How many equilibrium states does a 2D elliptical Debye cluster have?

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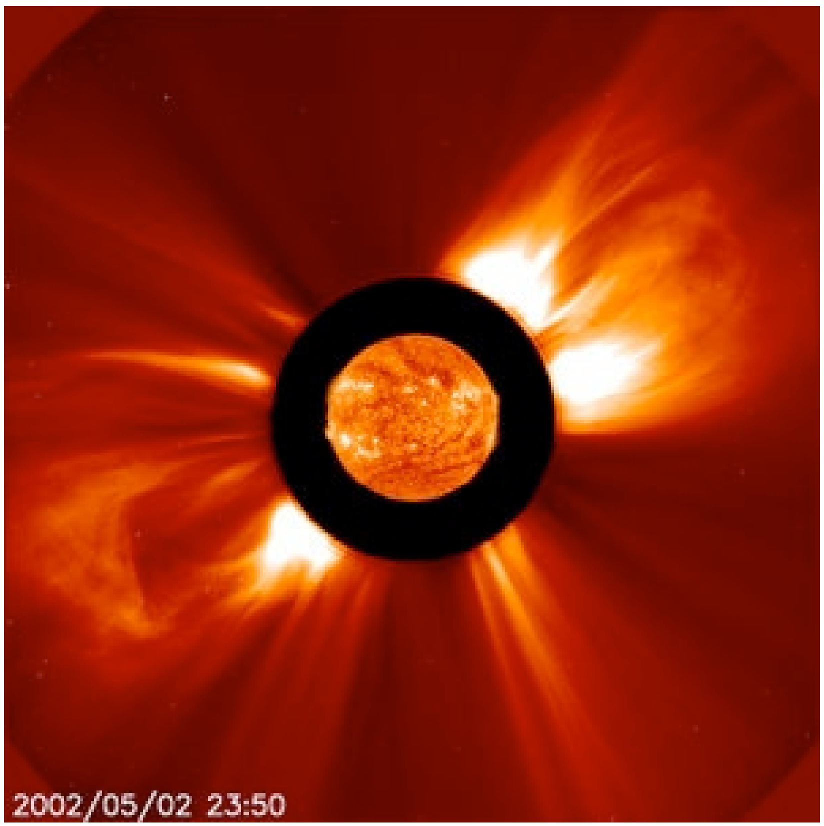
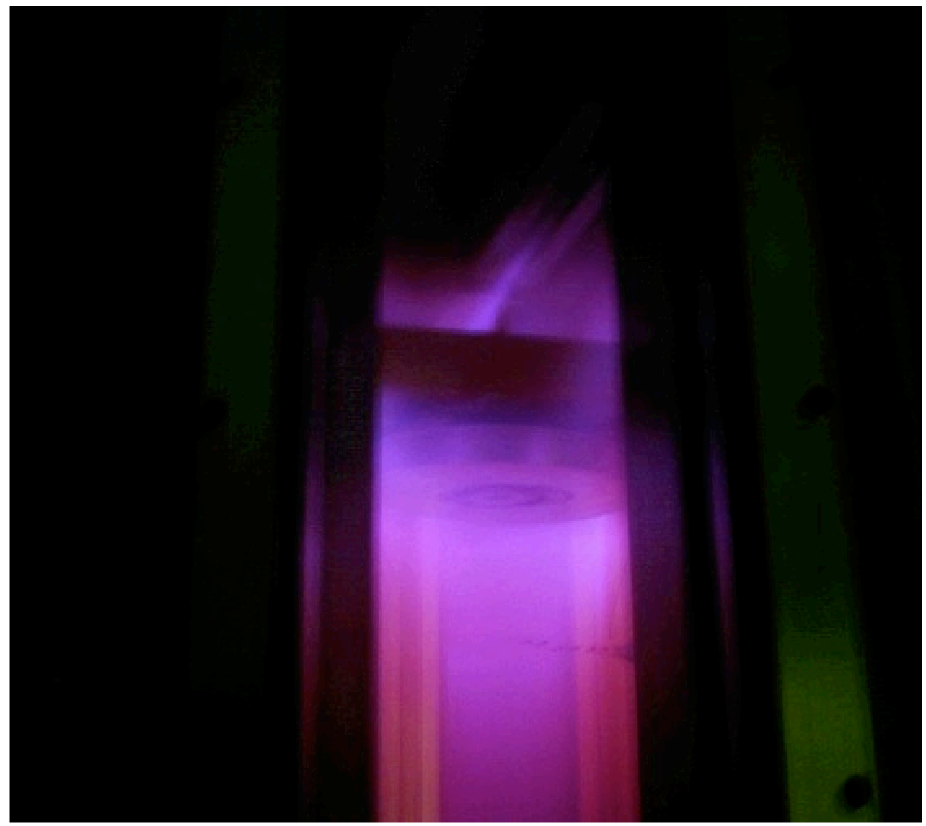
Plasma

- plasma - quasineutral gas of charged and neutral particles which exhibits collective behavior (F. F. Chen)
- physics definition - weakly-damped system of many particles which interact through long-range forces



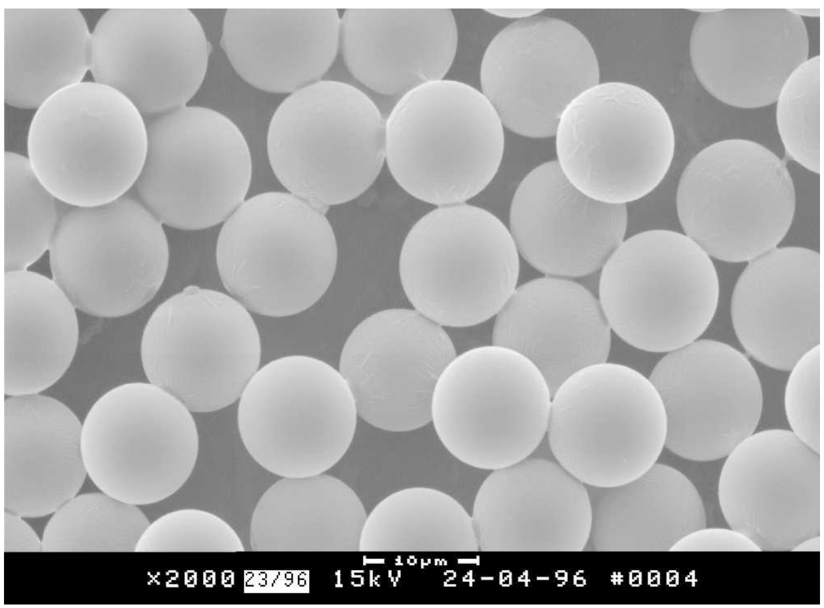


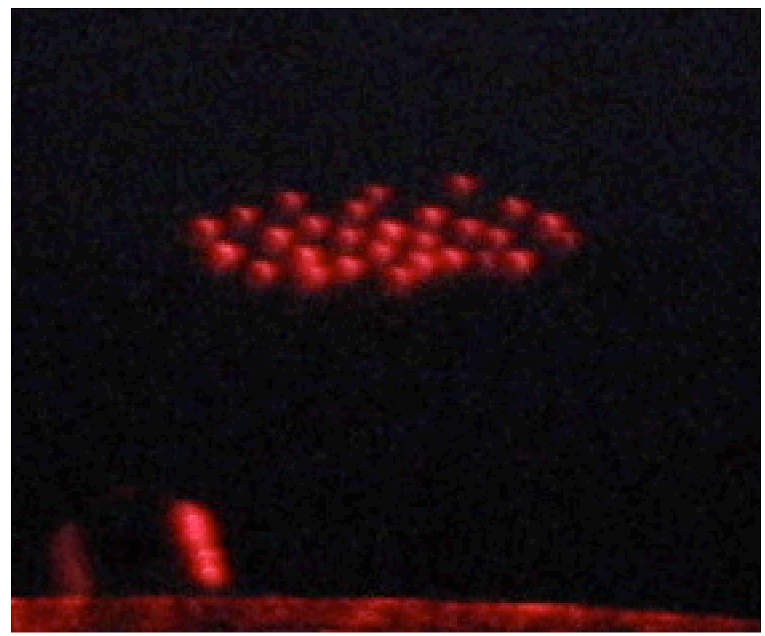
Dusty (complex) plasma

- microscopic dust particles (nm to μm) in electron-ion plasma
- particles acquire net charge, typically $q < 0$ in experiment
- particles interact through Debye potential

$$V(r) = \frac{1}{4\pi\epsilon_0}\frac{q}{r}e^{-r/\lambda_D}$$

- can form a strongly-coupled 2D system
- open, dissipative, weakly damped
- condensed matter system inside a plasma with separate physics





Is dusty plasma = plasma?

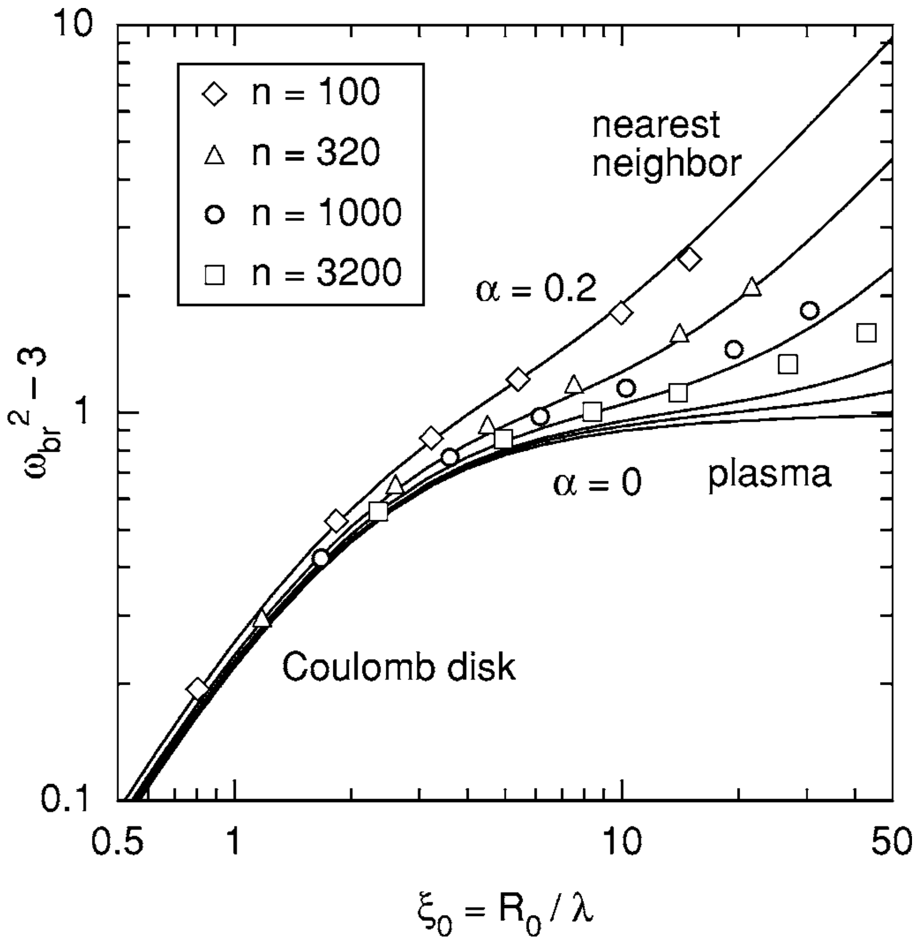


FIG. 4. Normalized, squared breathing frequency minus 3 vs the normalized disk radius $\xi_0 = R_0/\lambda$ for $n = 100$, 320, 1000, and 3200 particles with the Debye shielding parameter $\kappa = 0.2$, 0.5, 1, 2, 5, and 10. For a given n, κ increases from left to right. Solid lines are theory using Eqs. (5) and (15) with $\alpha = a/R_0 = 0$, 0.01, 0.02, 0.05, 0.1, and 0.2.

Sheridan, PoP **14**, 032108

- weakly-damped system of many particles which interact via long-range forces
- large 2d systems have 3 regimes: Coulomb, plasma, nearest neighbor
- for some parameters dusty plasma = physics plasma

Small dusty plasmas – Debye clusters

- assume (nearly) identical dust particles
- strongly-coupled 2D systems interacting through Debye potential
    - just happens to be embedded in electron-ion plasma
- isotropic harmonic well, equilibrium states depend on 2 param's:

    n - number of particles

    κ - Debye shielding parameter

    - limited experimental range: $0.5 < \kappa < 4$
    - why is κ always ~1?
- biharmonic well, equilibrium states depend on 3 parameters:

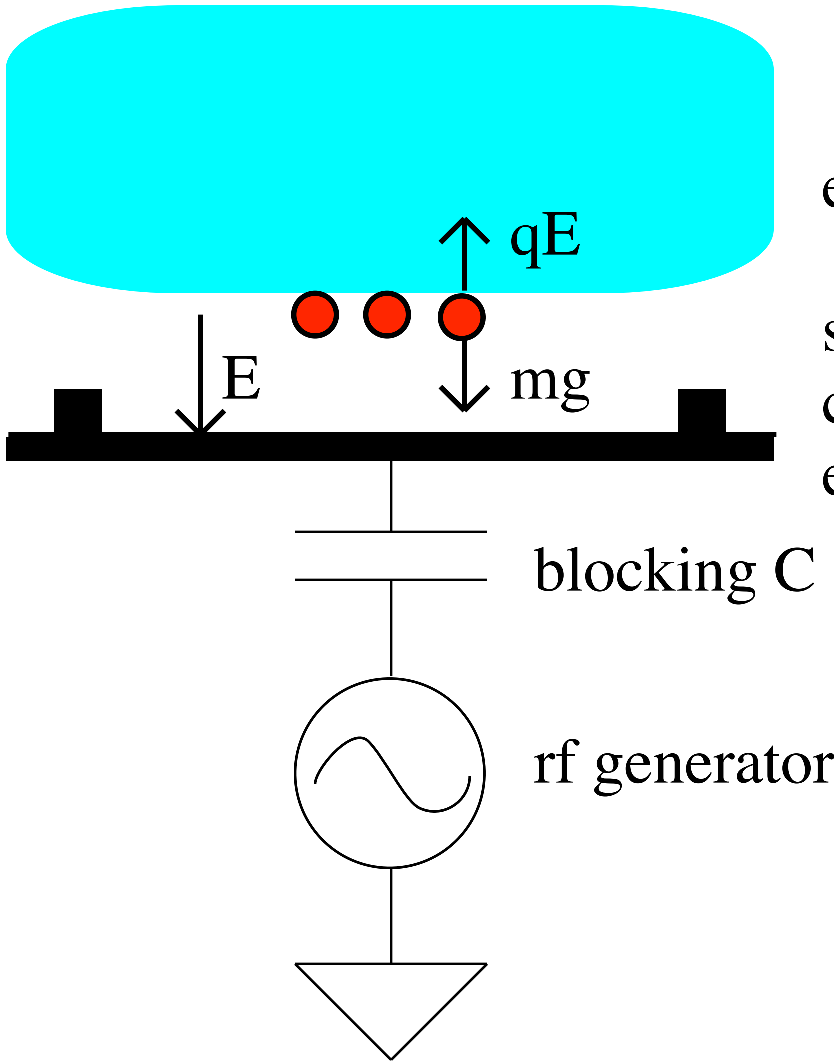
    n - number of particles

    κ - Debye shielding parameter

    α^2 - well anisotropy parameter

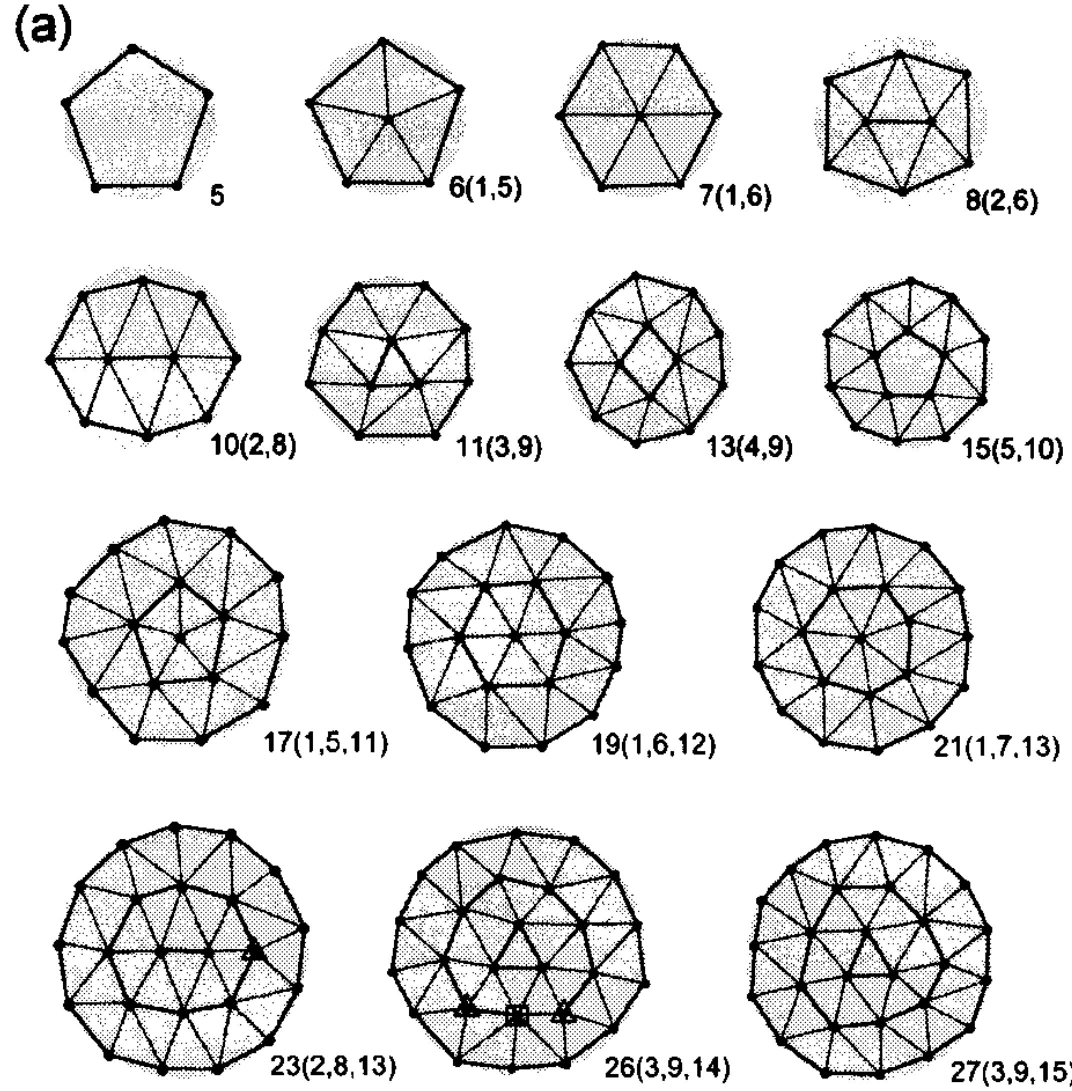
    - wide experimental range
    - rich structure of stable/metastable states

2D dust confinement



- motes float at sheath edge
    - 2D layer
- particles repel one another
- horizontal confinement
  from aperture on electrode
- aperture determines 2D well
  => exp'tal control of well shape

Isotropic well – circular Debye clusters



- observe ground states vs *n*
- Debye parameter fixed
  - hard to change in exp't
- *n* = 6, 7 have single central particle
- *n* = 8 is (1,7) for weak Debye shielding

Juan *et al*., PRE **58**, R6947 (1998)

Biharmonic well – elliptical clusters

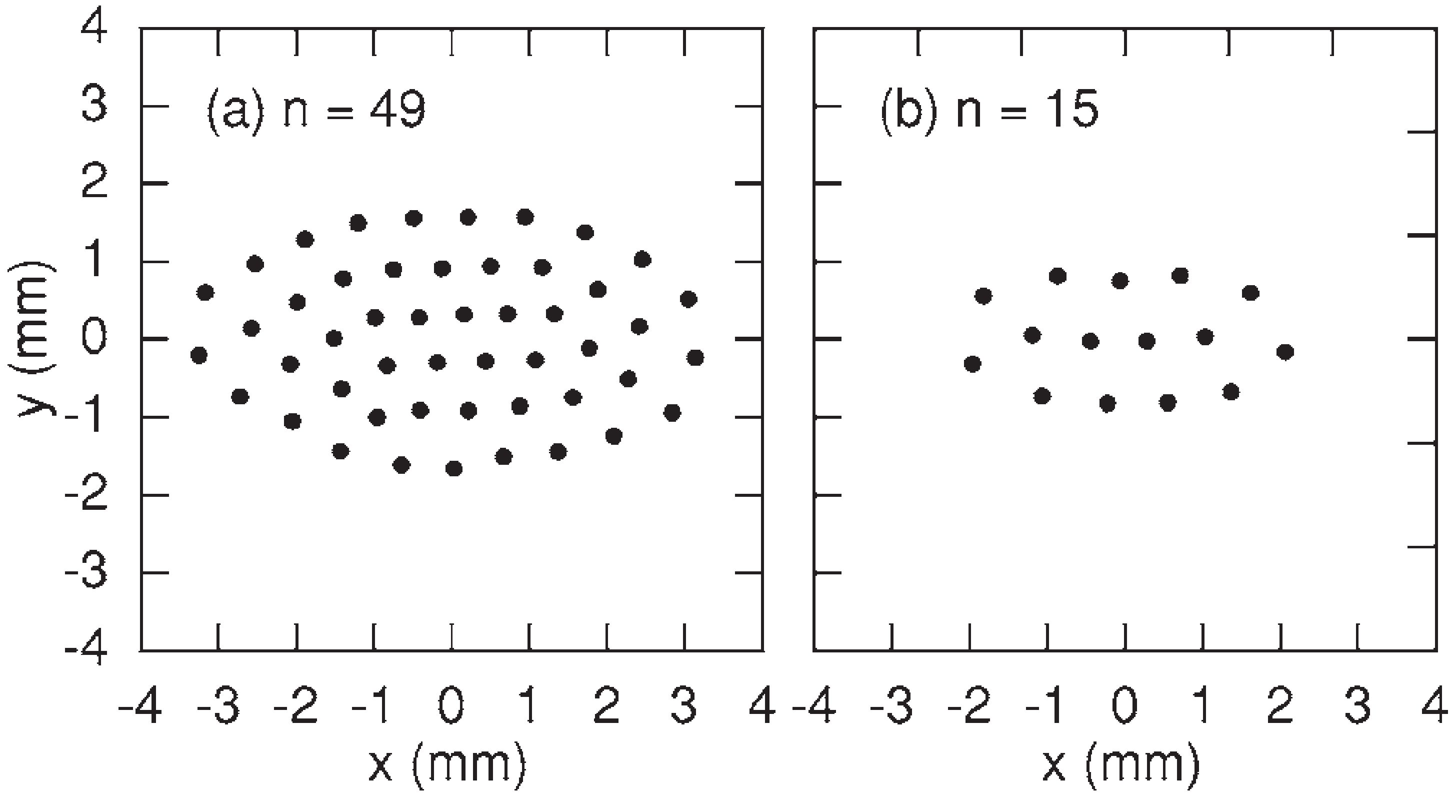


FIG. 4. Experimentally measured equilibrium configurations for (a) $n=49$ and (b) 15 particles.

Sheridan, Wells, Herrick and Garee, *J. Appl. Phys.* **101**, 113309

Biharmonic potential well

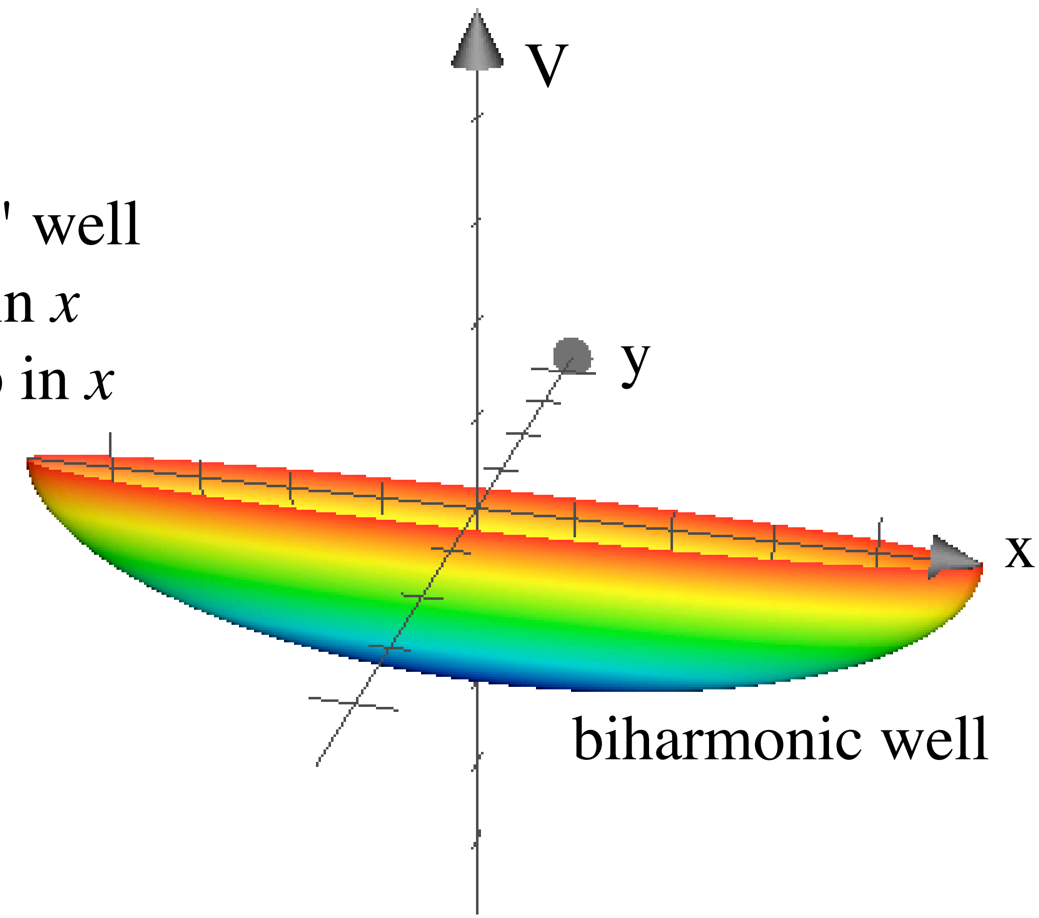
- generalize isotropic well to 2D biharmonic well

$$U_{\text{well}} = \frac{1}{2} m\omega_x^2 x^2 + \frac{1}{2} m\omega_y^2 y^2$$

- anisotropy parameter

$$\alpha^2 = \frac{\omega_y^2}{\omega_x^2} \geq 1$$

- larger α^2 gives "skinnier" well
  - forces in *y* >> forces in *x*
  - $\Rightarrow$ ptcls tend to line up in *x*



biharmonic well

Model

- n identical particles with charge q and mass m at positions $\{x_i, y_i\}$
- interact through shielded Coulomb potential
- confined in 2D biharmonic well
- total potential energy

$$U = \sum_{i=1}^{n} \left(\frac{1}{2} m \omega_{0x}^2 x_i^2 + \frac{1}{2} m \omega_{0y}^2 y_i^2 \right) + \sum_{j>i=1}^{n} \left(\frac{q^2}{4\pi\epsilon_0} \frac{e^{-r_{ij}/\lambda_D}}{r_{ij}} \right)$$

confining well energy

particle-particle interaction energy

- equilibrium configuration is $\{x_i, y_i\}$ which minimizes U
- 6 model parameters: $n, m, \omega_{0x}, \omega_{0y}, q, \lambda_D$

Non-dimensional model

- non-dimensional potential energy

$$\frac{U}{U_0} = \sum_{i=1}^{n} \left(\xi_i^2 + \alpha^2 \eta_i^2\right) + \sum_{j>i=1}^{n} \left(\frac{e^{-\kappa \rho_{ij}}}{\rho_{ij}}\right)$$

- distances

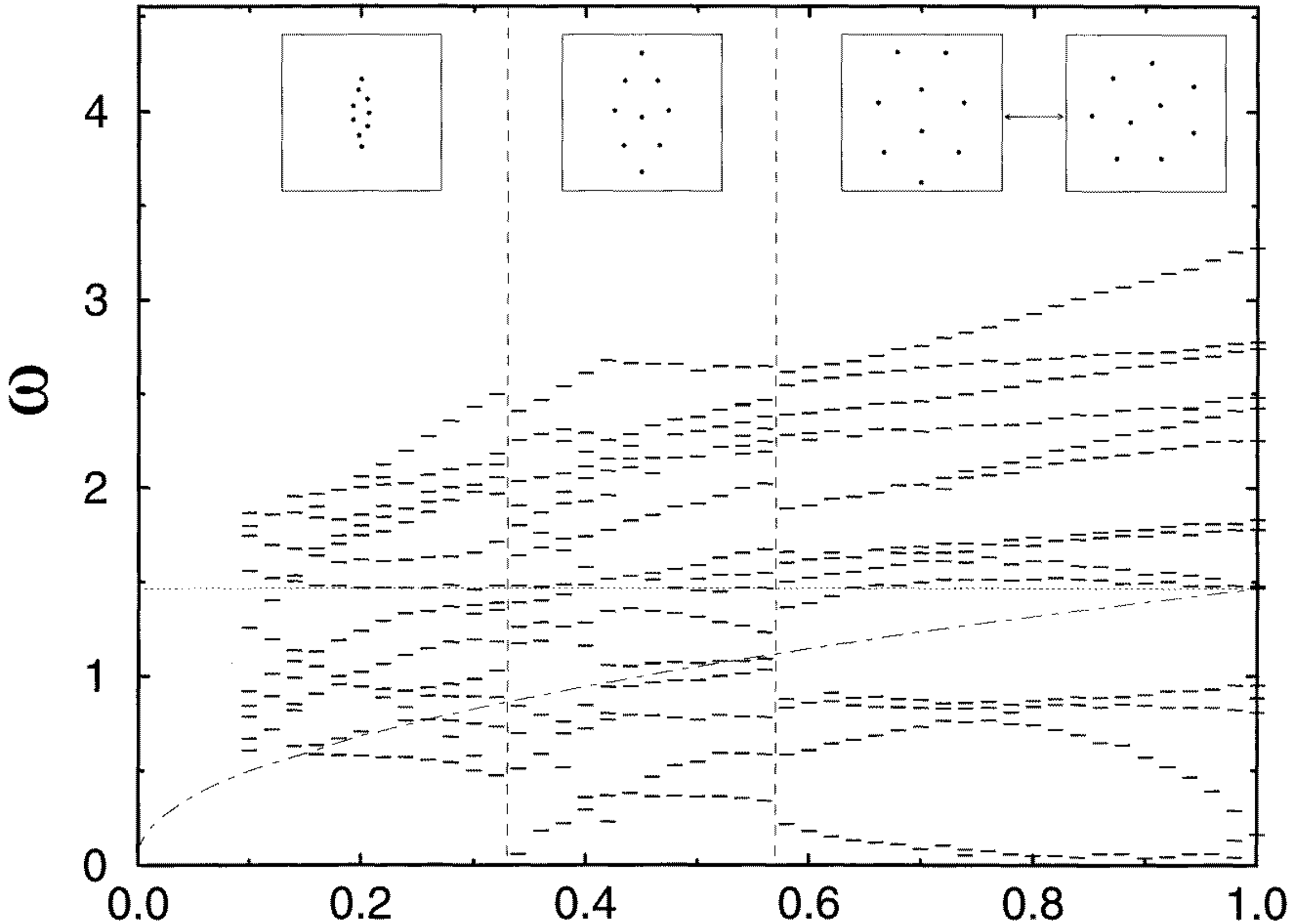
$$\xi_i = \frac{x_i}{r_0},\ \eta_i = \frac{y_i}{r_0},\ \rho_{ij} = \frac{r_{ij}}{r_0} \quad \text{where} \quad r_0 = \left(\frac{2}{m\omega_x^2} \frac{q^2}{4\pi\epsilon_0}\right)^{1/3}$$

- stable/metastable configurations minimize U
- 3 parameters:

$$n,\ \alpha^2 = \frac{\omega_y^2}{\omega_x^2},\ \kappa = \frac{r_0}{\lambda_D}$$

- α^2 - anisotropy parameters easily varied in experiment
  - rich structure of stable and metastable states is accessible
- search for stable/metastable states by rapidly quenching random initial configurations

Arrangement transitions in biharmonic well



- theoretical ground state, mode spectra vs anisotropy for $n = 9$
- distinct arrangement phases with change in symmetry
- Candido *et al*., *J. Phys: Condens. Matter* **10**, 11627

Arrangement transitions in ∞ harmonic trough

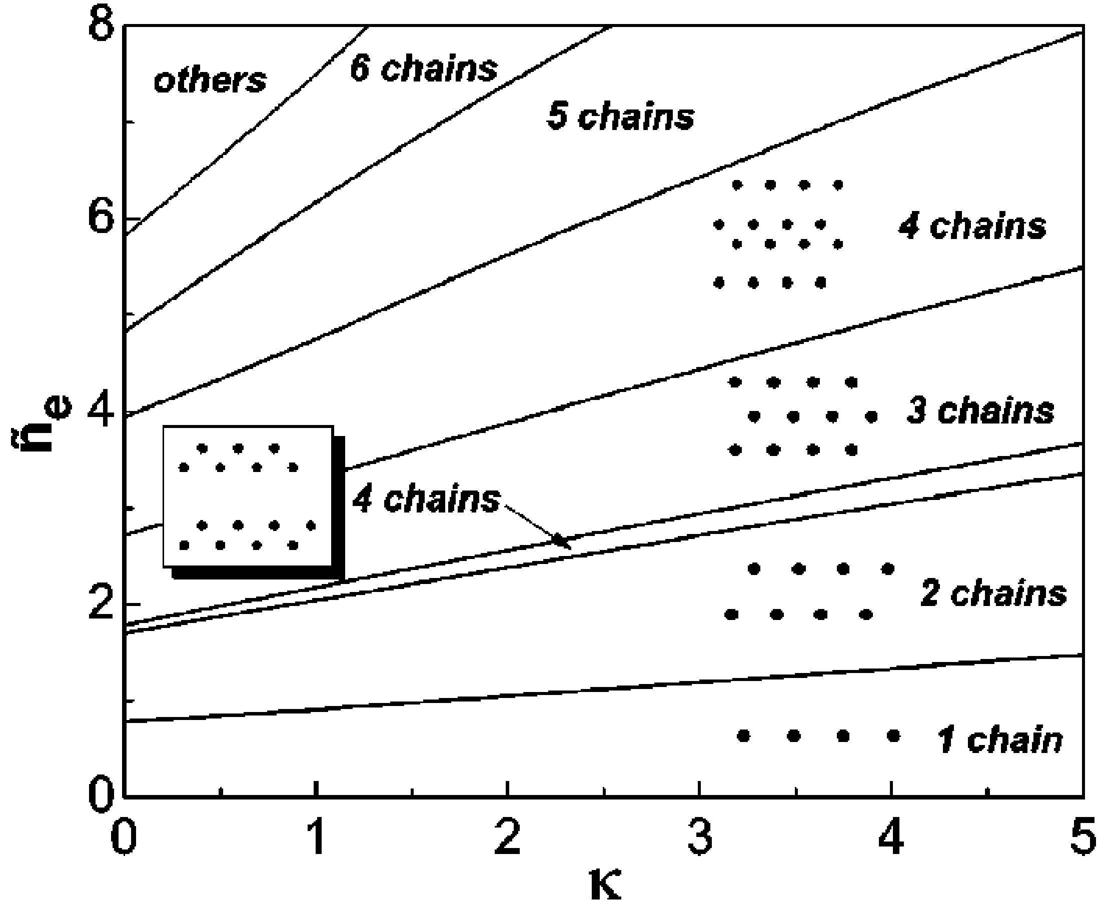


FIG. 1. The T=0 structural phase diagram as a function of the inverse screening length κ and the density $\tilde{n}_e$. The plotted quantities are dimensionless, as for all the figures in the paper.

- 4-chain separates 2-chain and 3-chain
- Piacente *et al*., *PRE* **70**, 036406

Zigzag arrangement transition

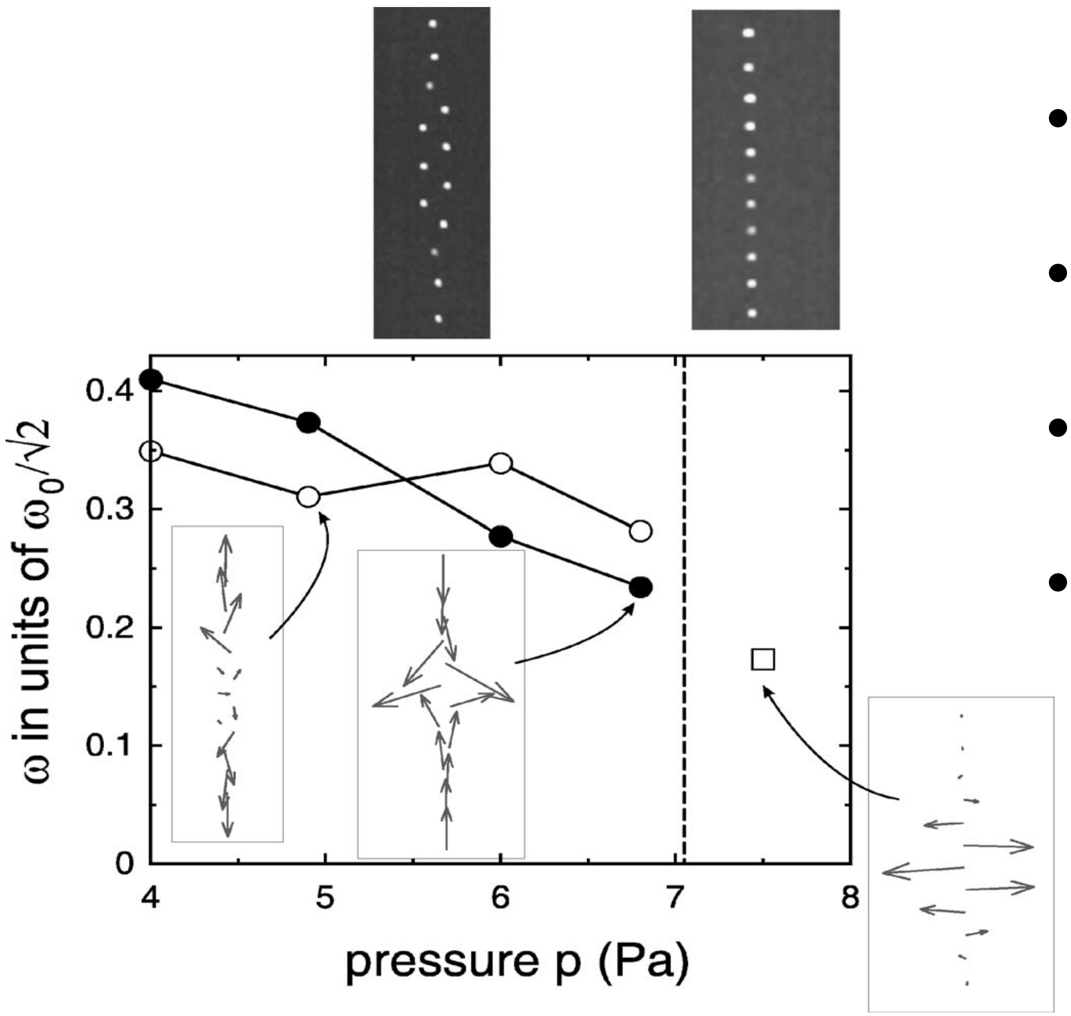


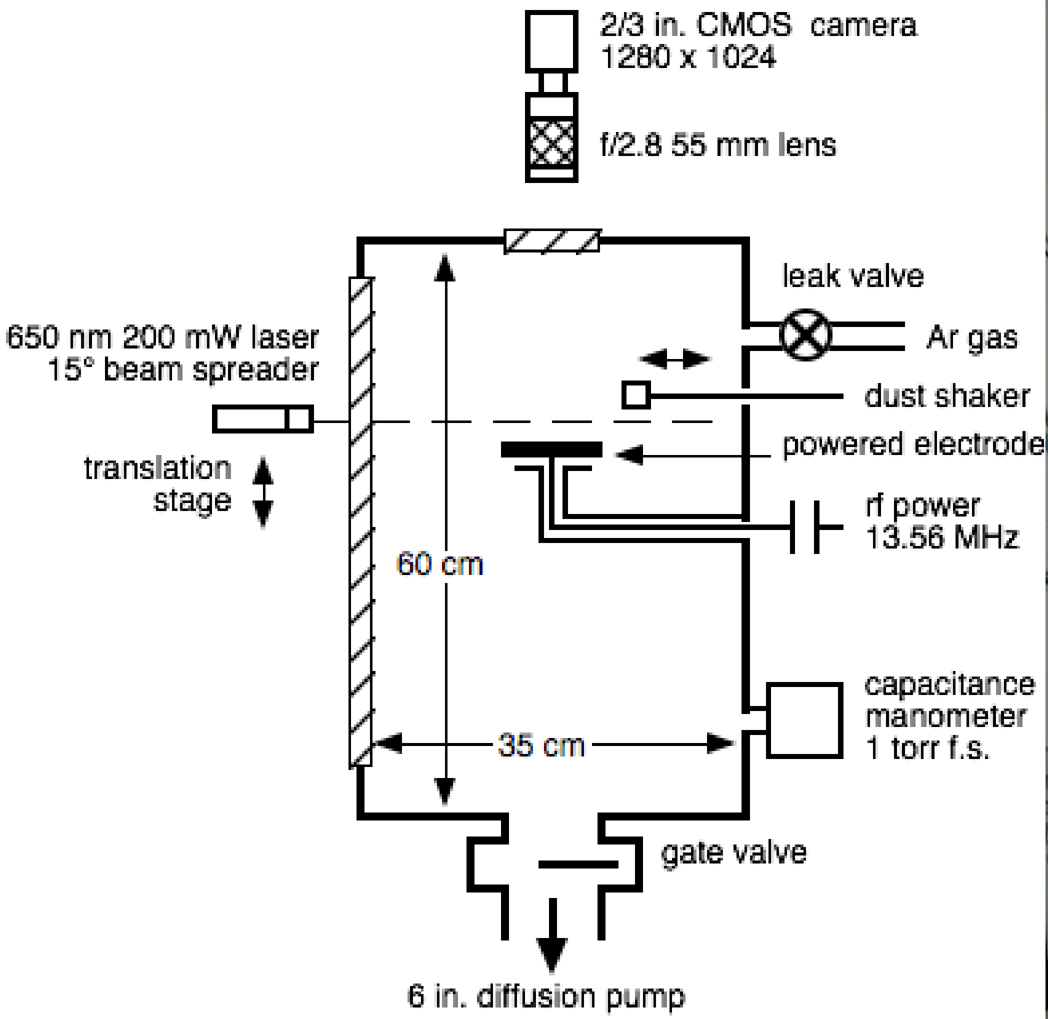
FIG. 5. Frequency of the lowest eigenmodes during pressure variation. The oscillation pattern of the corresponding eigenmodes is indicated in the insets. Below 7.5 Pa the cluster is in the zigzag state, above that pressure it forms a 1D chain.

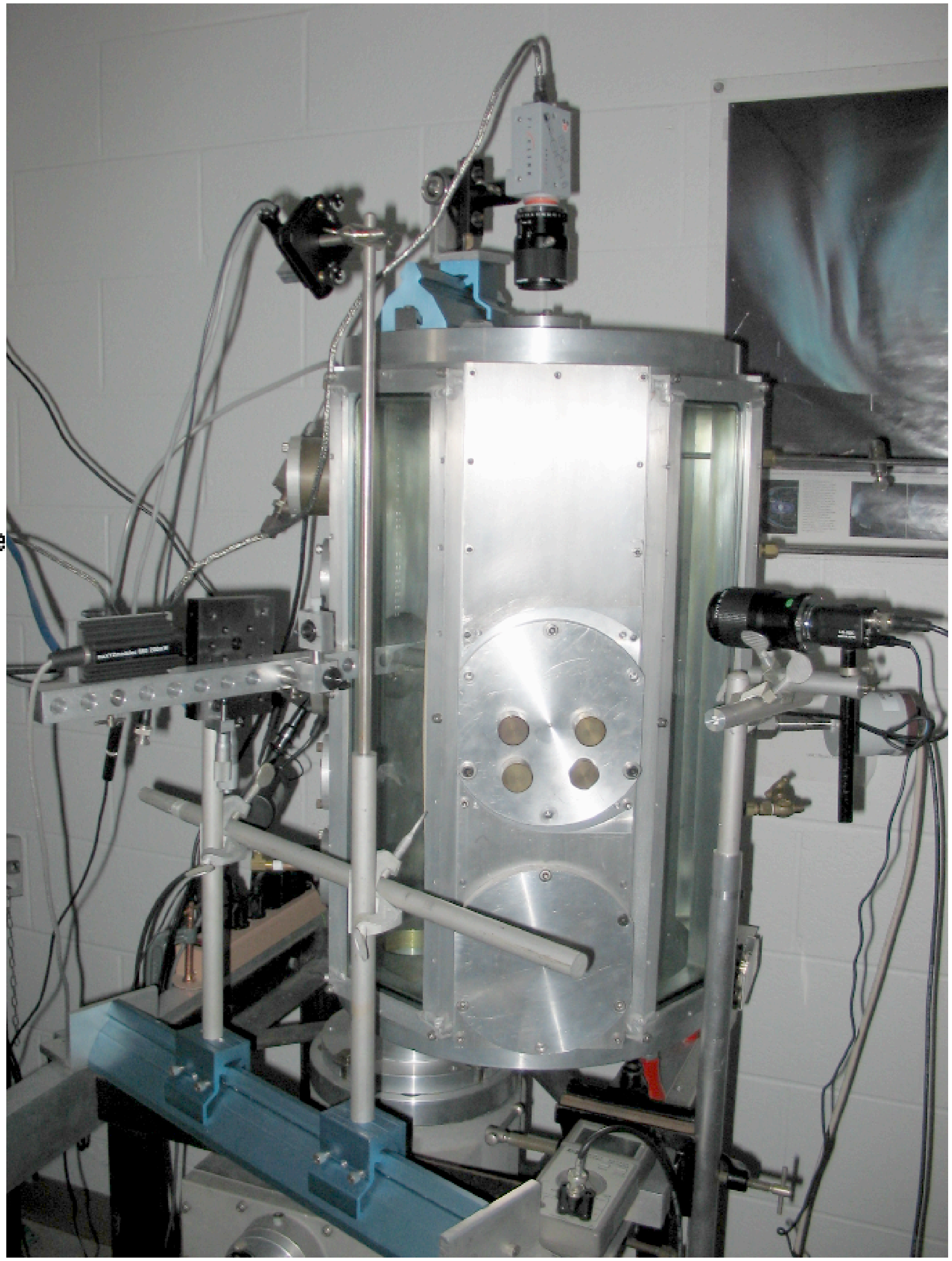
- transition from 1D straight line to 2D "zigzag"
- finite analog of 1-chain to 2-chain transition
- shortest wavelength tranverse mode unstable
- caused by change in *n*, anistropy, shielding

Melzer, *PRE* **73**, 056404

D.ONU.T experiment

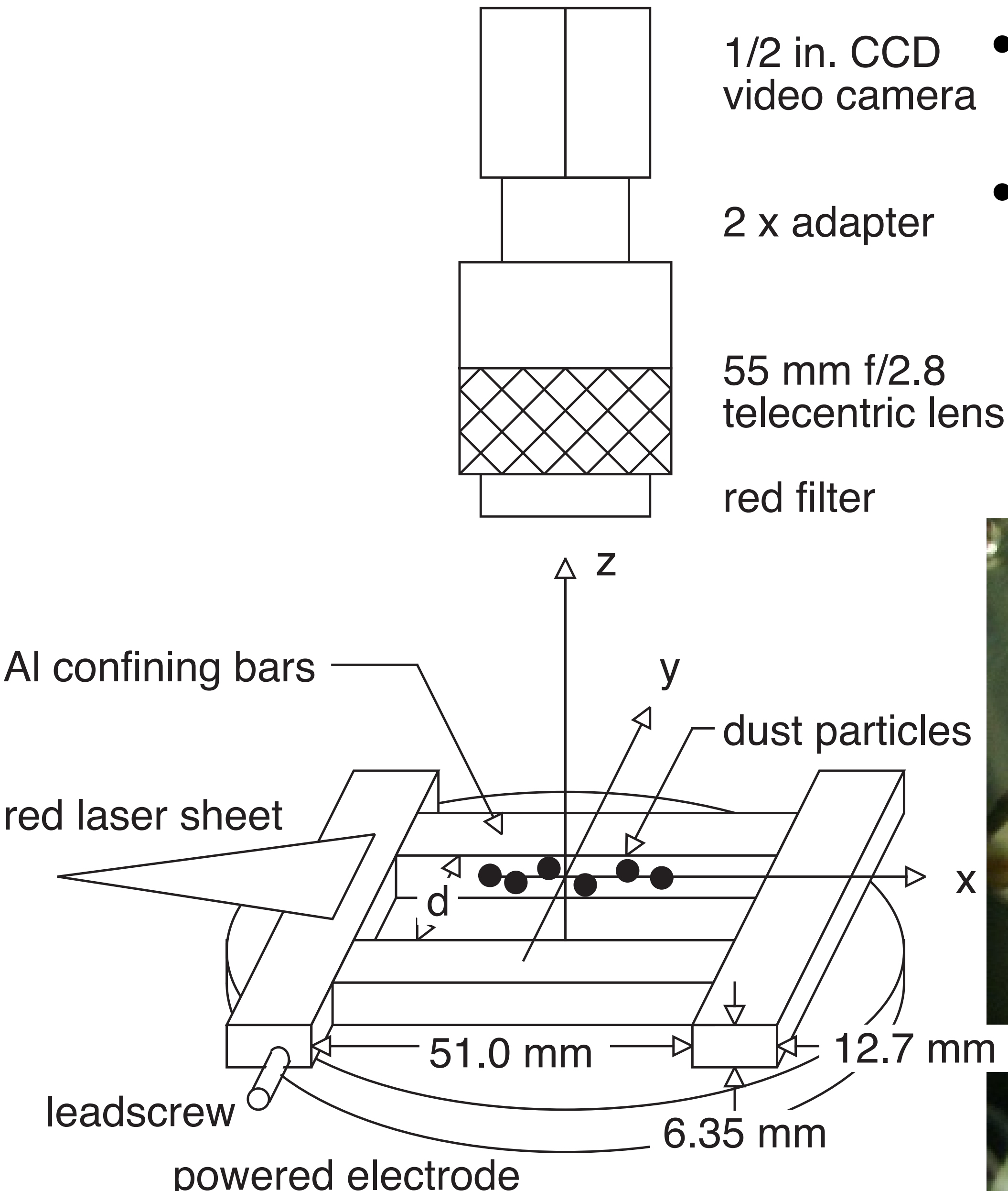
- Dusty O.N.U. experimenT

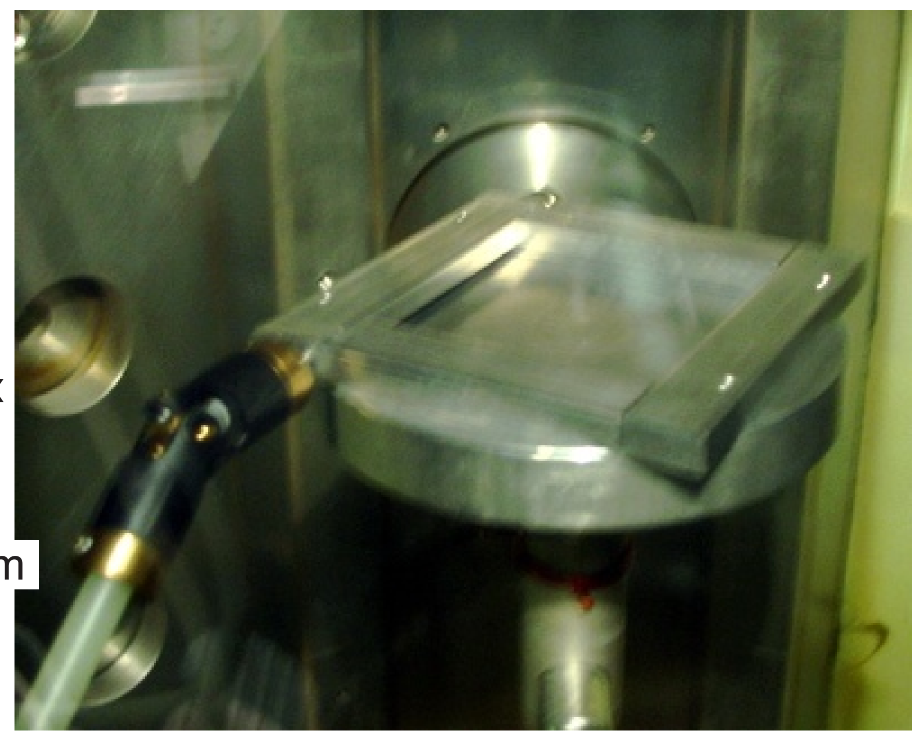




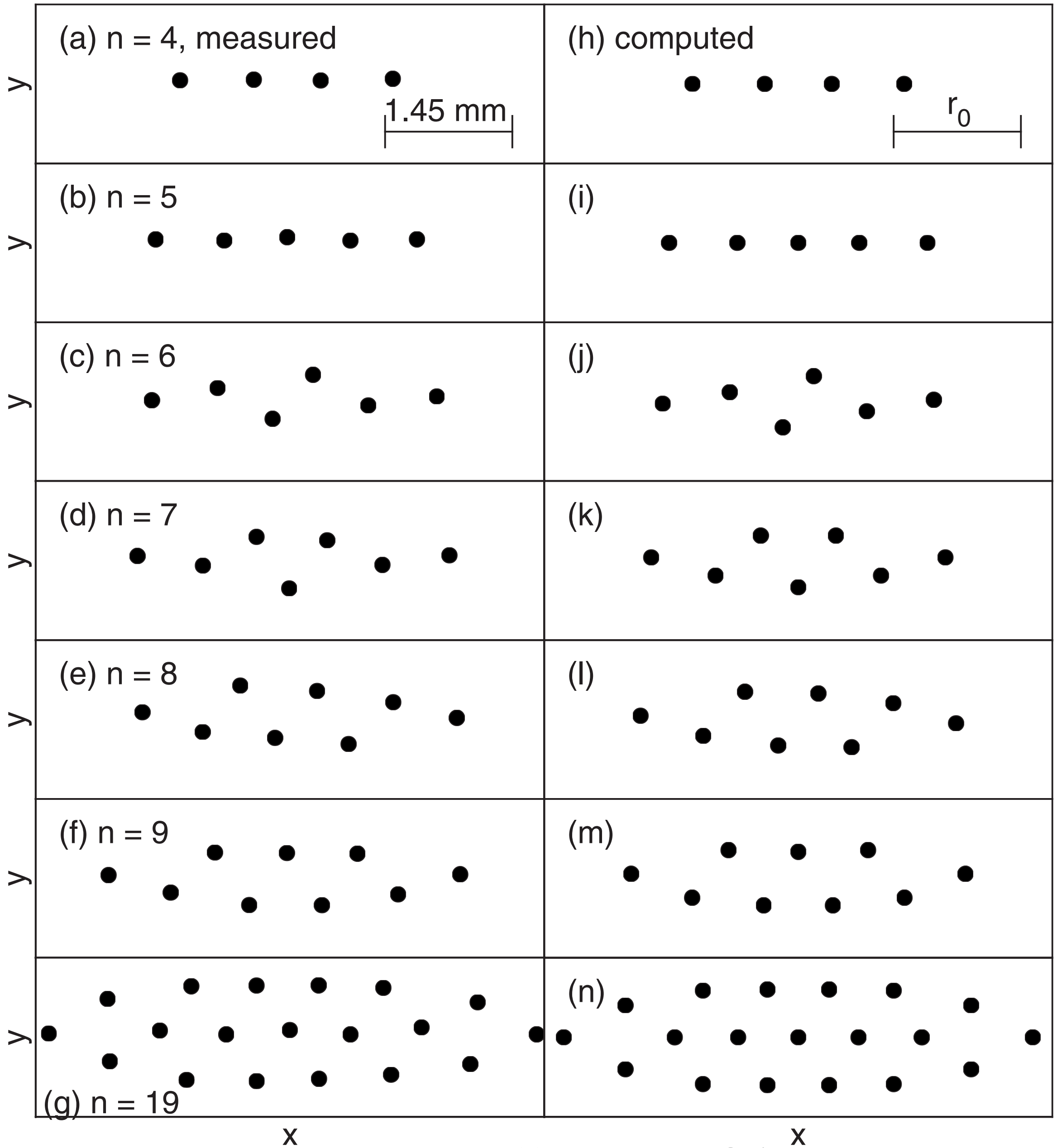
Adjustable confining well

- confining well due to 4 Al bars
  - biharmonic near center
- *d* adjustable using leadscrew
  - vary well anisotropy





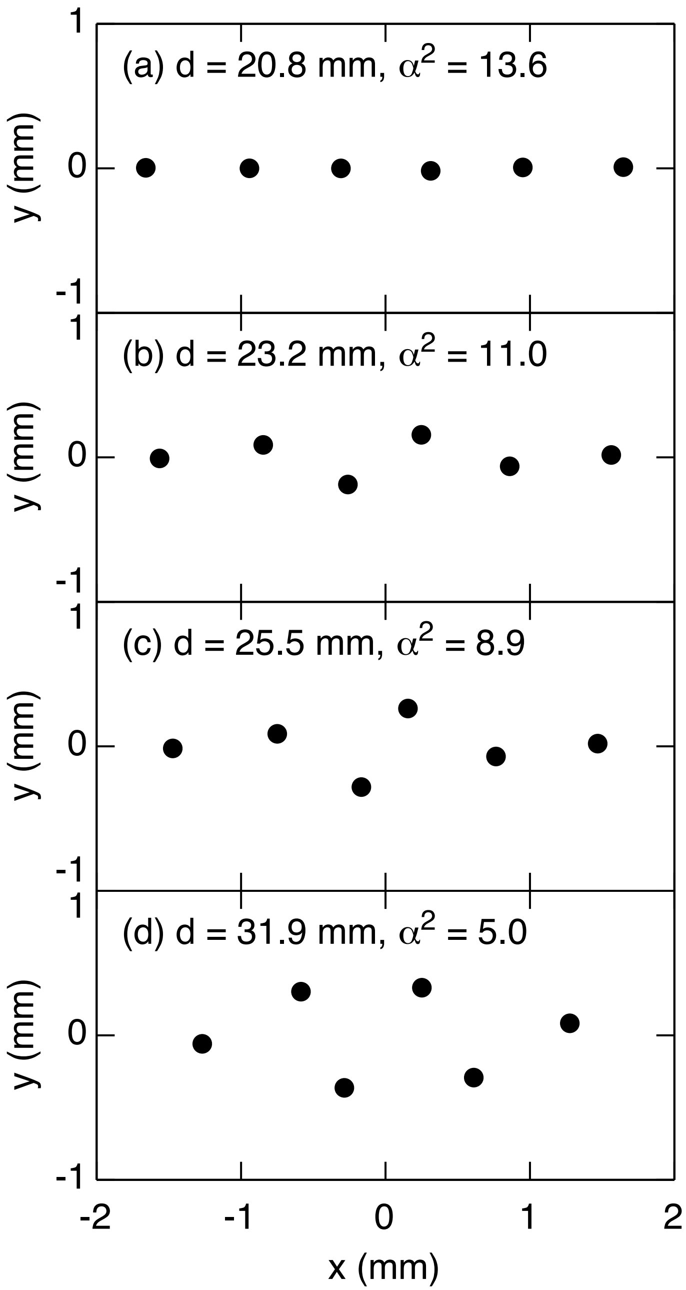
Equlibrium arrangements vs *n*



- fixed rectangular well 25.4 mm x 50.8 mm
- straight line
- zigzag
- internal ptcls, 3-chain?

Sheridan and Wells, *PRE* **81**, 016404

Measured zigzag vs anisotropy



- anisotropy changed by adjusting *d*
- straight line for large anisotropy

- zigzag - 1D to 2D transition

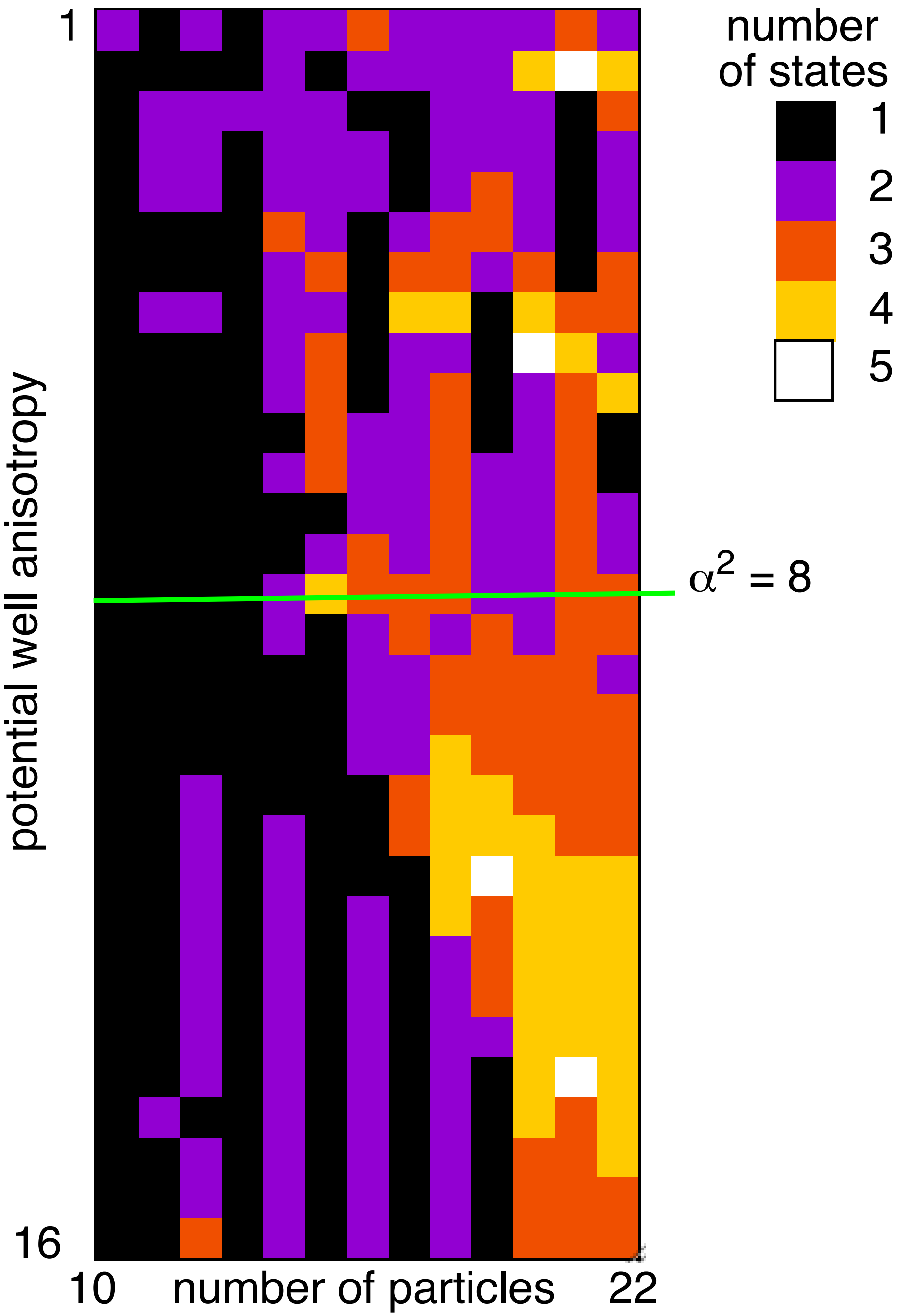
- fat zigzag - all particles on "outside" of cluster

Sheridan and Magyar, *PoP* **17**, 113703

What about metastable states?

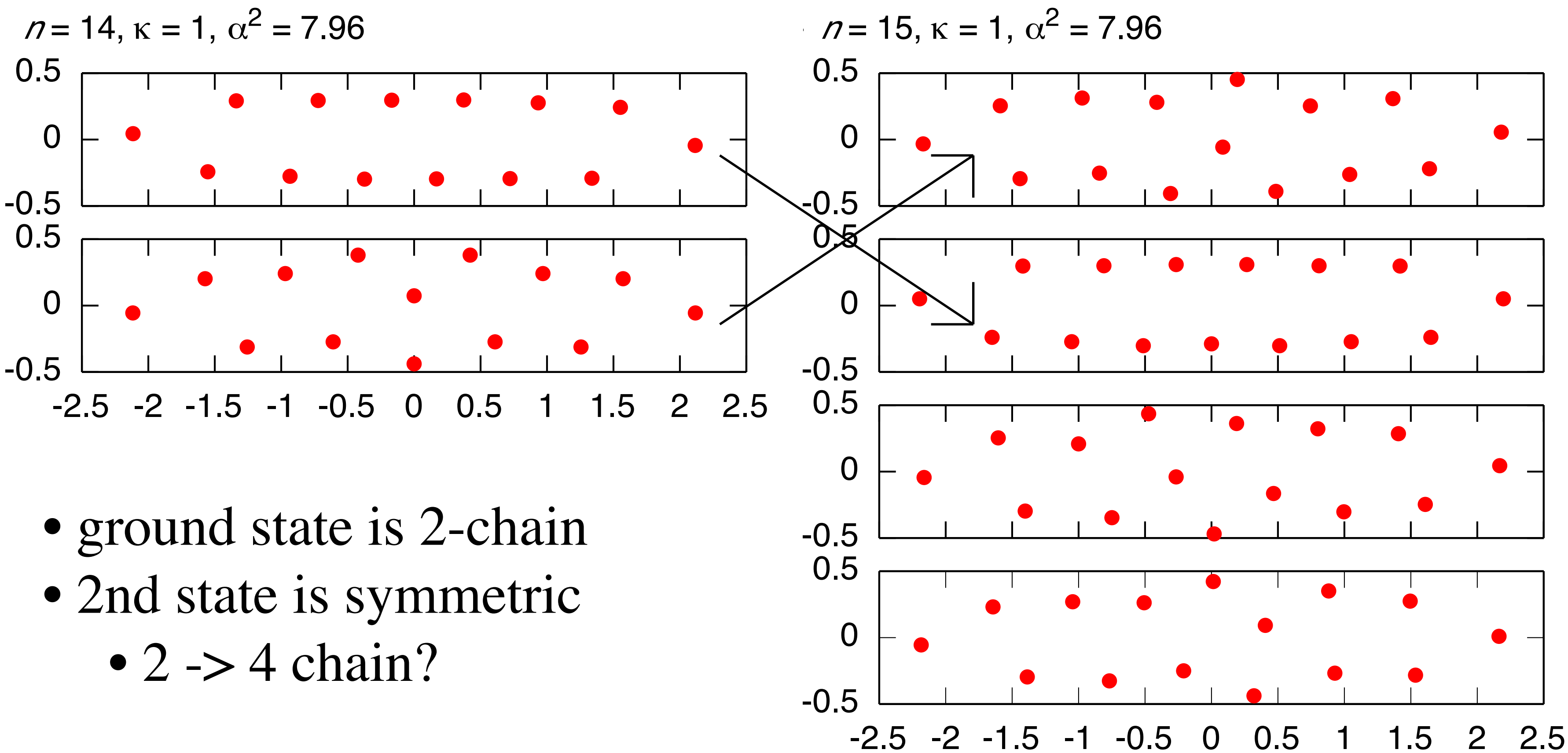
- metastable - potential barrier blocks rearrangement into lower energy state
- 3 parameter model: n, α^2, κ
- effect of α^2?
- large anisotropy $\alpha^2 >> 1$
  - straight line arrangement, no metastable states
- decrease α^2
  - line -> zigzag (1-chain -> 2-chain)
  - all particles on cluster "edges"
    - no "internal" barriers to rearangement (no "jammed" dust)
      => no metastable states?
- decrease α^2 some more
  - zigzag -> double zigzag (2-chain -> 4-chain)
    - appearance of inside particles
      => interior jammed ptcls lead to metastable configurations?

Computed number of states for $\kappa = 1$



- 1-2 stripes for higher α^2
- nr. of states increases roughly with n, α^2
- up to 5 states found
- overall rich structure
  - rich = unpredictable
  - fractal?

Computed states for n = 14 and 15



- ground state is 2-chain
- 2nd state is symmetric
    - 2 -> 4 chain?

- ground + 3 metastable configes
- ground state asymmetric 4-chain
- 2-chain zigzags in differing ways

Conclusions

- considered Debye clusters in 2D biharmonic well
- arrangement transitions characterized
  - as a function of *n* for fixed anisotropy and Debye shielding
  - as a function of anisotropy for fixed *n* and shielding
  - good agreement between experiment and model
- number of metastable states explored with model
  - rich structure vs *n* ($n > 10$) and well anisotropy
  - no clear patterns
  - metastable states may caused by internal "jamming"