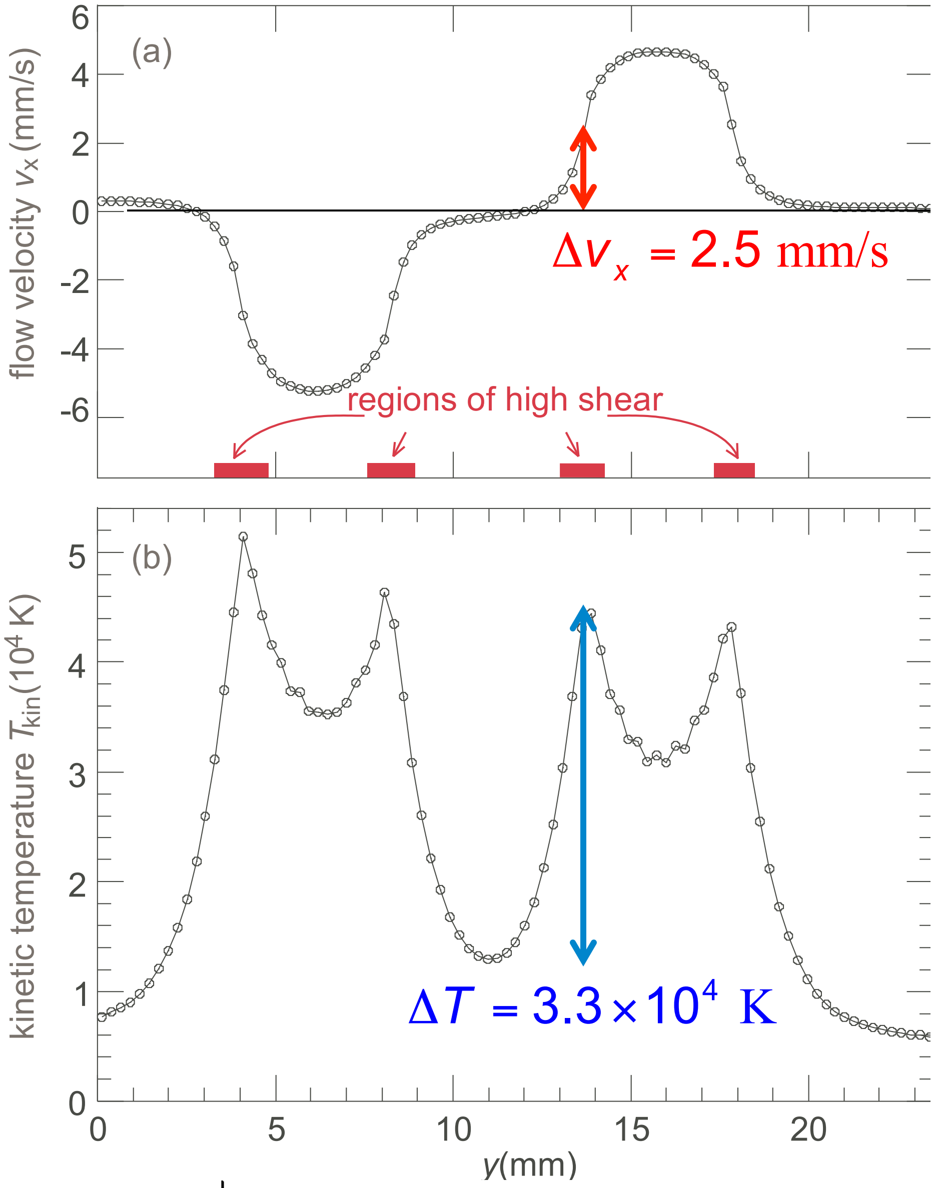
$$\left.\begin{array}{l}\eta \\ \kappa\end{array}\right\}\ \text{Material parameters}$$

Calculate the Brinkmann number

Measure the flow parameters:

Δv_x

ΔT



$\Delta v_x = 2.5$ mm/s

$\Delta T = 3.3 \times 10^4$ K

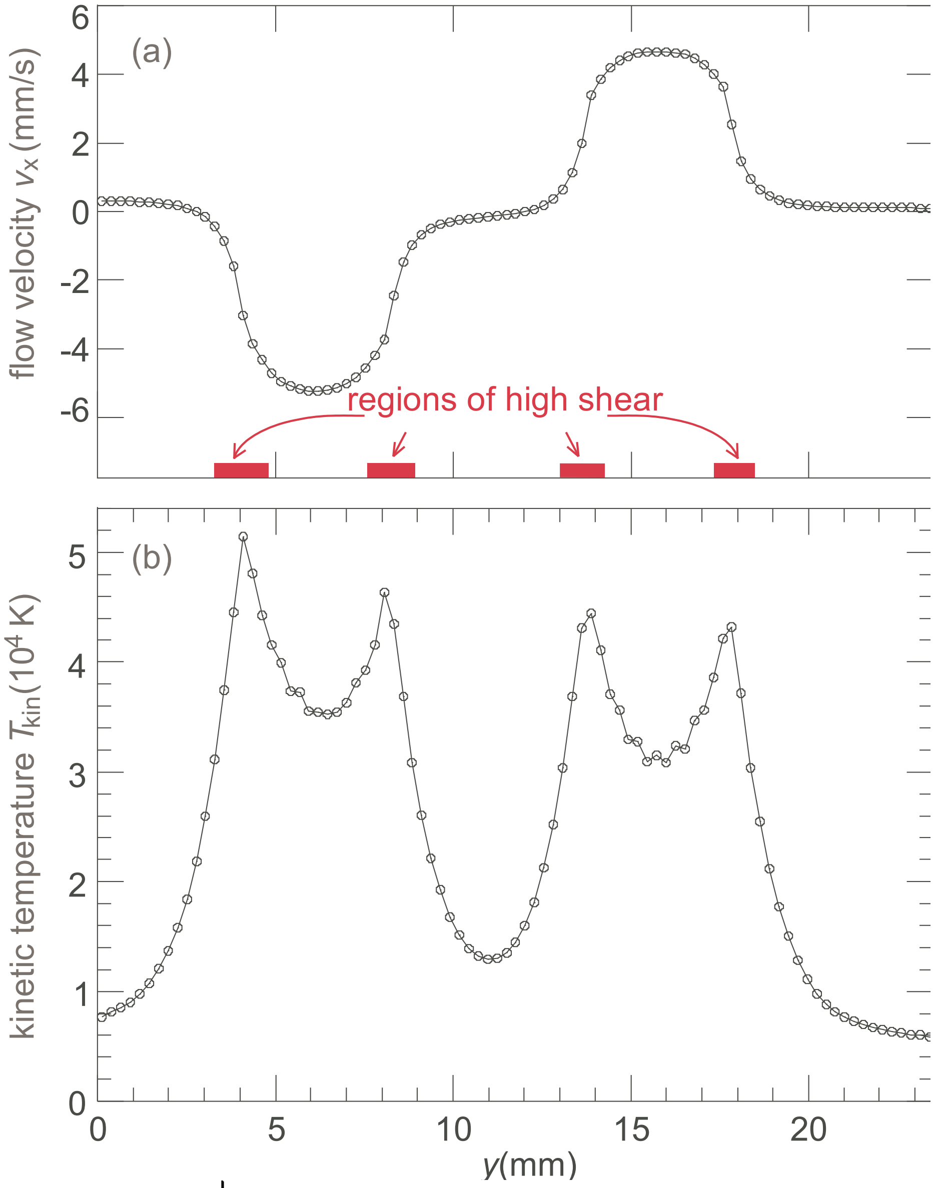
Calculate the Brinkmann number

Measure the material properties:

η

κ

We do this by fitting the profiles to the hydrodynamic equations.



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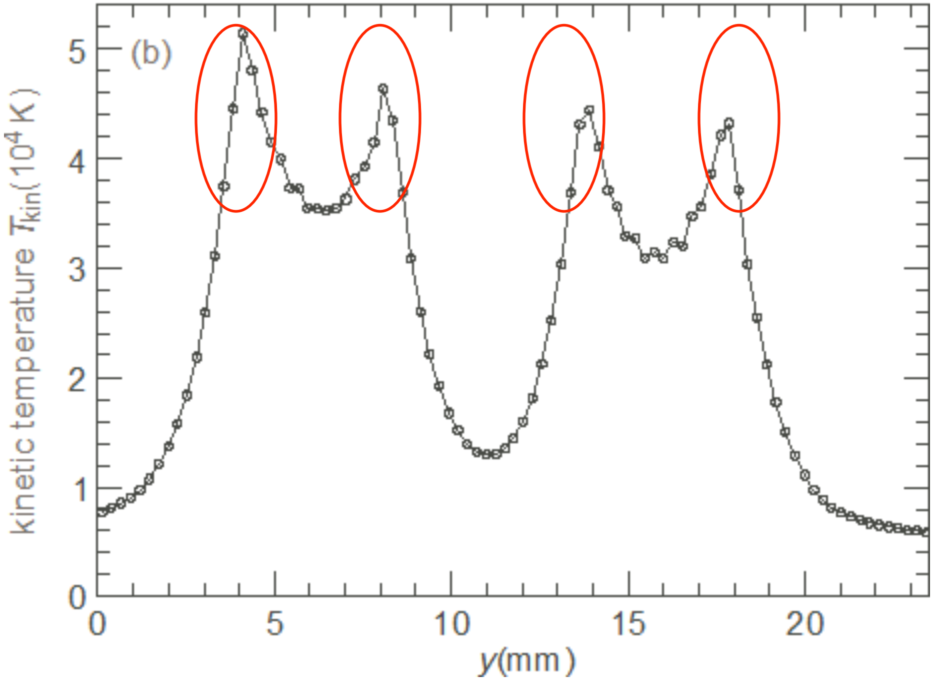
Calculate the Brinkmann number

We find the Brinkmann number

$$\mathrm{Br} \equiv \frac{\eta}{\kappa} \frac{\Delta v_x{}^2}{\Delta T} \approx 0.5$$

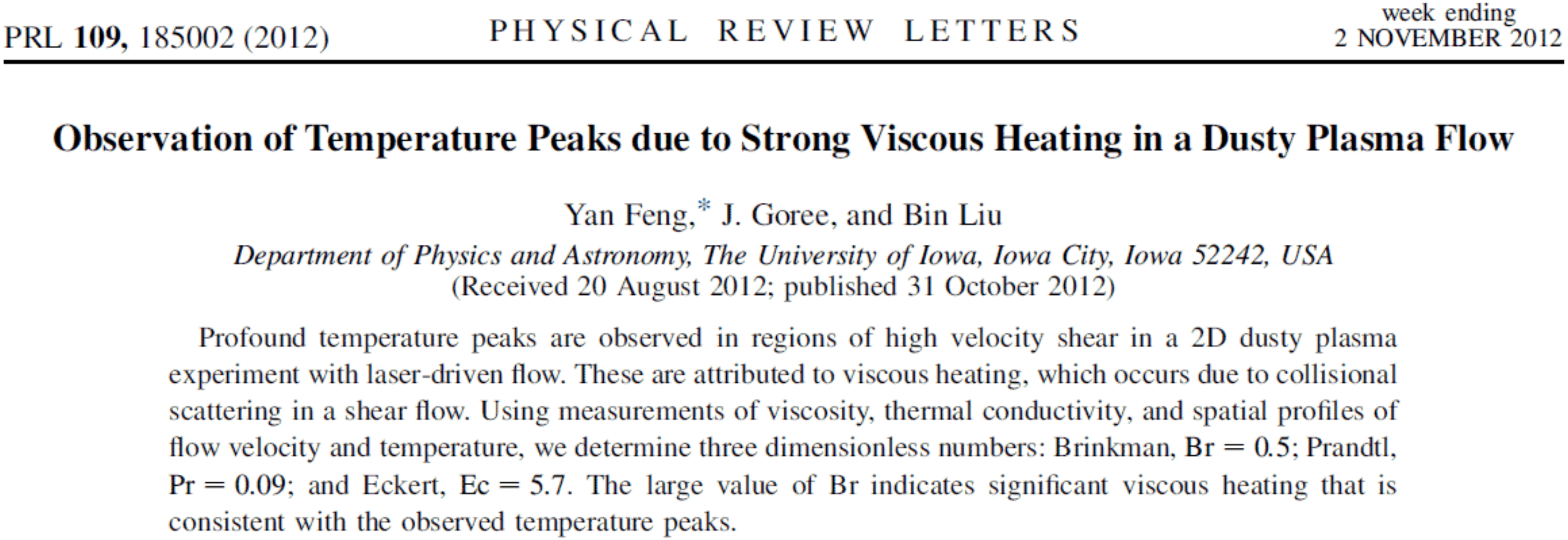
Since Br ~ unity, this result confirms the conclusion:

We have observed spatially-localized viscous heating.



Our temperature peaks are the elusive “hot spots” of viscous heating.


DUSTY
PLASMAS



PRL **109,** 185002 (2012) PHYSICAL REVIEW LETTERS week ending
2 NOVEMBER 2012

Observation of Temperature Peaks due to Strong Viscous Heating in a Dusty Plasma Flow

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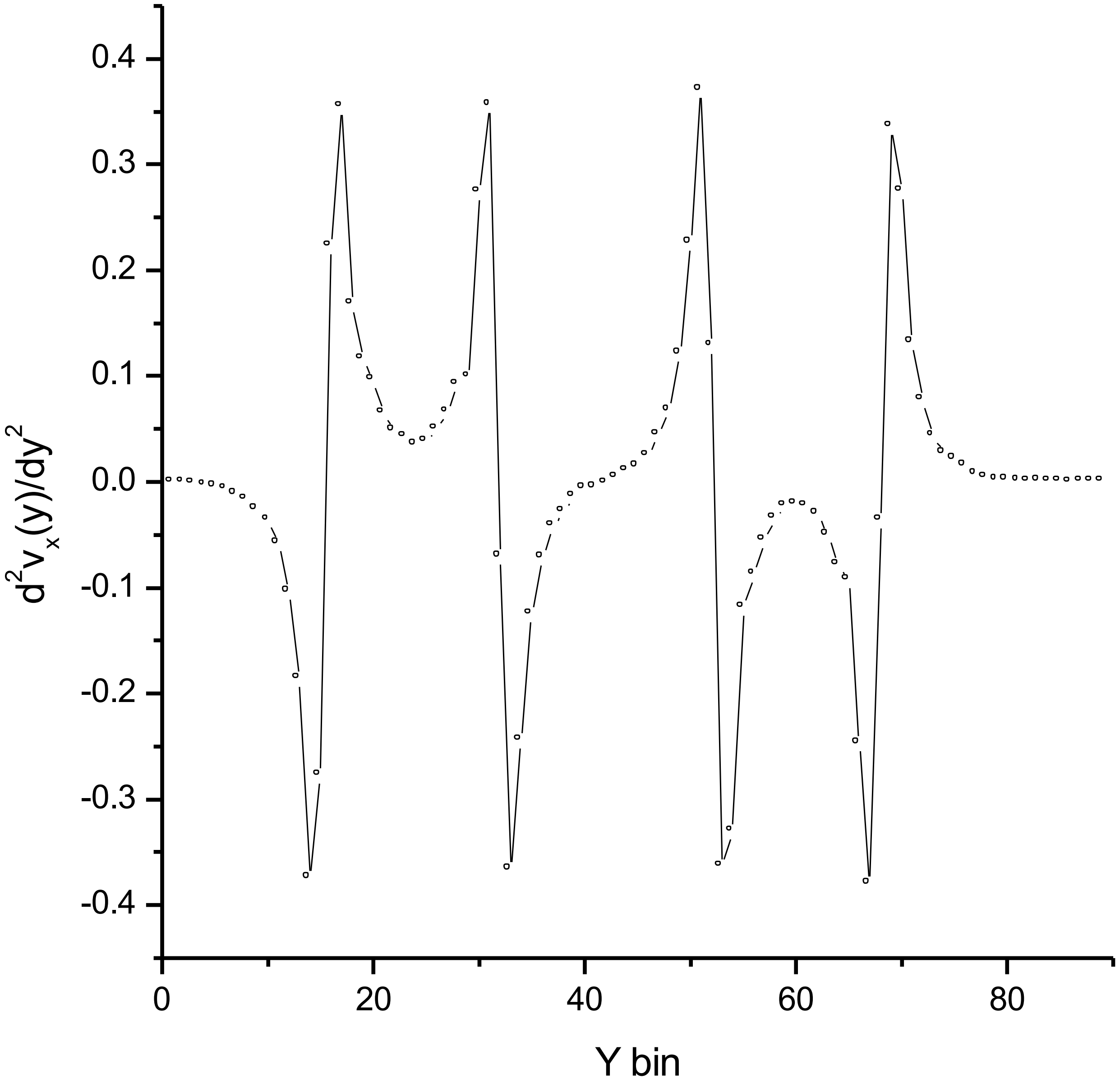
(Received 20 August 2012; published 31 October 2012)

Profound temperature peaks are observed in regions of high velocity shear in a 2D dusty plasma experiment with laser-driven flow. These are attributed to viscous heating, which occurs due to collisional scattering in a shear flow. Using measurements of viscosity, thermal conductivity, and spatial profiles of flow velocity and temperature, we determine three dimensionless numbers: Brinkman, Br = 0.5; Prandtl, Pr = 0.09; and Eckert, Ec = 5.7. The large value of Br indicates significant viscous heating that is consistent with the observed temperature peaks.

Second derivative of the flow velocity



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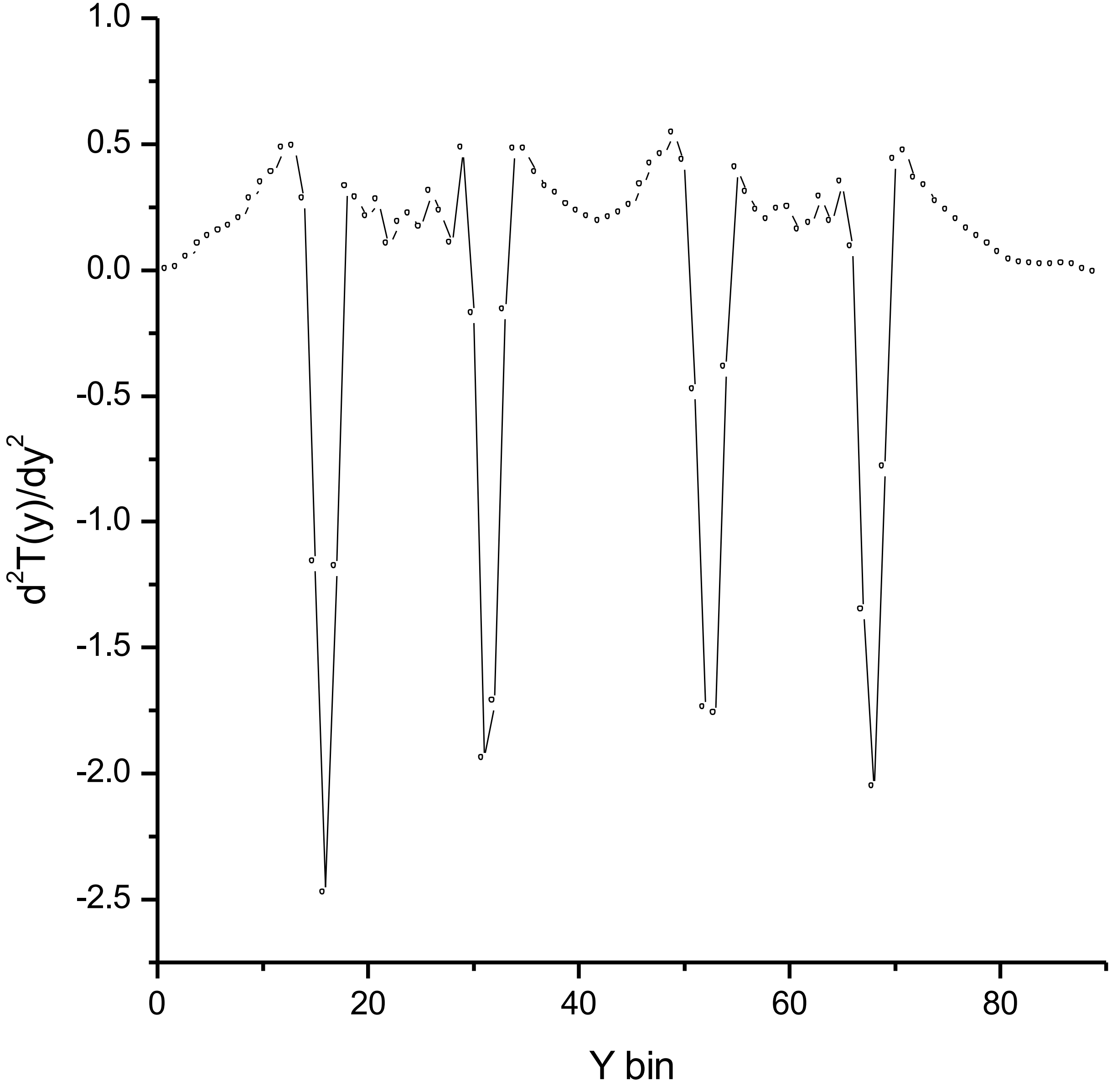
Second derivative of the kinetic temperature



Temperature dependent thermal conductivity κ(T)

All transport coefficients are temperature dependent, for example κ

$$\frac{\eta}{2\rho}\left(\frac{\partial v_x}{\partial y}\right)^2 + (1/\rho)\frac{\partial}{\partial y}\left(\kappa\frac{\partial T}{\partial y}\right) - \nu\,\overline{\mathbf{v}}^2 = 0$$

$$(\eta/2\rho)(\partial v_x/\partial y)^2 + (\kappa/\rho)\partial^2 T/\partial^2 y + (1/\rho)\frac{\partial T}{\partial y}\frac{\partial \kappa}{\partial y} - \nu\,\overline{\mathbf{v}}^2 = 0$$

Temperature variation is not extreme, so we can assume $\kappa = \kappa_0 + \alpha T$

$$(\eta/2\rho)(\partial v_x/\partial y)^2 + [(\kappa_0 + \alpha T)/\rho]\partial^2 T/\partial^2 y + \alpha(1/\rho)\left(\frac{\partial T}{\partial y}\right)^2 - \nu\,\overline{\mathbf{v}}^2 = 0$$

Temperature dependent thermal conductivity κ(T)

Minimizing the residual of the energy equation in no laser region

$$\text{Residual}\,(\kappa) = (\eta/2\rho)(\partial v_x/\partial y)^2 + [(\kappa_0 + \alpha T)/\rho]\,\partial^2 T/\partial^2 y + \alpha(1/\rho)\left(\frac{\partial T}{\partial y}\right)^2 - \nu v^2$$

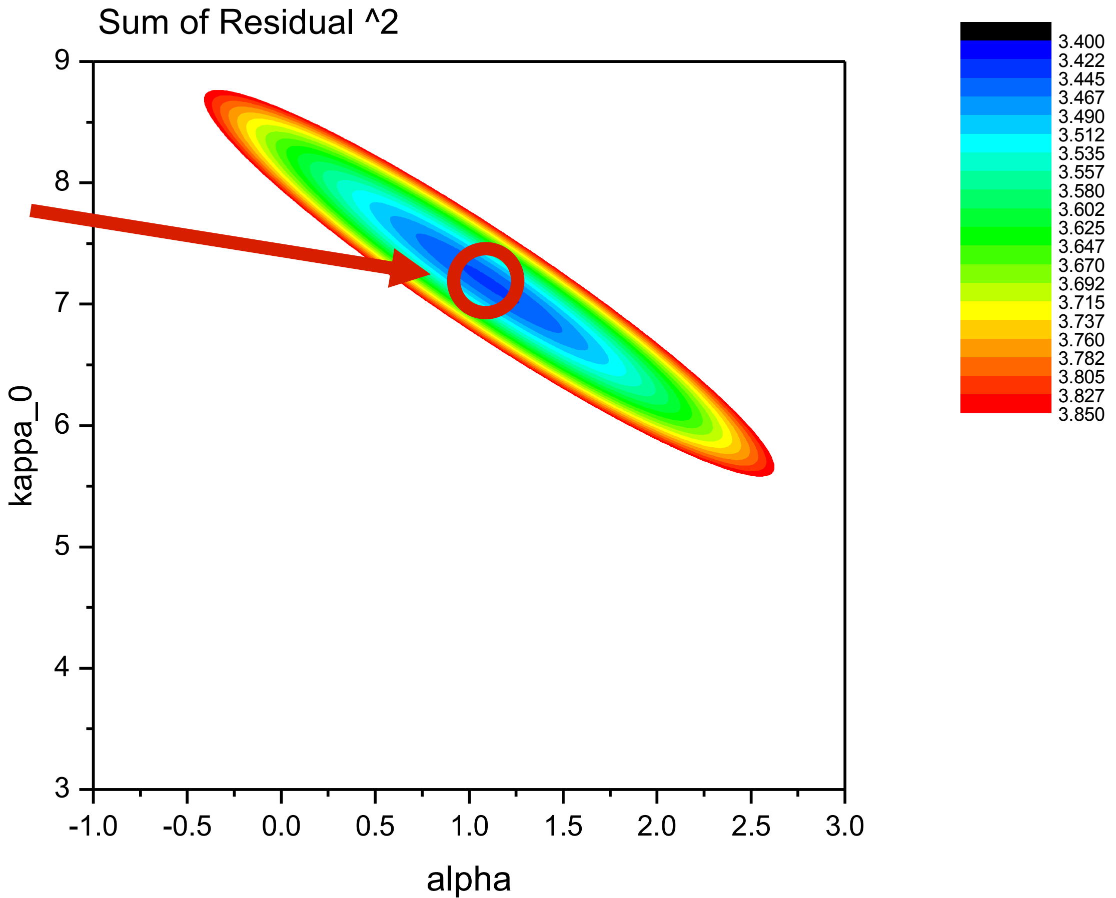
For the region between flows:

$$\frac{\kappa(T)}{\rho} = (7.17 + 1.12 v^2_{thermal\ y})\,\mathrm{mm^2/s}$$

$$v^2_{thermal\ y} \in [0.42, 0.84]$$

$$\frac{\kappa(T)}{\rho} \in [7.64, 8.11]$$

$$\frac{\kappa}{\rho} = 8.3\,\mathrm{mm^2/s}$$



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Transport coefficients using previous methods

	Our method of minimizing residuals	Previous methods *Nosenko and Goree, PRL (2004)* *Nosenko et al., PRL (2008)*
η/ρ		
κ/ρ		
$\kappa(T)/\rho$		

The transport coefficients using our method are consistent with those using different methods.

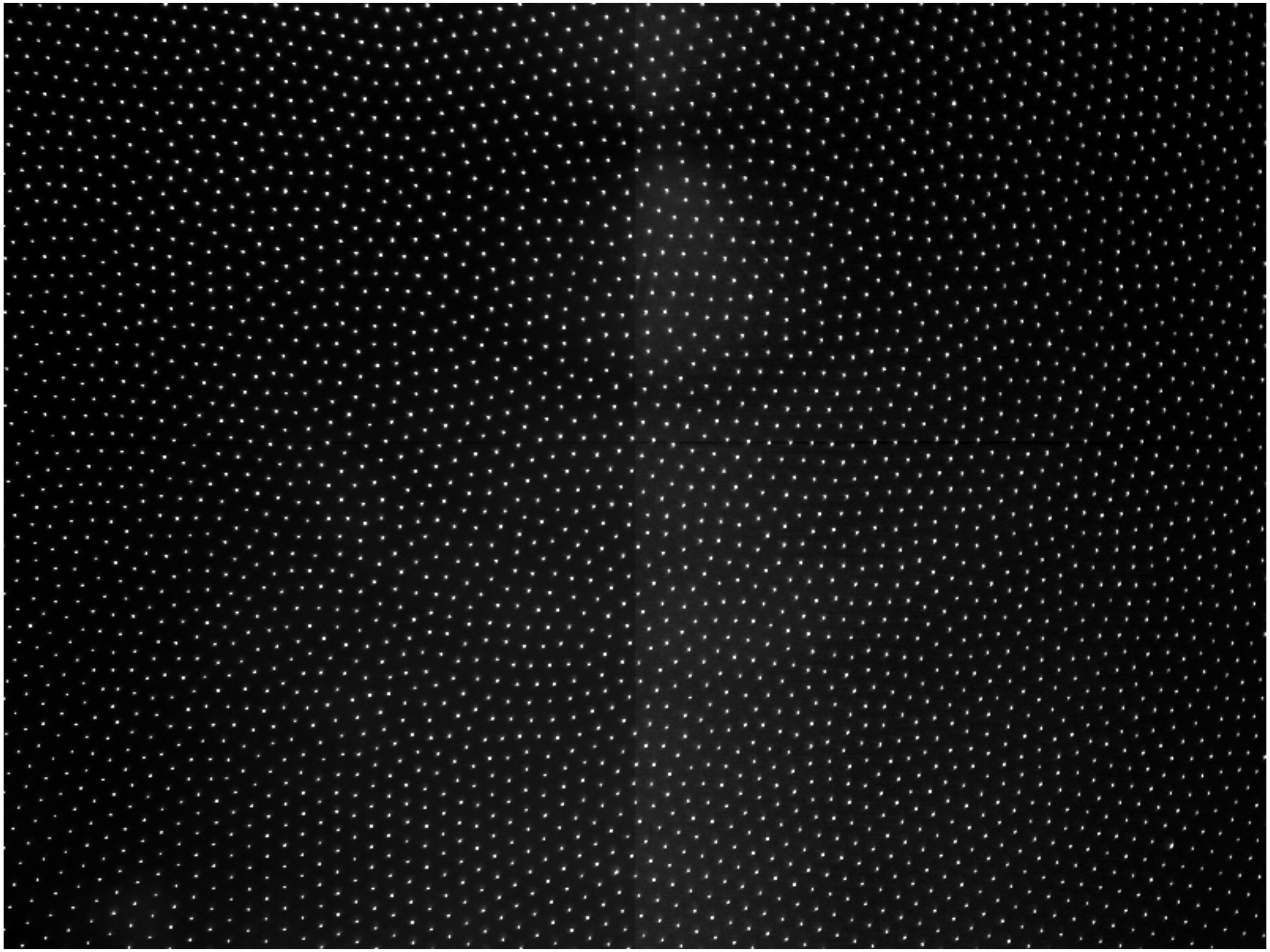
Validity of transport coefficients in 2D systems

Some theorists argue that transport coefficients in 2D system does not exists, due to non-converging integrals of the Green-Kubo relation for **uniform** system.

For **non-uniform** systems with a gradient, the analogous test would be to determine whether transport coefficients is independent of the scale length.

But, our experiment cannot give a clear answer to this question.

Movie of the laser-driven flow in dusty plasma



laser 1

analyzed region

laser 2

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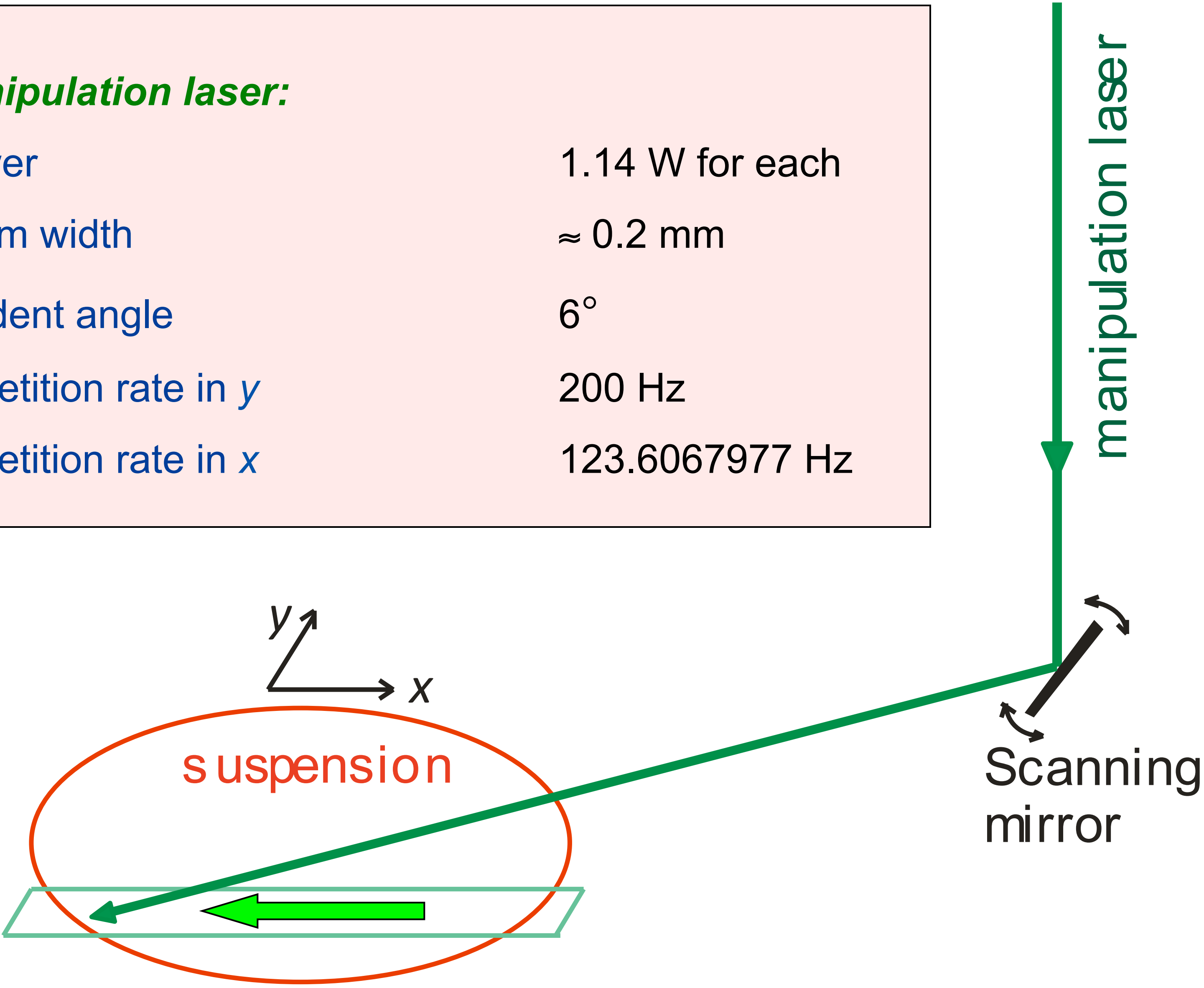
Parameters for laser manipulation

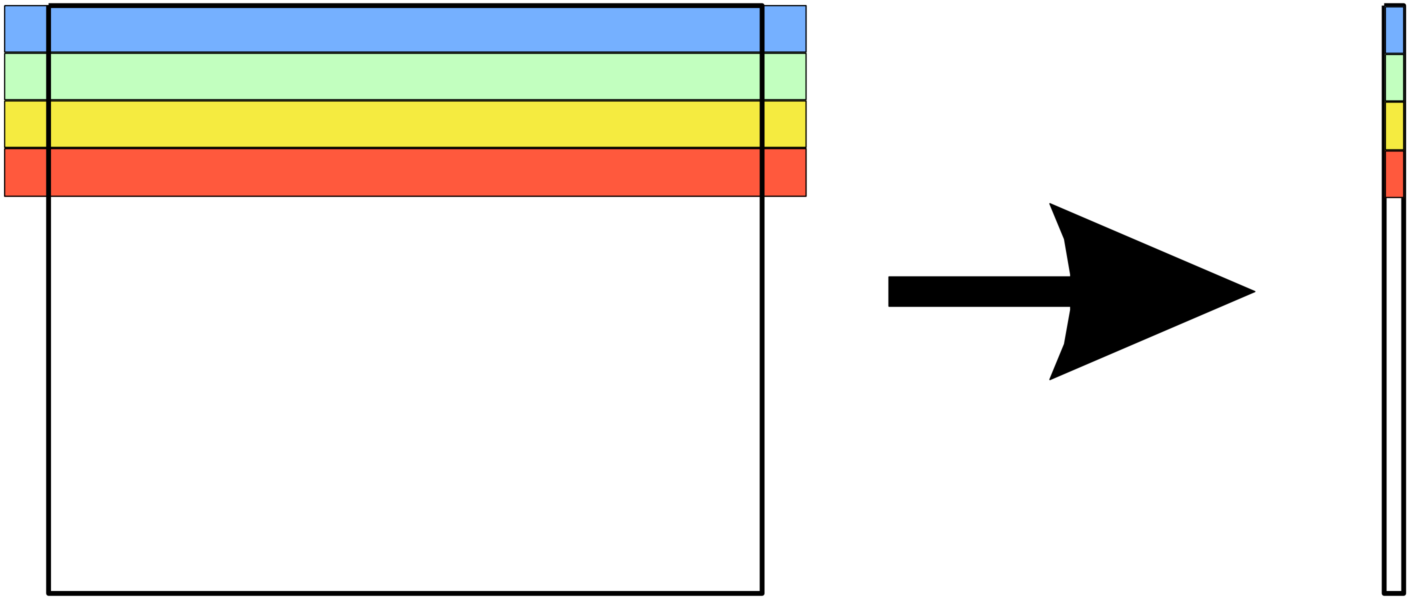
Manipulation laser:

Power	1.14 W for each
Beam width	≈ 0.2 mm
Incident angle	6°
Repetition rate in *y*	200 Hz
Repetition rate in *x*	123.6067977 Hz

Improve



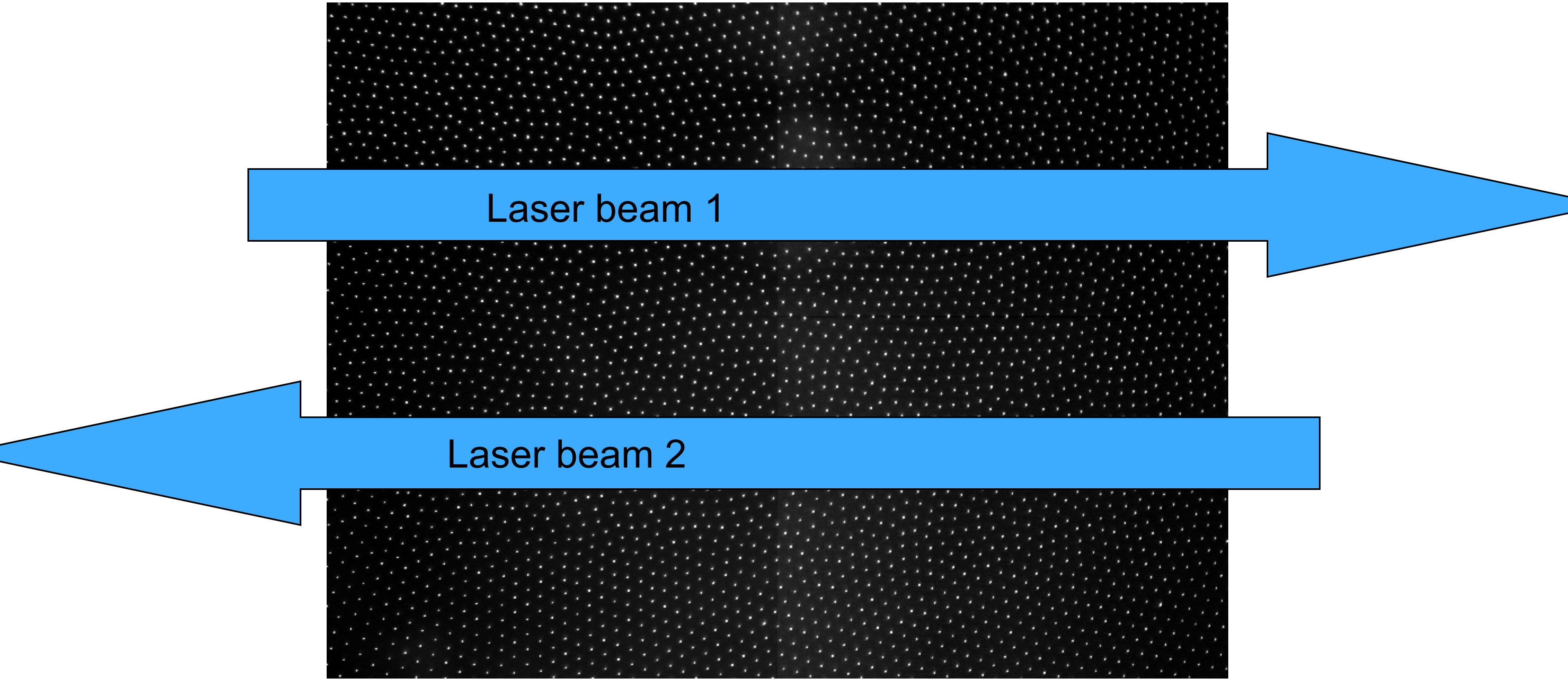
Data analysis: characterize nonuniformity in *y*



(1) Divide the field of view into 89 bins (of width $\Delta d = r_{ws}$)

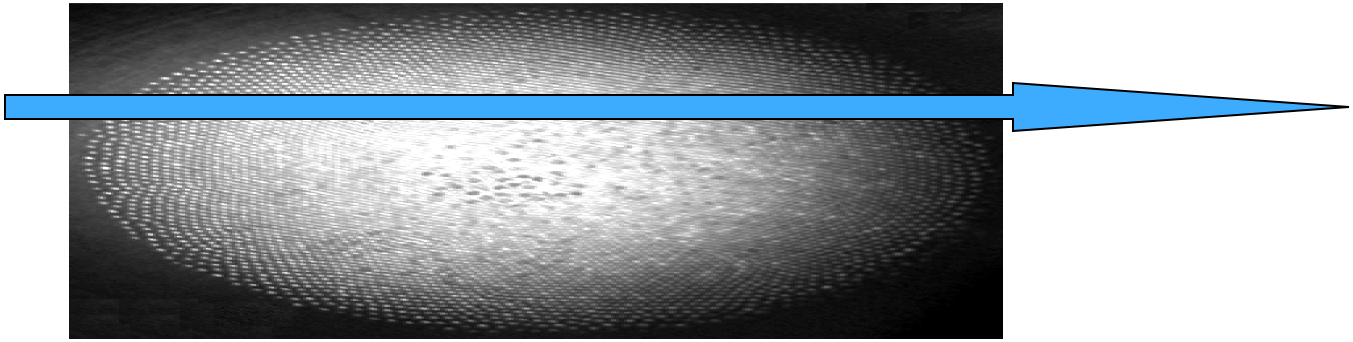
(2) For each frame, **bin** the information (e.g. velocity) of all particles in a stripe

- Each particle contributes to two stripes, according to a “weight”
- weighting factor: Cloud-in-Cell algorithm



Scheme for producing counterpropagating flows

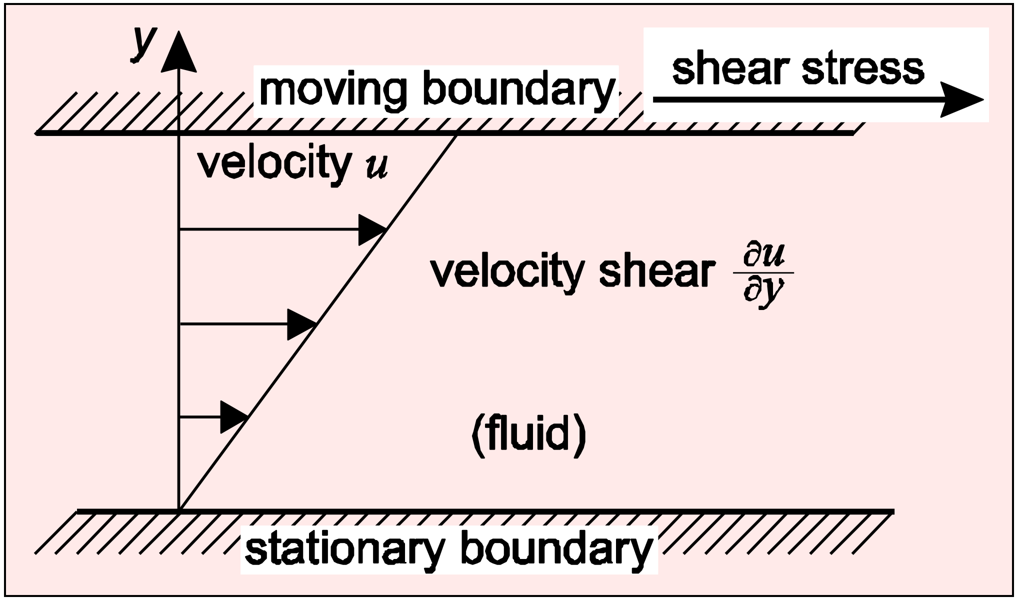
F_laser is zero everywhere except within two stripes



What is viscosity?

Viscosity (η)

A measure of how a fluid flows under the shear stress.



$$\eta = \text{shear stress} \Big/ \frac{\partial u}{\partial y}$$

$$\text{kinematic viscosity} = \frac{\eta}{\rho}$$

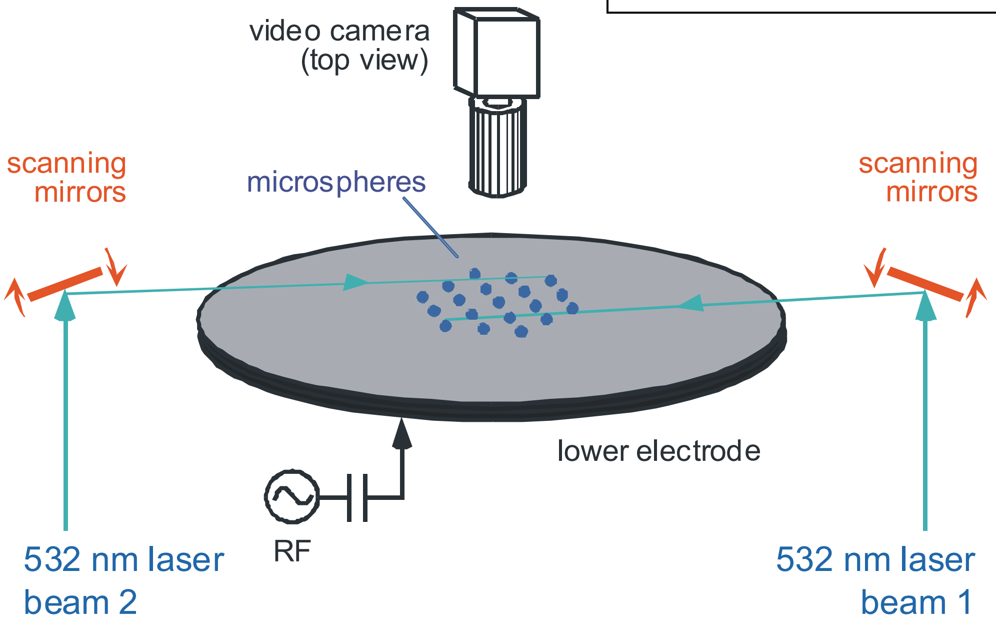
Measuring η with shear: Setup

Experiment setup

V. Nosenko and J. Goree, PRL (2004)

Two laser beams:

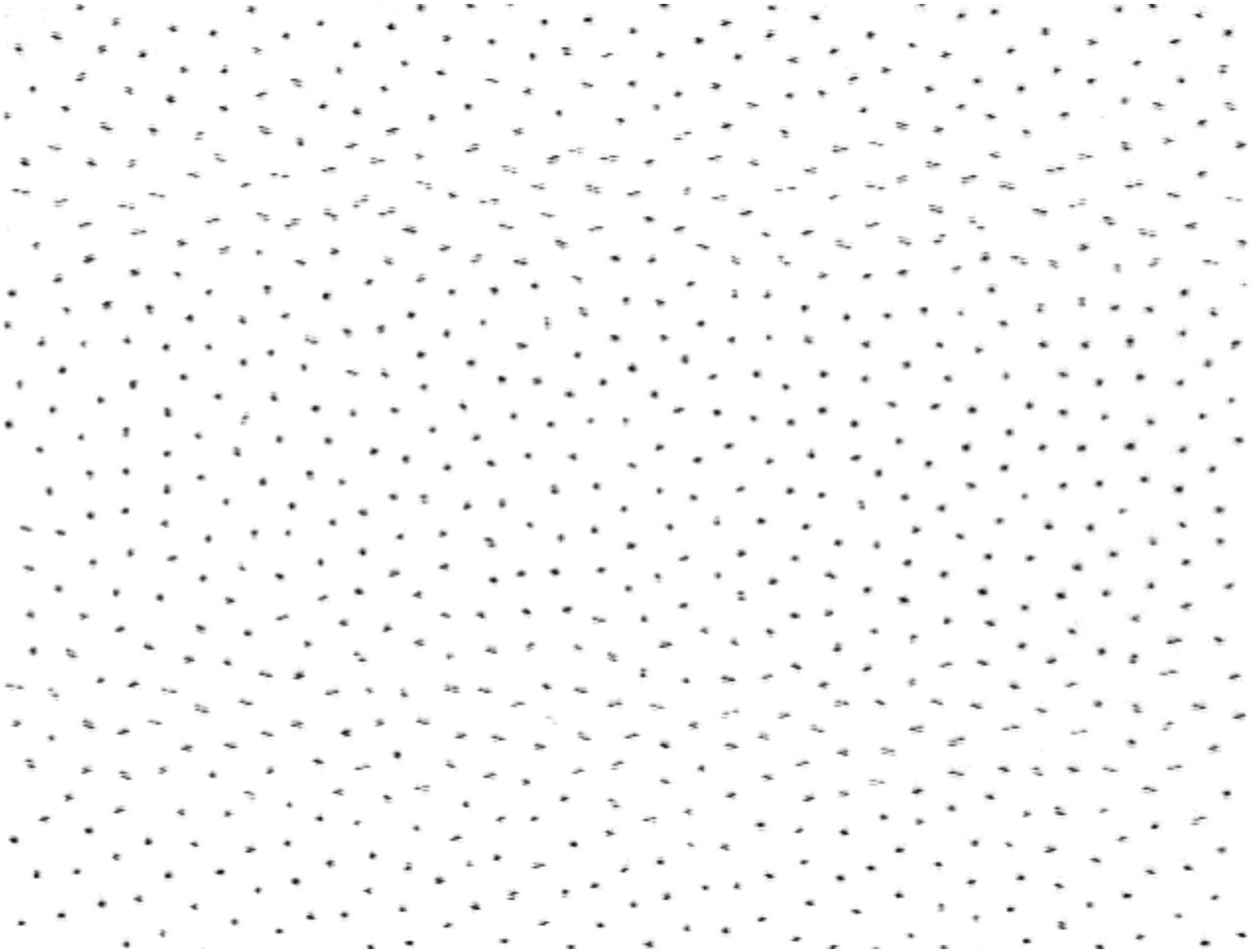
apply shear by two counter propagating flows



shear-induced melting

Measuring η with shear: Movie

Movie of suspension with high shear



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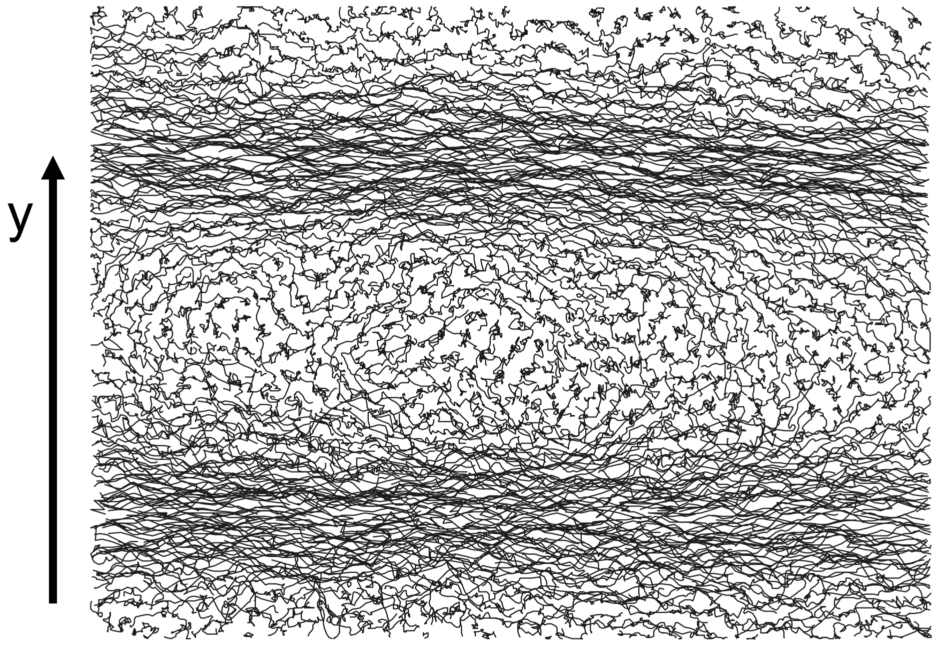
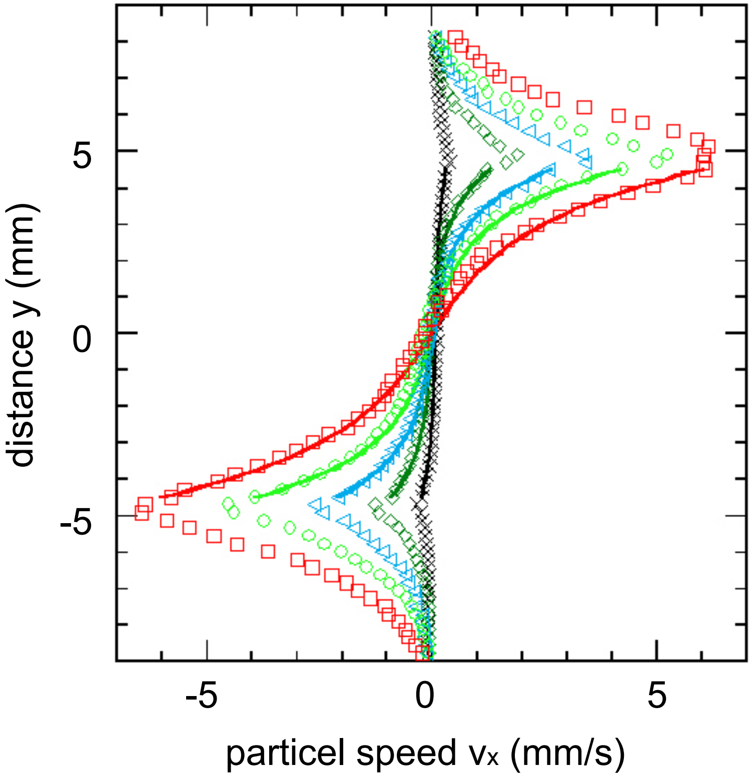
Measuring η with shear

Particle trajectories





256 frames averaged in steady-state regime

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Measuring η with shear: Result

Model the velocity profile to continuum, using Navier-Stokes equation

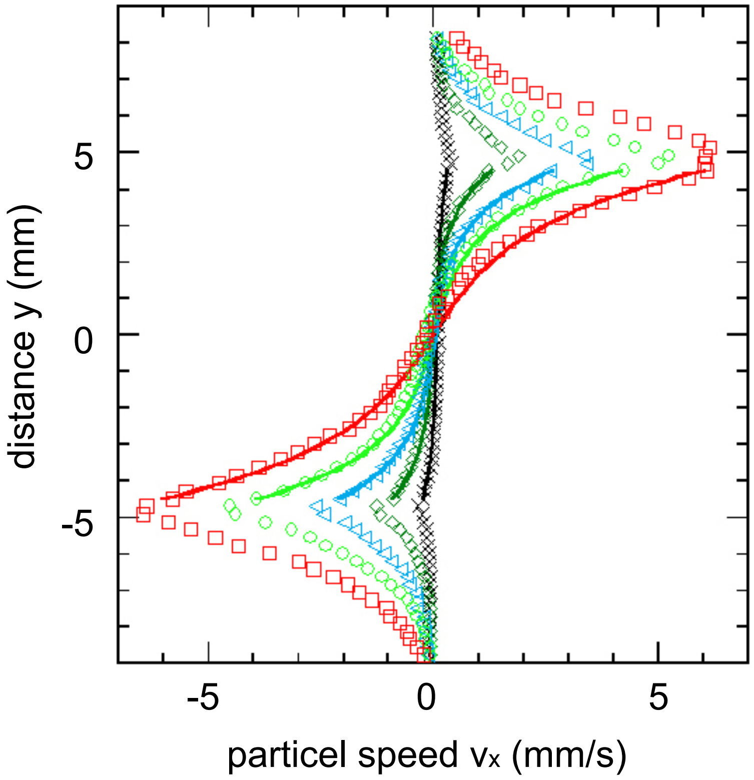
$$\cancel{\frac{\partial \mathbf{v}}{\partial t}} + \cancel{(\mathbf{v}\nabla)\mathbf{v}} = \cancel{\frac{\nabla p}{\rho}} + \frac{\eta}{\rho}\nabla^2\mathbf{v} + \left[\frac{\zeta}{\rho}+\frac{\eta}{3\rho}\right]\cancel{\nabla(\nabla\cdot\mathbf{v})} - \nu_{gas}\mathbf{v}$$

$$\frac{\eta}{\rho}\frac{\partial^2 v_x(y)}{\partial^2 y} - \nu_{gas} v_x(y) = 0$$

$$v_x(y) = V\ sinh(\alpha y)/sinh(\alpha h)$$

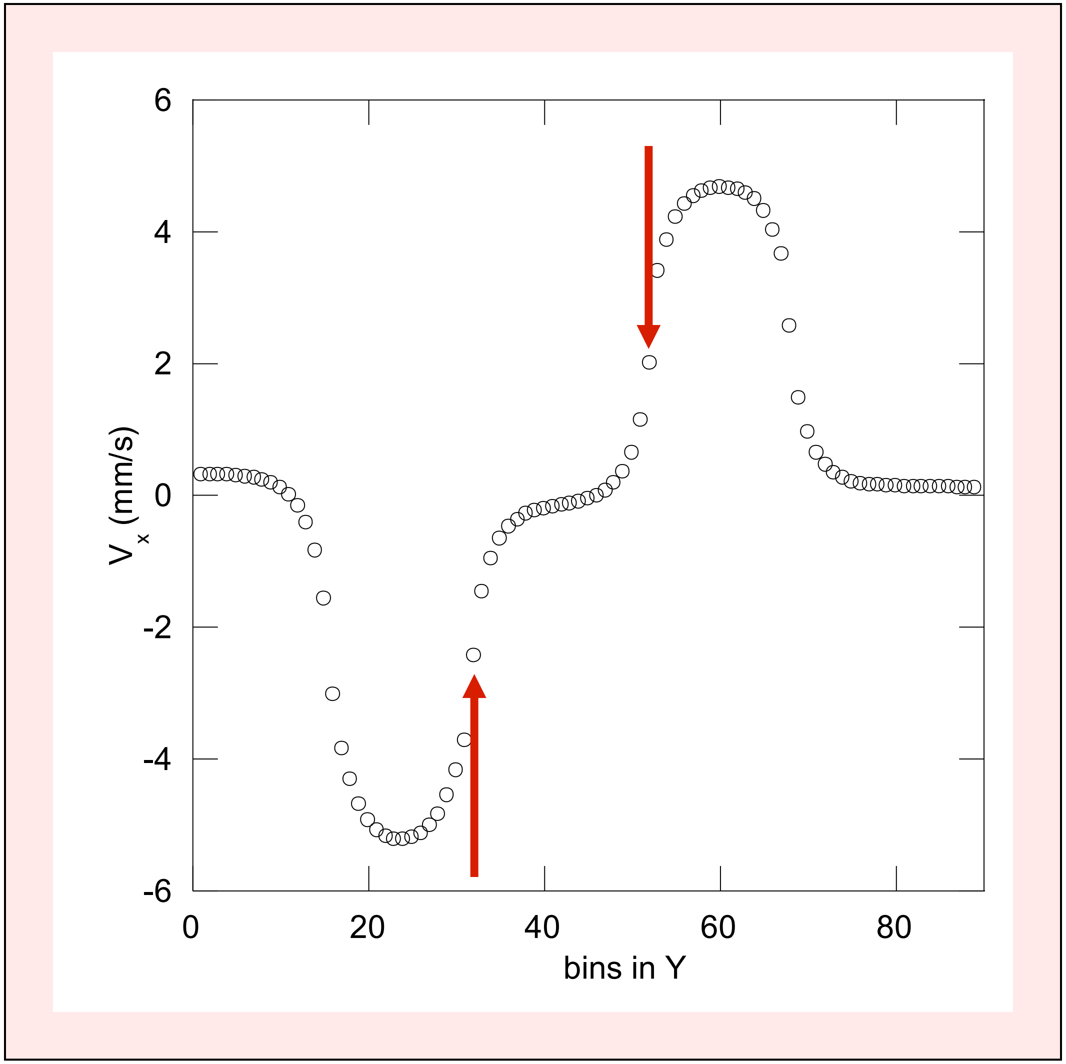
$$\alpha = \sqrt{\nu_{gas}\rho/\eta}$$

h is the width of between flows



Shear viscosity determination in our experiment

Flow velocity profile $<v_x>$



$$v_x(y) = V \sinh(\alpha y)/\sinh(\alpha h)$$

$$\alpha = \sqrt{\nu_{gas}\rho/\eta}$$

h is the width of between flows

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Shear viscosity determination in our experiment

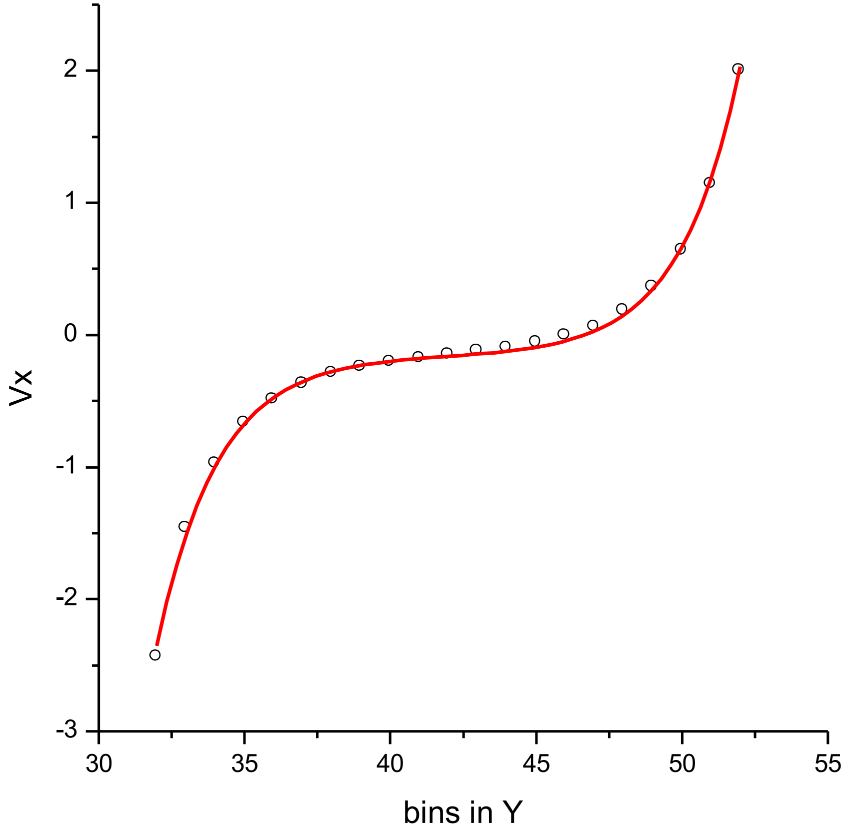
$$v_x(y) = V \sinh(\alpha y) / \sinh(\alpha h)$$

The fitting result of $\alpha = \sqrt{\nu_{gas}\rho/\eta}$

is not affected by the input value of h

The determined viscosity is

$\eta/\rho = 0.8\ mm^2/s$



What is thermal conductivity?

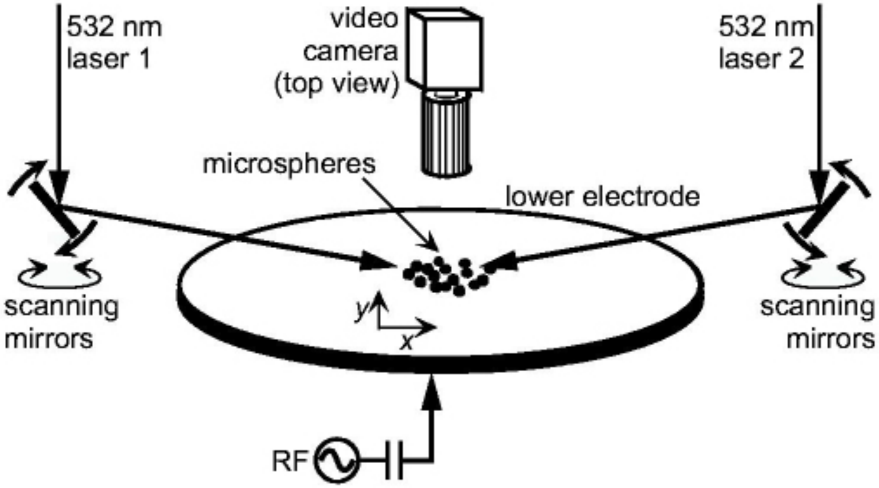
Thermal conductivity (κ)

The property of a material’s ability to conduct heat.

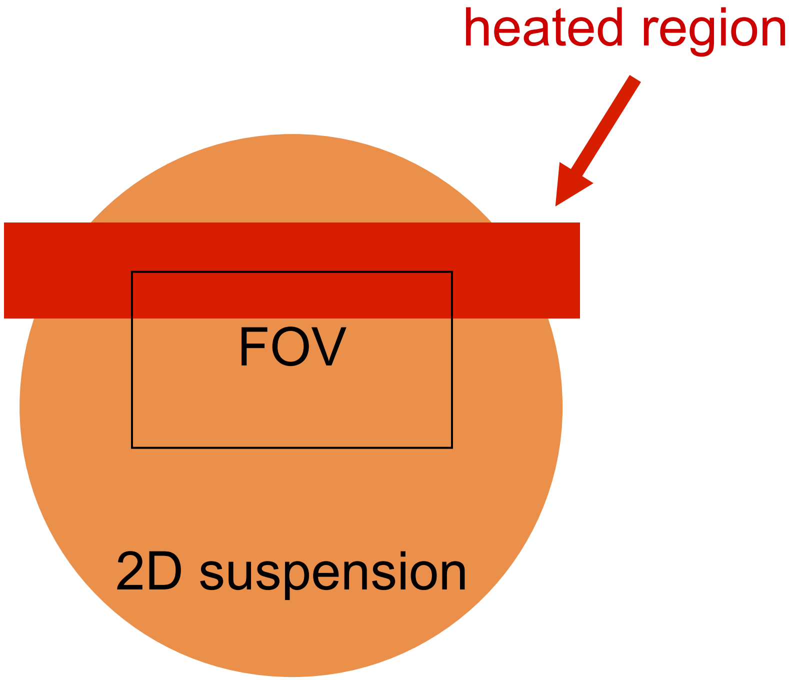
Heat flux $\dot{q} = \kappa \nabla T$

Measuring κ in dusty plasmas: Setup

Experiment setup



Nosenko et al., PRL (2008)

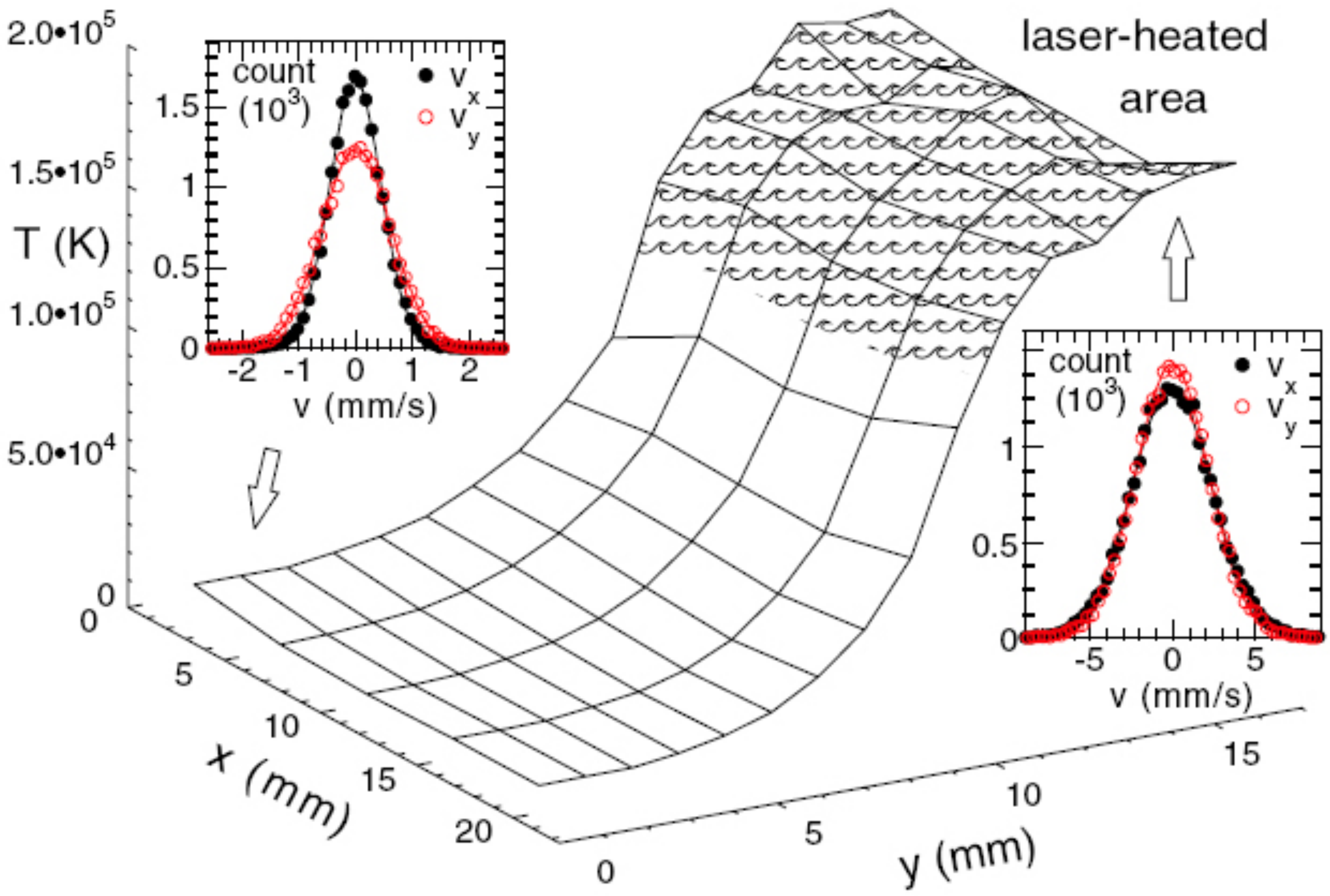


heated region

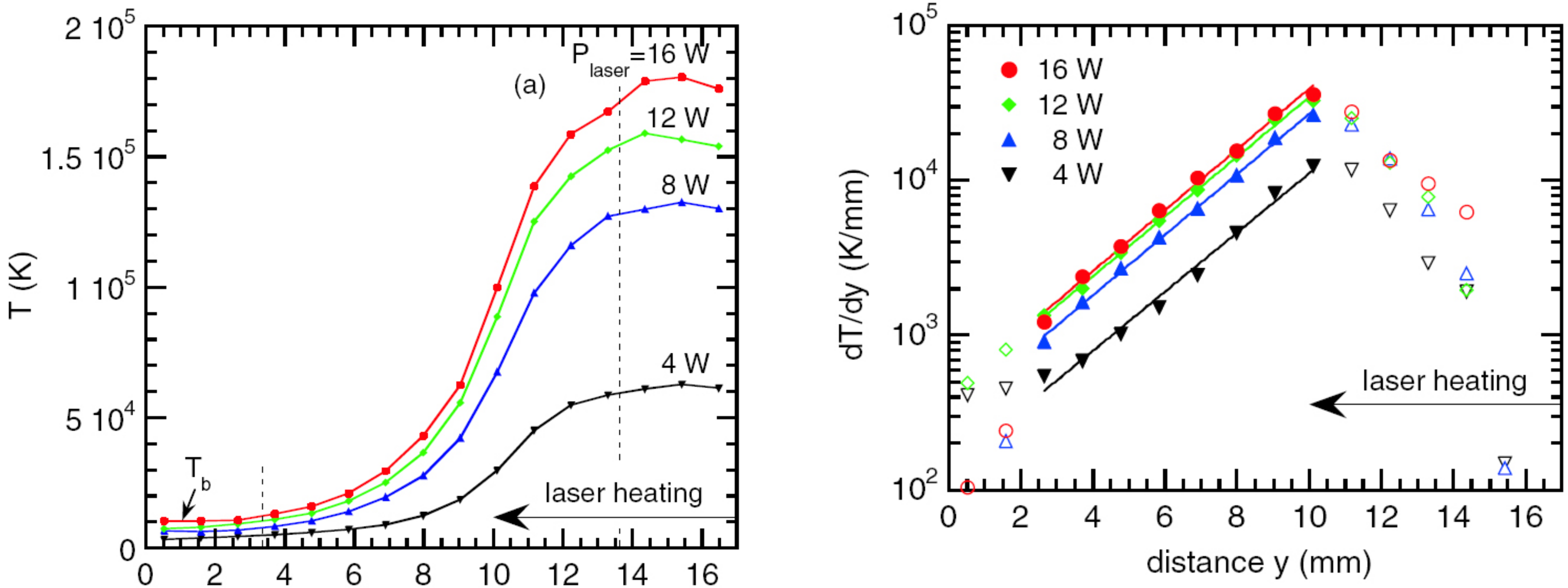
FOV

2D suspension

Measuring κ in dusty plasma



Measuring κ in dusty plasma



Outside laser-heated region, the temperature is about exponential decay.

$$dT/dy \propto \exp(y/L_{heat})$$

Measuring κ in dusty plasma

Model the dusty plasma suspension to continuum, using fluid equation

$$cn\cancel{\mathbf{v}\cdot\nabla}\boldsymbol{T}k_B = \nabla\cdot(\kappa\nabla\boldsymbol{T}) - 2\nu n(T - T_0)k_B + \cancel{S}_{viscous}$$

$$\nabla\cdot(\kappa\nabla\boldsymbol{T}) = 2\nu n(T - \cancel{T_0})k_B$$

$$dT/dy \propto \exp(y/L_{heat})$$

$$\kappa = 2\nu n L_{heat}^2 k_B$$

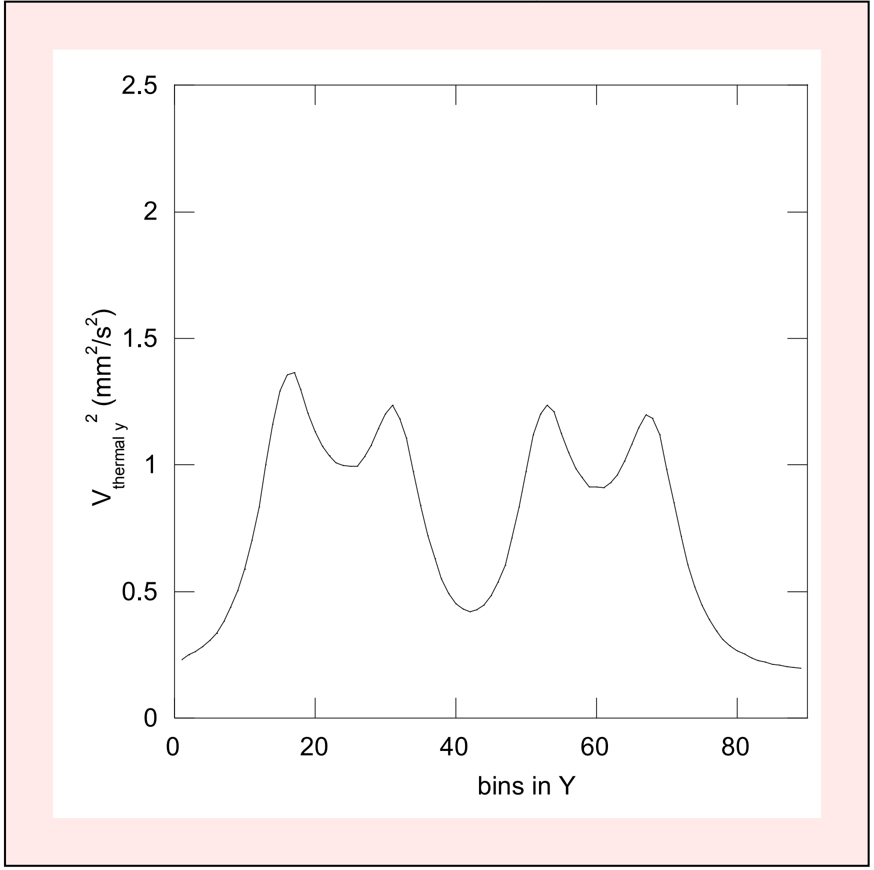
Thermal diffusivity χ is

$$\chi = \kappa/cn = 2\nu L_{heat}^2 k_B/c = 2\nu L_{heat}^2 \cancel{k_B}$$

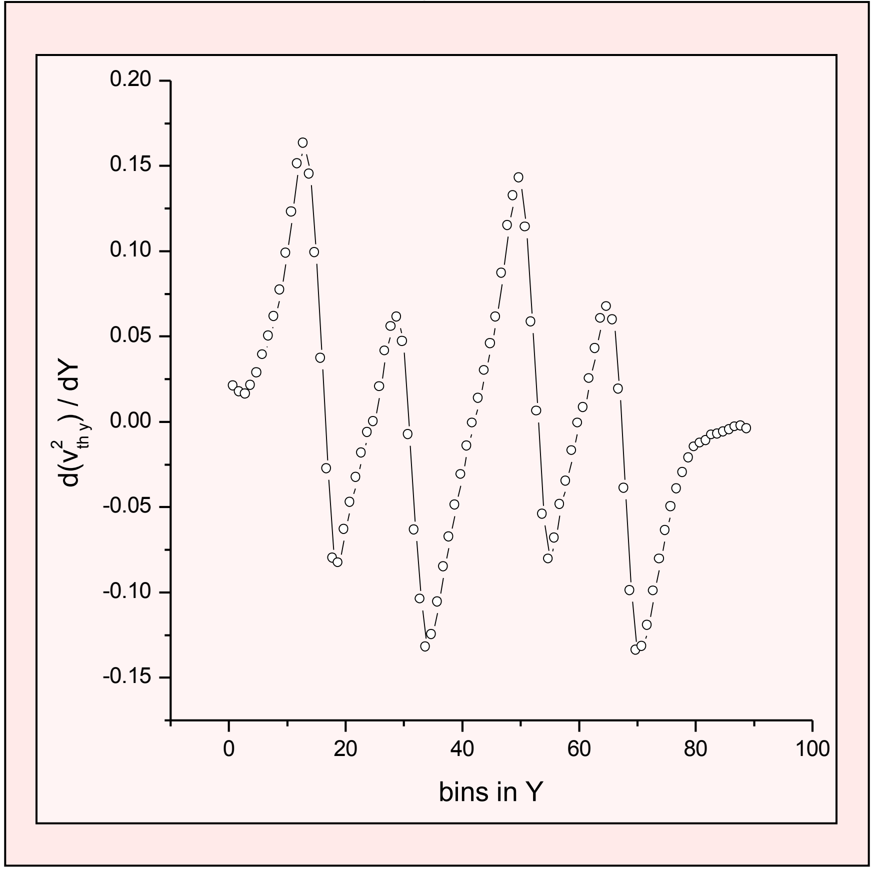
Thermal conductivity determination in our experiment

We use KT_y as the kinetic temperature to determine κ.

$<v_{thermal\ y}^2>$



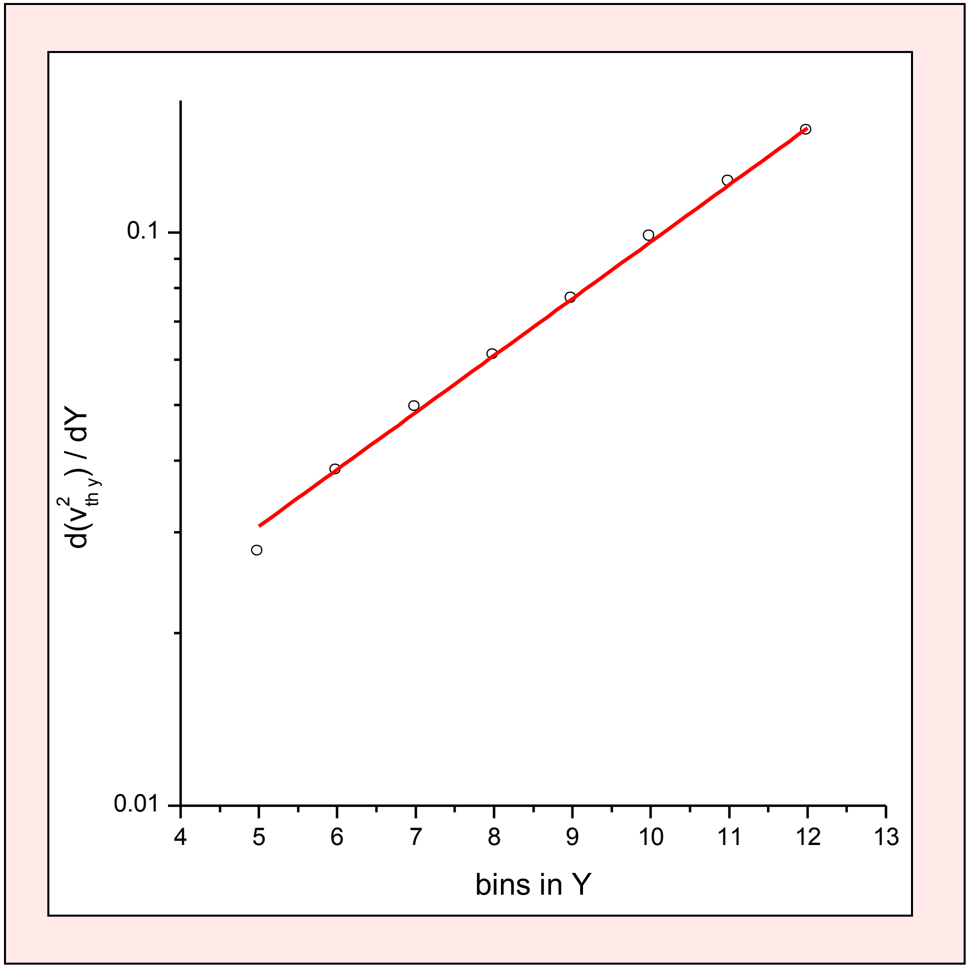
$d<v_{thermal\ y}^2>/dY$



Thermal conductivity determination in our experiment

Thermal diffusivity

$$\chi = \kappa / cn = 2\nu L_{heat}^2$$



Based on our fitting results:

$$L_{heat} = 4.45\, bins$$

The determined thermal diffusivity is

$$\chi = \kappa / cn = 2\nu L_{heat}^2 = 7.4\ mm^2 / s$$

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Equation of energy exchange of fluids

For 2D fluids which has a symmetry in the *x* direction, the energy equation is:

$$T\frac{DS}{Dt} = \frac{1}{2}\frac{\eta}{\rho}\left(\frac{\partial v_x}{\partial y}\right)^2 + \frac{\kappa}{\rho}\frac{\partial^2}{\partial y^2}(T)$$

energy dissipation per unit mass

Batchelor, An Introduction to Fluid Dynamics (1979)

viscous heat

heat conduction

For our dusty plasma, we also need to consider:

- The dissipation due to gas damping
- The energy input due to laser manipulation

Dissipation due to gas damping

Gas drag friction coefficient

$\nu = F / m_{dust}\, v_{dust} = 2.7\ \mathrm{s}^{-1}$

Gas damping force

$F = \nu\, m_{dust}\, v_{dust}$

Energy dissipation due to gas damping per unit mass

$P = F\, v_{dust} / m_{dust} = \nu\, v^2_{dust}$

Hydrodynamic Equations

Navier-Stokes equation

momentum

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\nabla)\mathbf{v} = -\frac{\nabla p}{\rho} + \frac{1}{\rho}\nabla \cdot (\eta \nabla \mathbf{v}) + \left[\frac{\zeta}{\rho} + \frac{\eta}{3\rho}\right]\nabla(\nabla \cdot \mathbf{v})$$

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\nabla)\mathbf{v} = -\frac{\nabla p}{\rho} + \frac{\eta}{\rho}\nabla^2 \mathbf{v} + \left[\frac{\zeta}{\rho} + \frac{\eta}{3\rho}\right]\nabla(\nabla \cdot \mathbf{v})$$ force per unit mass

energy

$$\frac{DH}{Dt} = \frac{1}{\rho}\frac{Dp}{Dt} + \frac{1}{\rho}\nabla \cdot (\kappa \nabla \mathrm{T}) + \frac{1}{\rho}\Phi$$

energy dissipation per unit mass

Batchelor, An Introduction to Fluid Dynamics (1979)

Forces in the momentum equation

Delete?

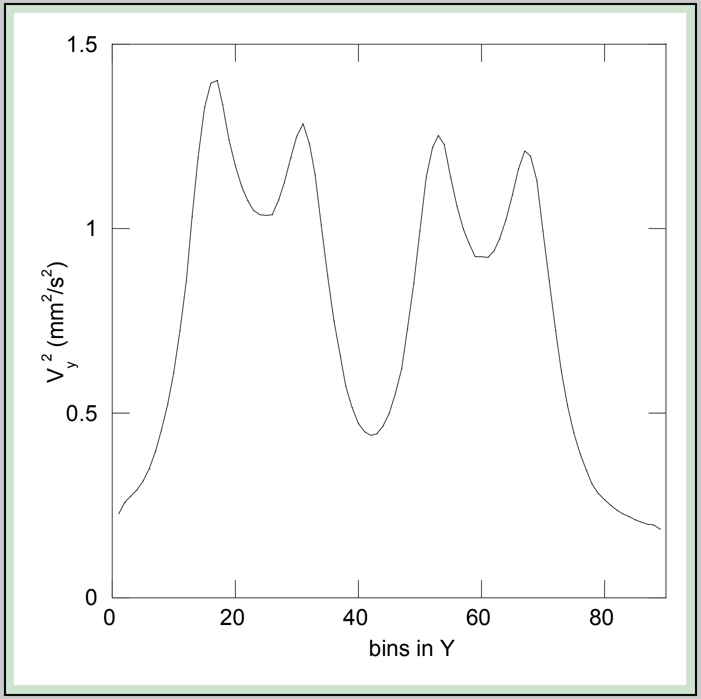
Momentum equation

The main horizontal forces acting on the dust particles:

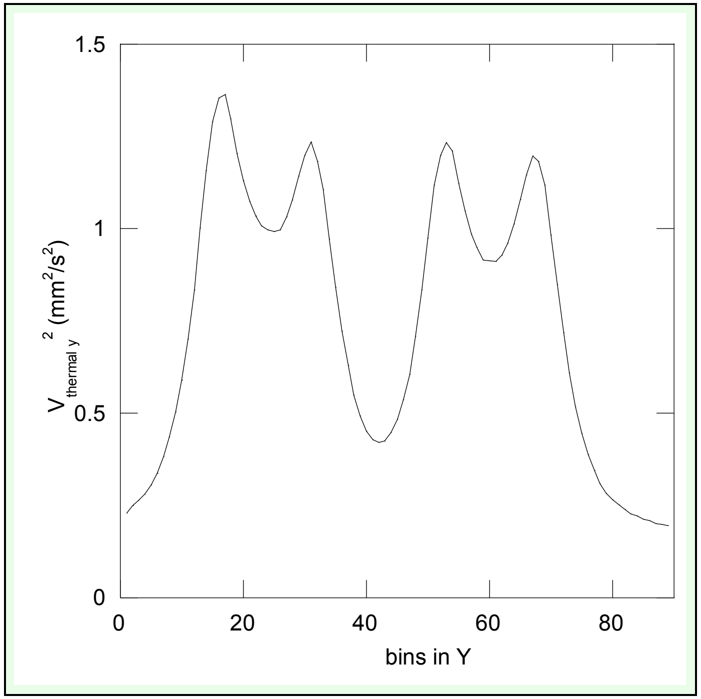
- Inter-particle electric repulsion (Coulomb collisions)
  - Responsible for transport coefficients eta kappa
- radiation pressure force from laser manipulation
- gas drag (friction) force

Profile of kinetic energy (contribution due to y-component of particle velocity)

with the contribution of the flow velocity



without the contribution of the flow velocity



$$2k_BT\,/\,m_{part} = \frac{1}{N}\sum_{i=1}^{N}[v_{part\ i} - v_{flow}(y)]^2$$

$v_{flow}(y)$ is flow velocity at *y*, interpolated between grid centers

THE UNIVERSITY OF IOWA

What is thermal conductivity?

Thermal conductivity (κ)

The property of a material's ability to conduct heat.

Heat flux $\dot{q} = \kappa \nabla T$

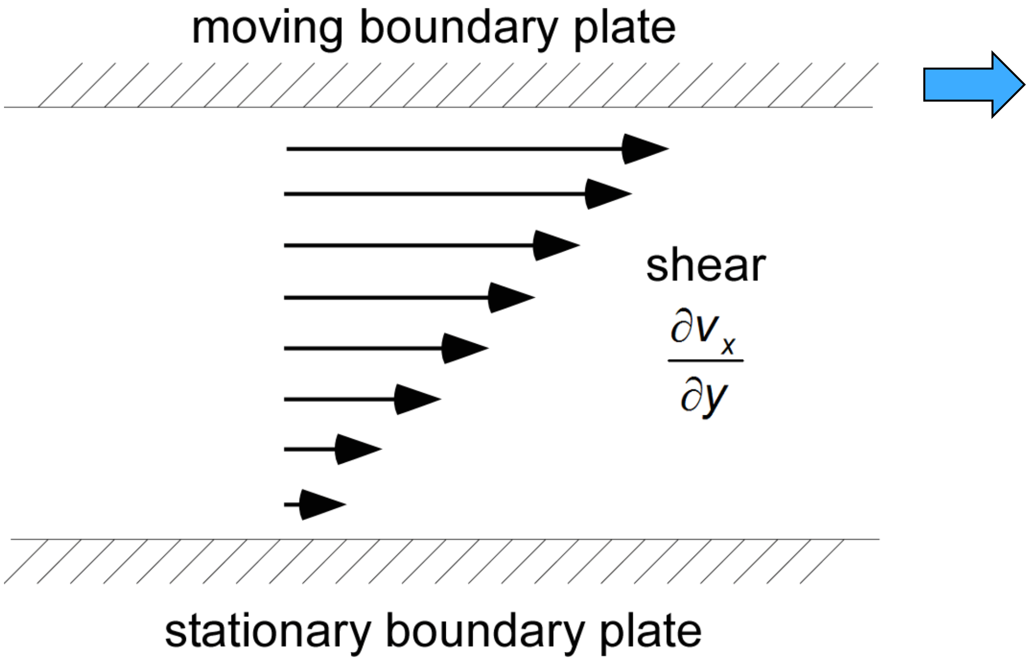
What is viscosity?

Viscosity (η)

A measure of how a fluid flows under a shear force (shear stress).

$$\eta = \text{shear stress} \Big/ \frac{\partial v_x}{\partial y}$$

Example:
Couette flow, apply a stress (force per unit area) to the top plate:



Pull upper plate with
shear stress = force/area