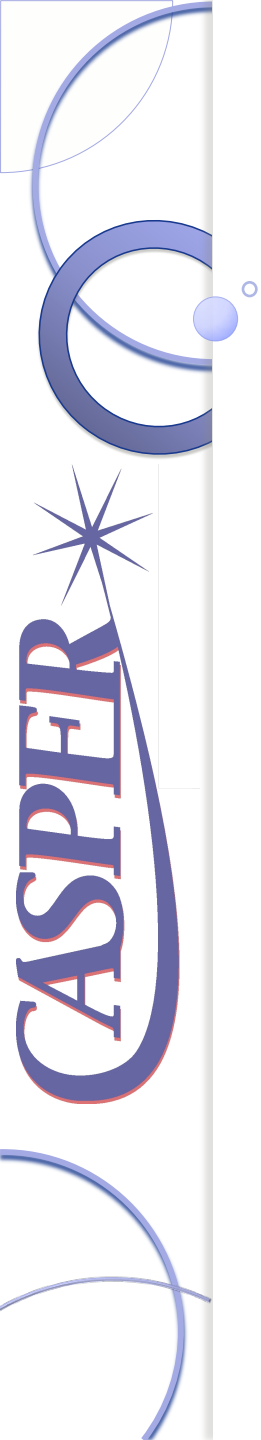


Mode coupling and resonance instabilities in small dust systems

Ke Qiao, Jie Kong, Lorin Matthews, and Truell Hyde

Center for Astrophysics, Space Physics and Engineering Research, Baylor University, Waco, Texas 76798-7310, USA

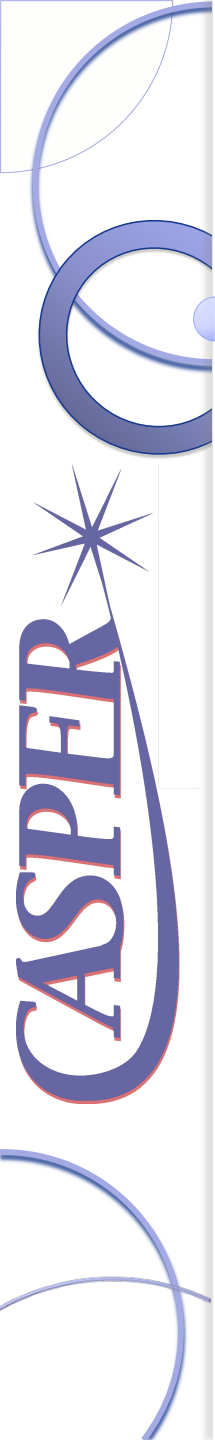


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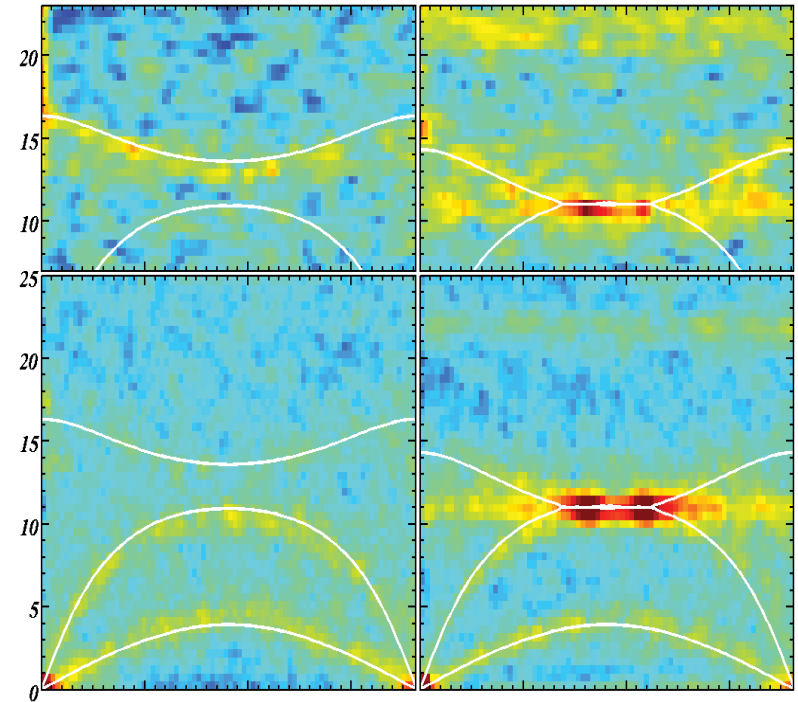
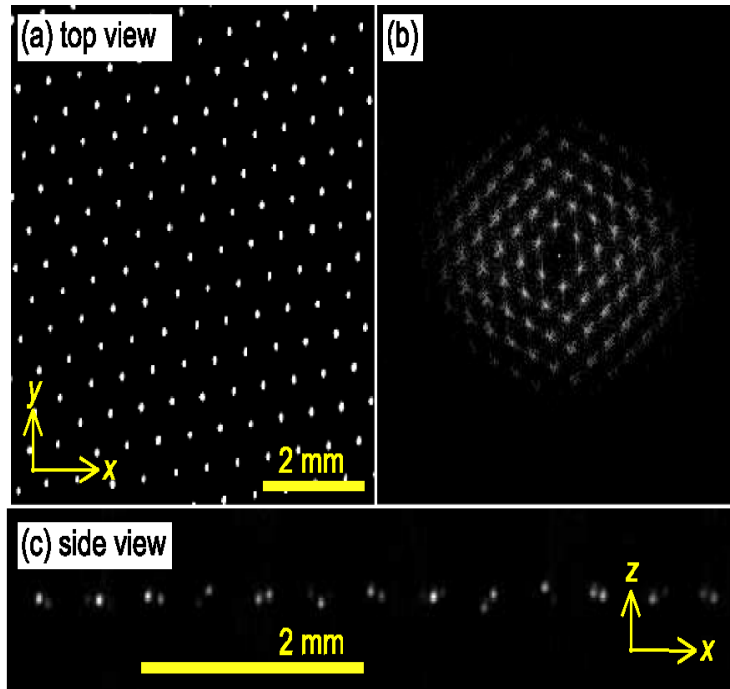


Outline

- Motivation: Research in large crystals
- Small circular clusters
 - experimental
 - simulation
- Small horizontal chains
 - Mode coupling and instabilities
 - Mode spectra and dispersion relations



Mode coupling in a one-layer (2D) large crystal



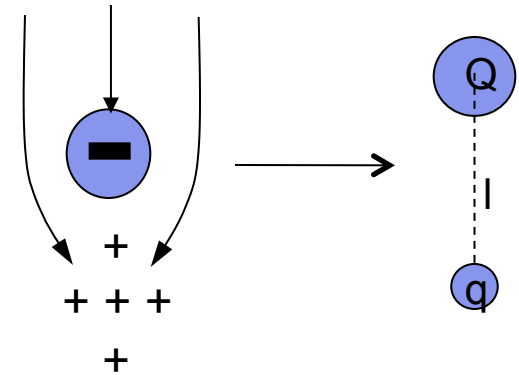
Couëdel, Nosenko, Zhdanov, Ivlev, Thomas and Morfill 2009 *Phys. Rev. Lett.* **103** 215001.
Couëdel, Nosenko, Zhdanov, Ivlev, Thomas and Morfill 2010 *Phys. Rev. Lett.* **104** 195001.
Liu, Goree and Feng 2010 *Phys. Rev. Lett.* **105** 085004.

Theoretical model

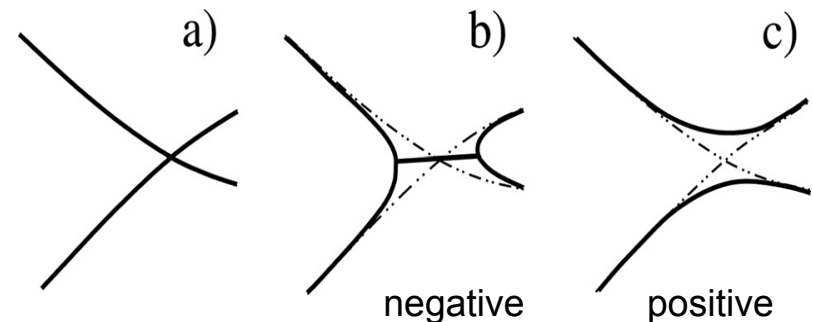
Non-Hamiltonian interaction due to the wakefield

Ion focusing → Wake field → Virtual particle

$$U(r) = Q \frac{\exp(-|r - r_Q|/\lambda)}{|r - r_Q|} - q \frac{\exp(-|r - r_q|/\lambda)}{|r - r_q|}$$



Three types of resonance in ω - k space
(dispersion relations)



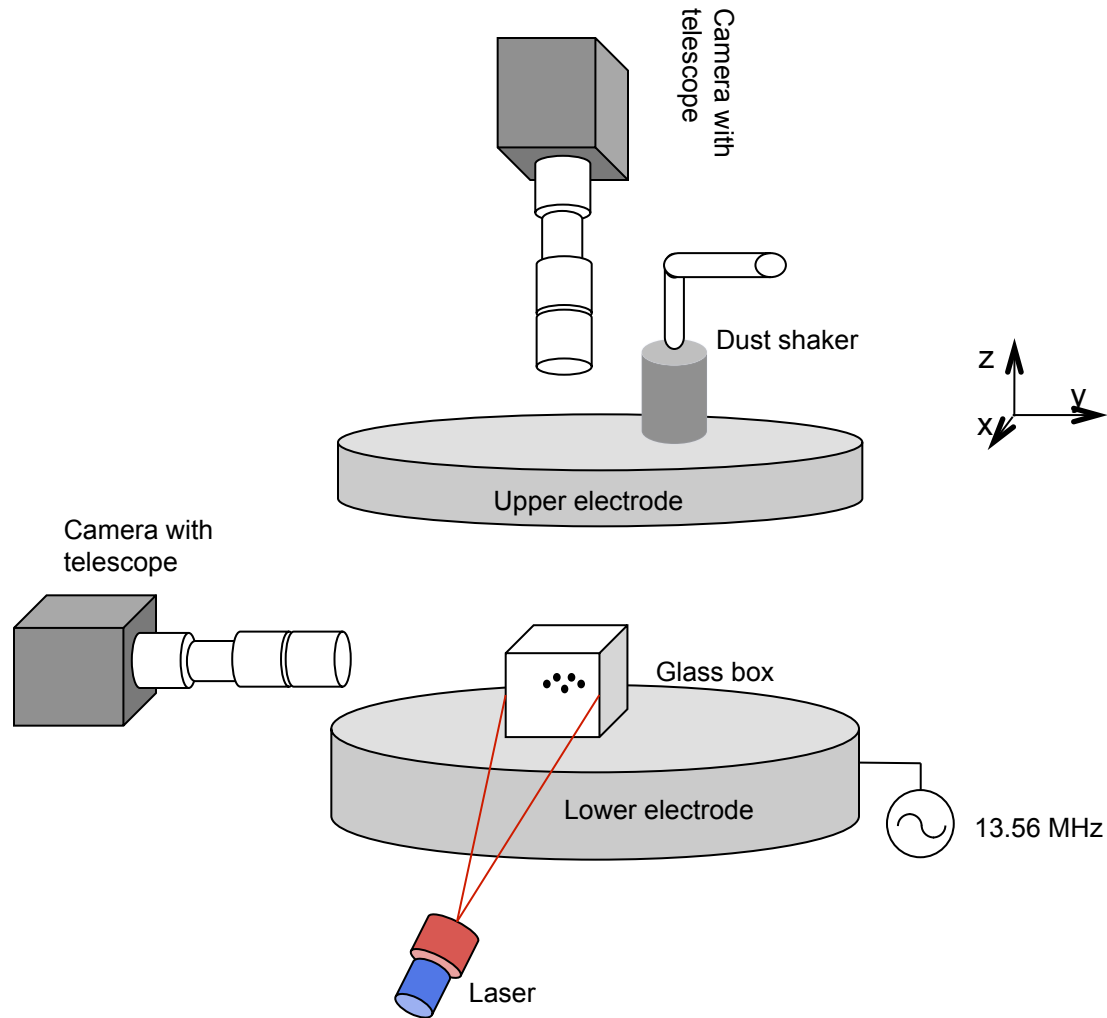
Schweigert et al. 1996 *Phys. Rev. E* 54 4155

Vladimirov and Samarian 2002 *Phys. Rev. E*, **65** 046416

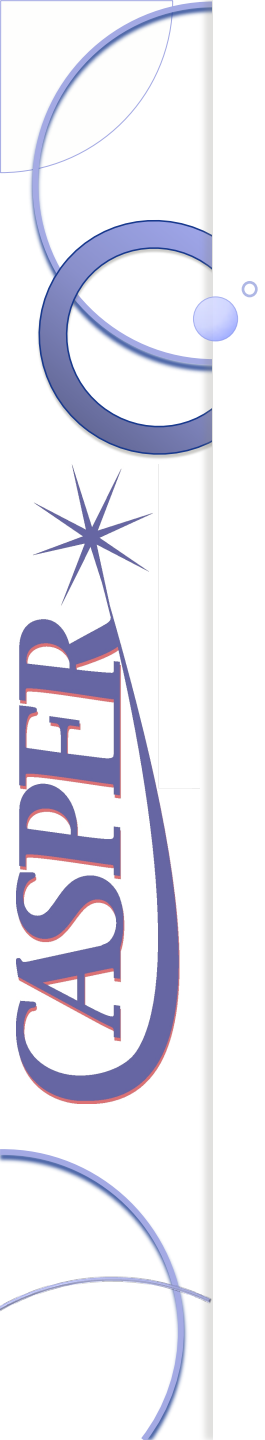
Yaroshenko, Ivlev and Morfill 2005 *Phys. Rev. E* **71** 046405

Couédel, Zhdanov, Ivlev, Nosenko, Thomas and Morfill 2011 *Phys. Of Plasmas* **18** 083707

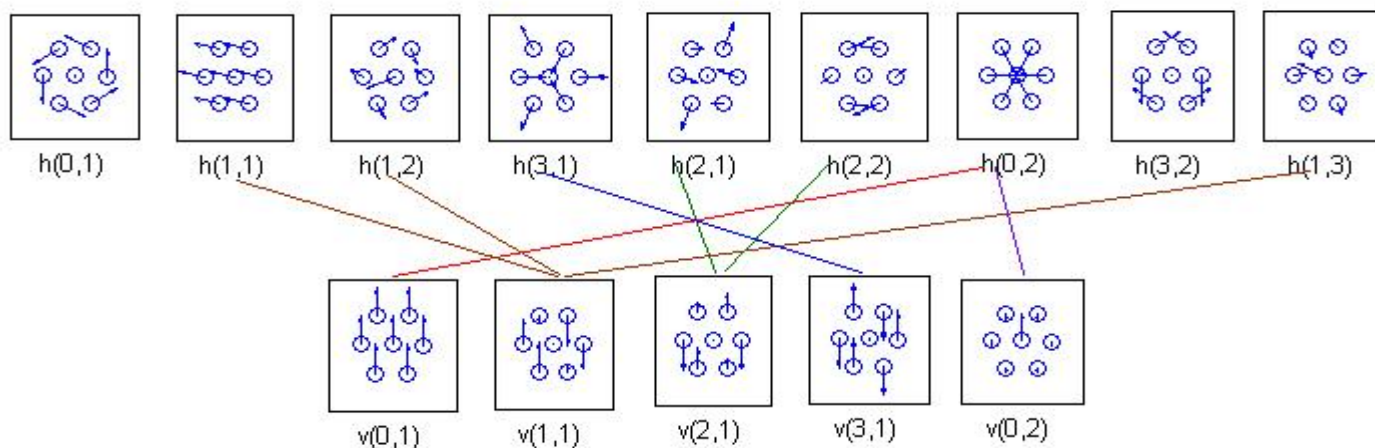
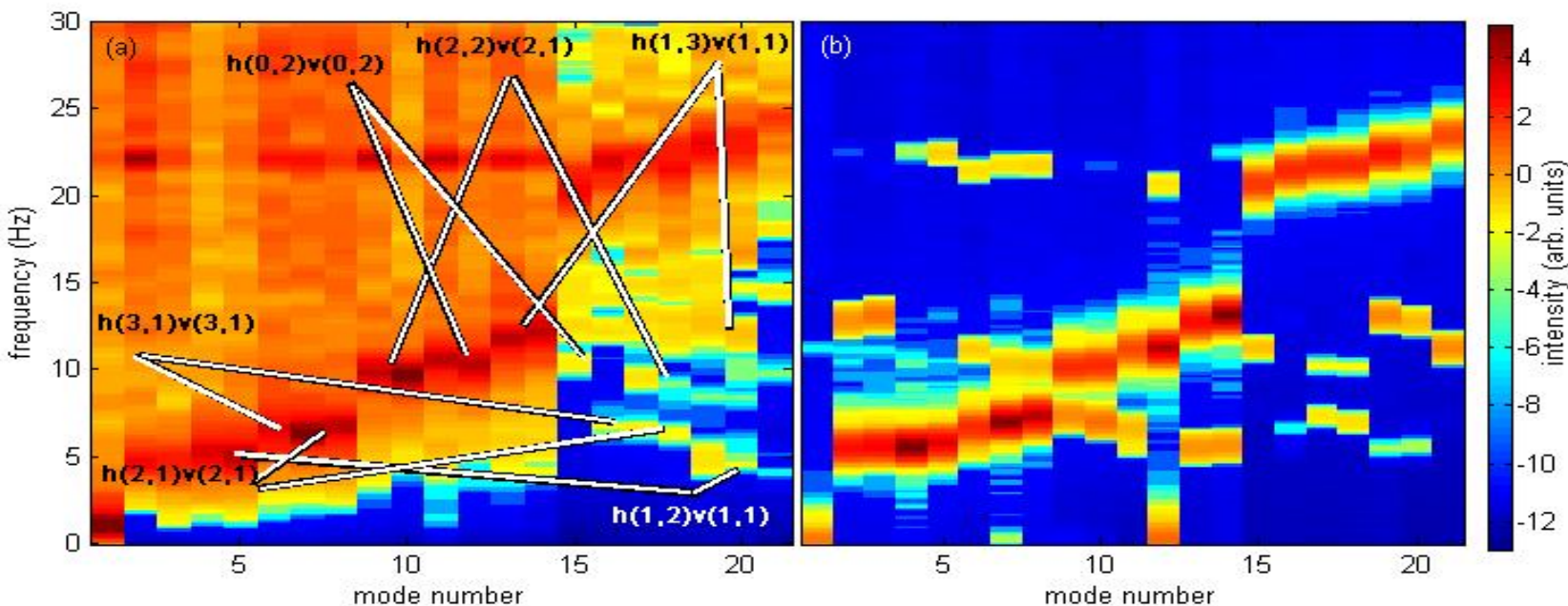
Experimental set up



Ke Qiao, J. Kong, J. Carmona, L.S. Matthews and T.W. Hyde, Phys. Rev. E 90, 033109 (2014)



Mode coupling in a seven particle circular cluster

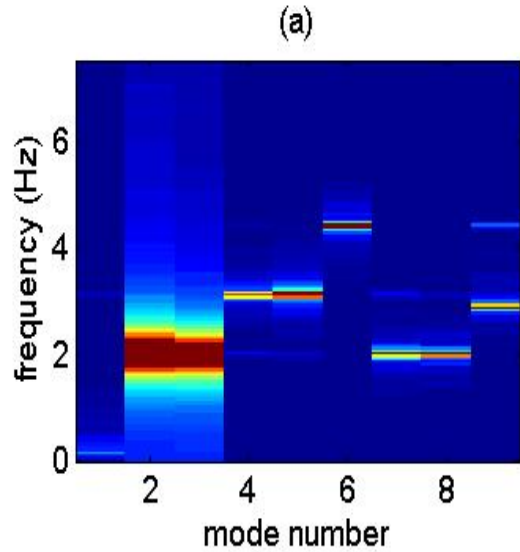


Ke Qiao, J. Kong, Eric Van Oeveren, L.S. Matthews and T.W. Hyde, Phys. Rev. E 88, 043103 (2013)

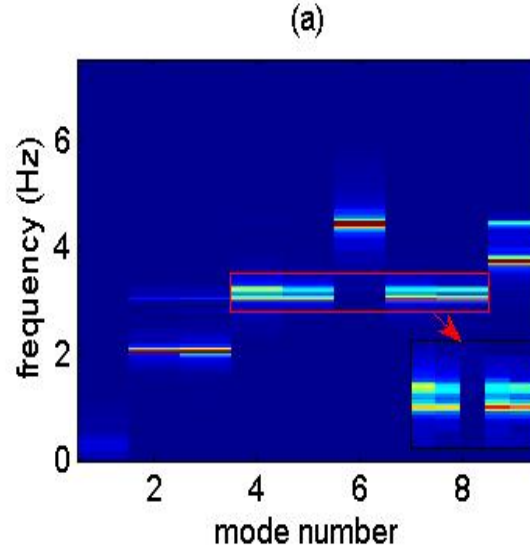
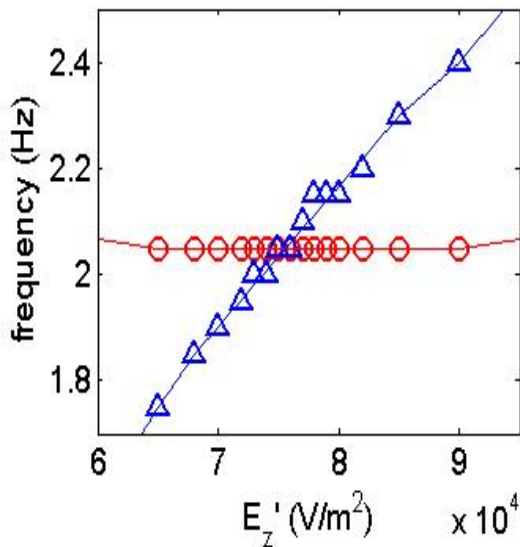
Ke Qiao, J. Kong, J. Carmona, L.S. Matthews and T.W. Hyde, Phys. Rev. E 90,, 033109 (2014)

Melzer, Physical Review E 67, 016411 (2003)

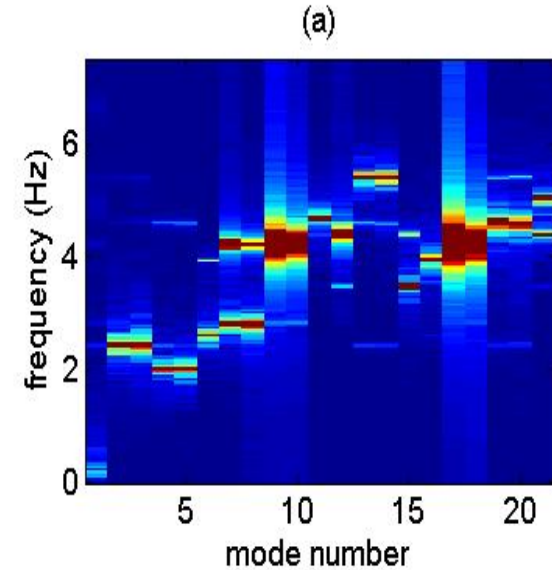
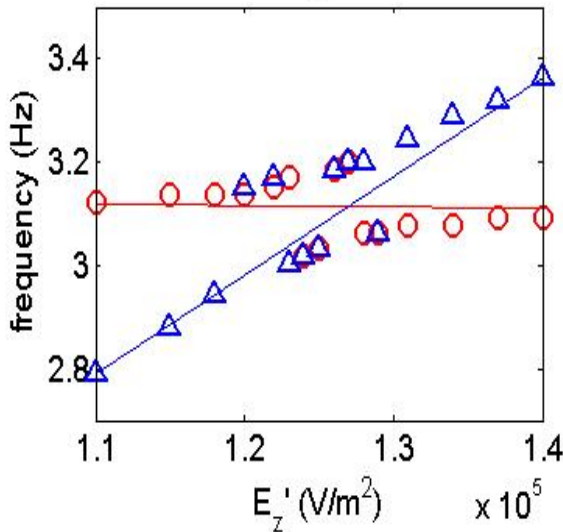
Three type of resonances (simulation)



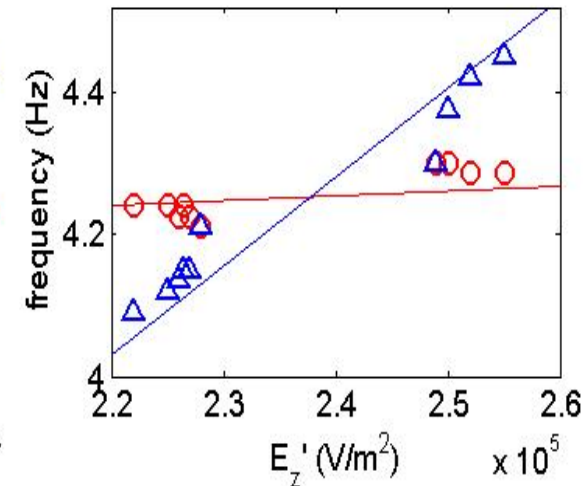
(b)



(b)

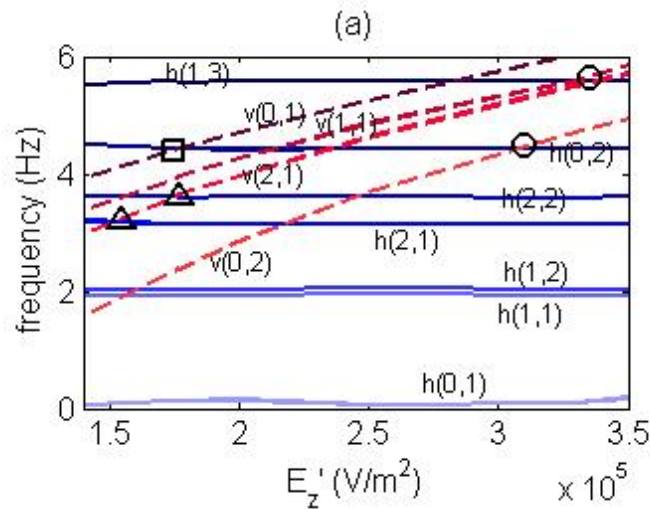


(b)

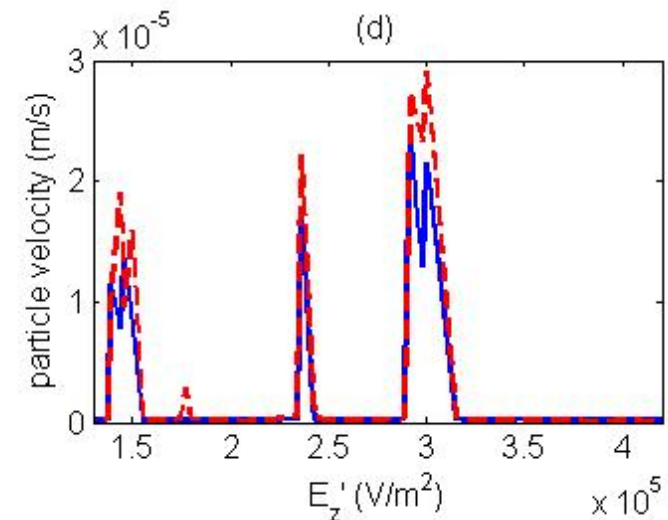
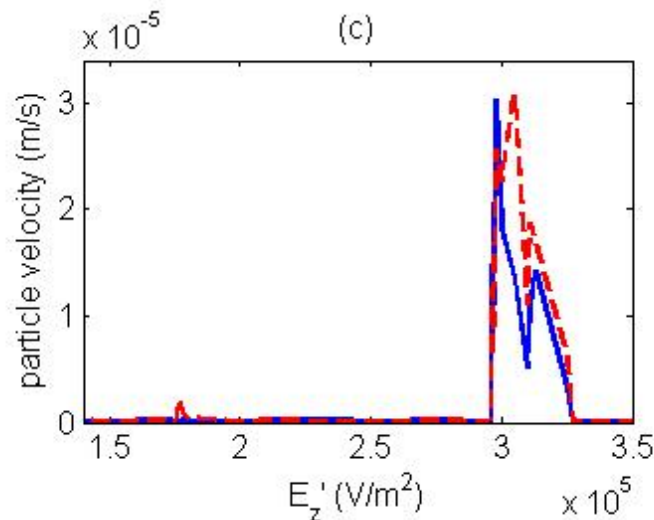
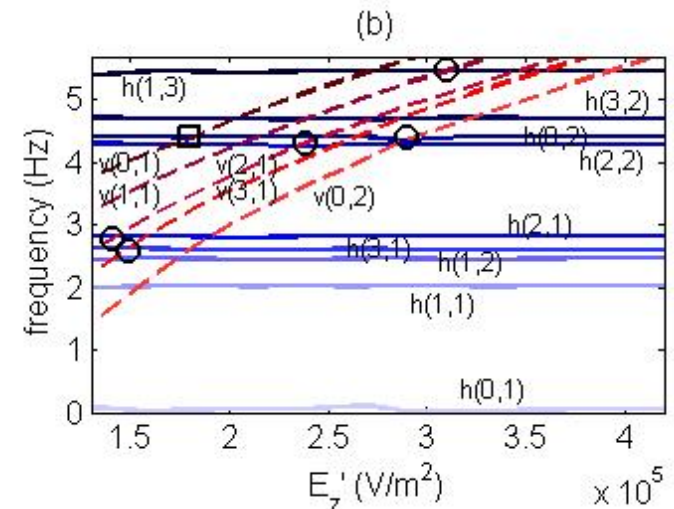


Discrete resonance instabilities (simulation)

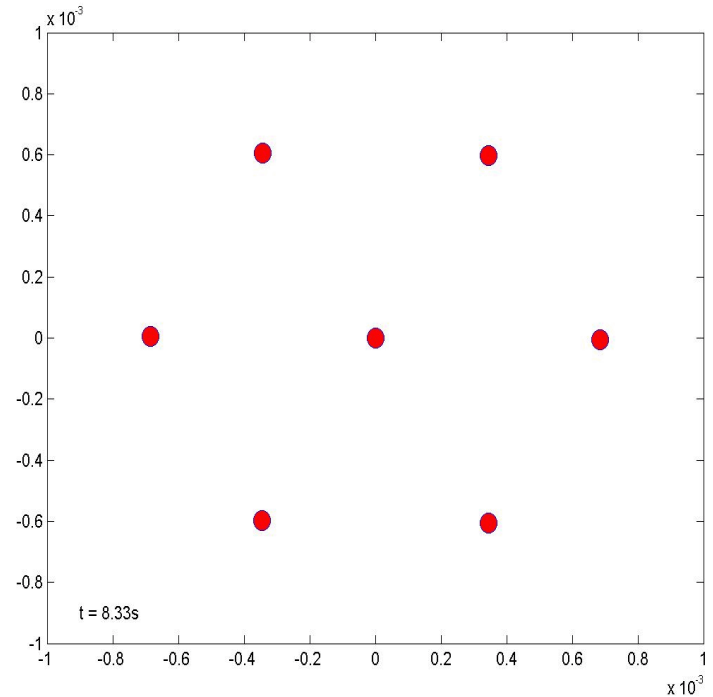
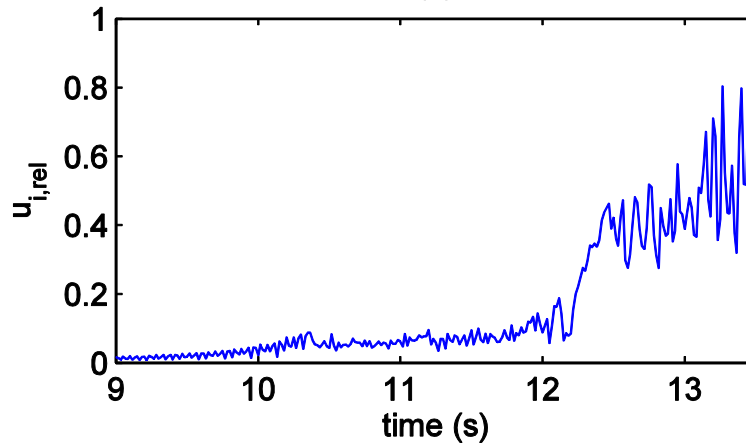
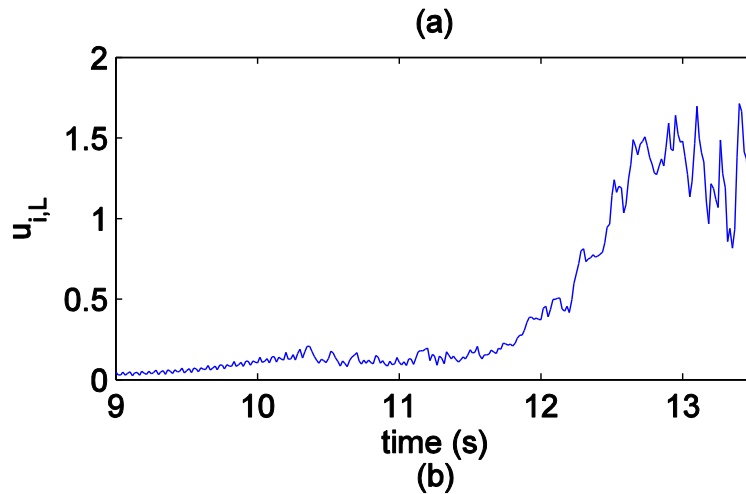
Six particles



seven particles



Instability induced Melting (simulation)



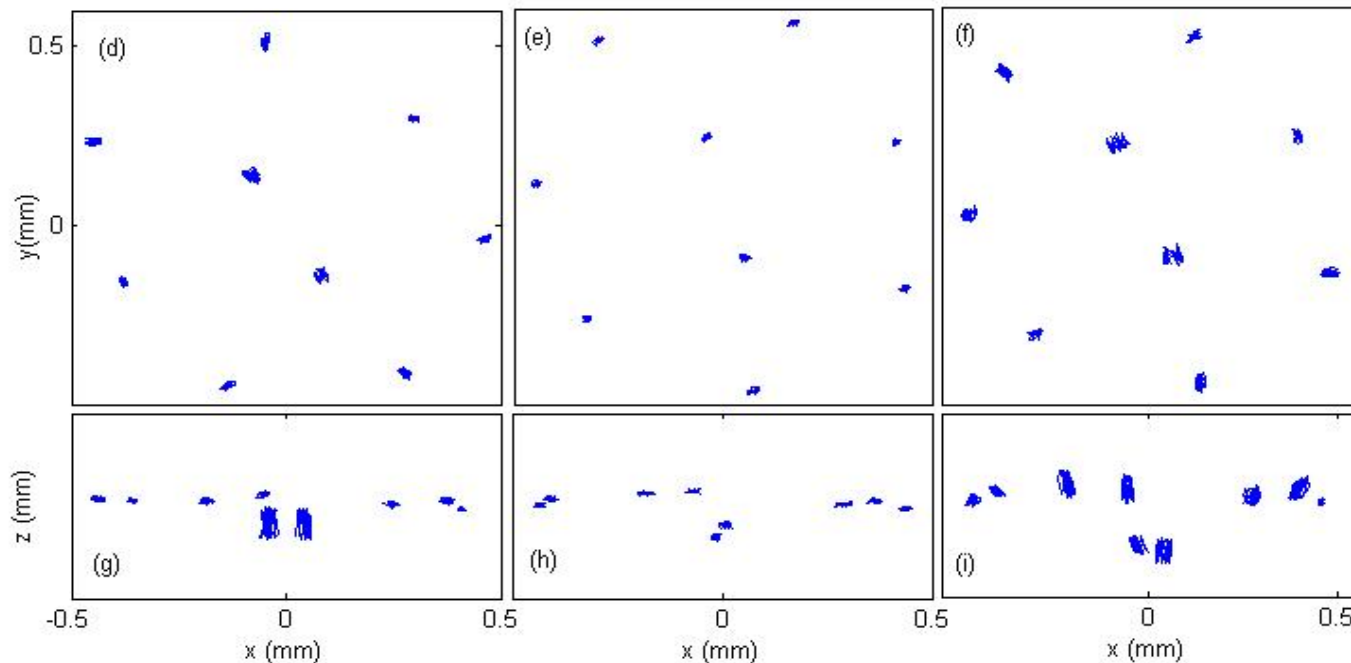
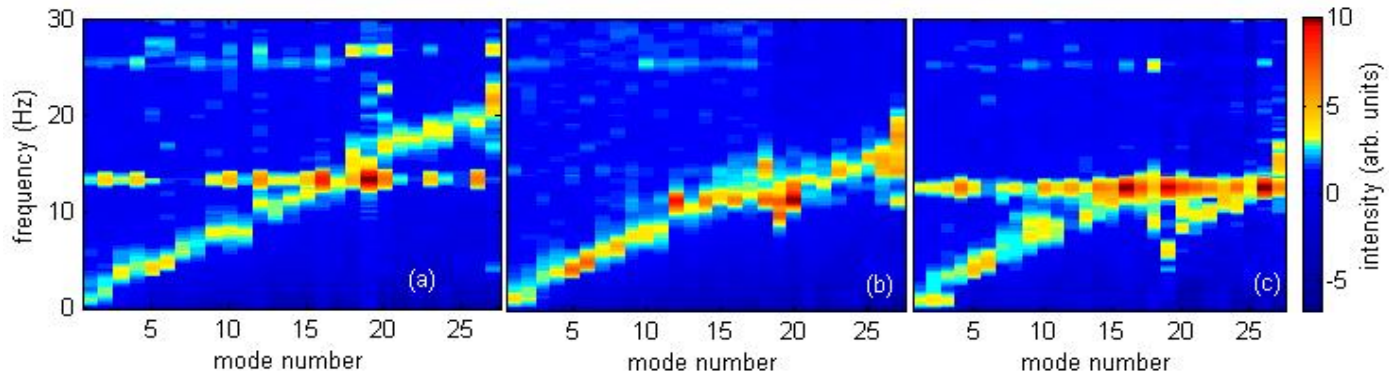
Lindemann criterion:

$$u_{i,rel(x)} \approx u_{i,rel(z)} \approx 0.1$$

F. A. Lindemann, Physik. Z. (1910) 11, 609–612.

Resonance instabilities (experiment)

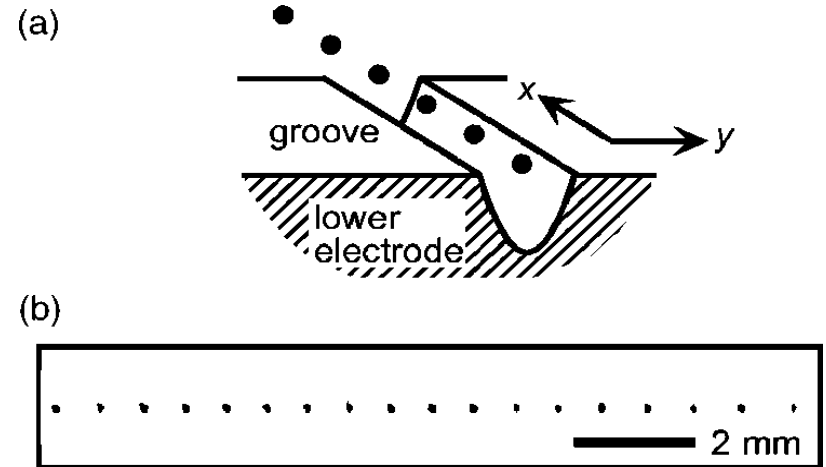
rf power decrease



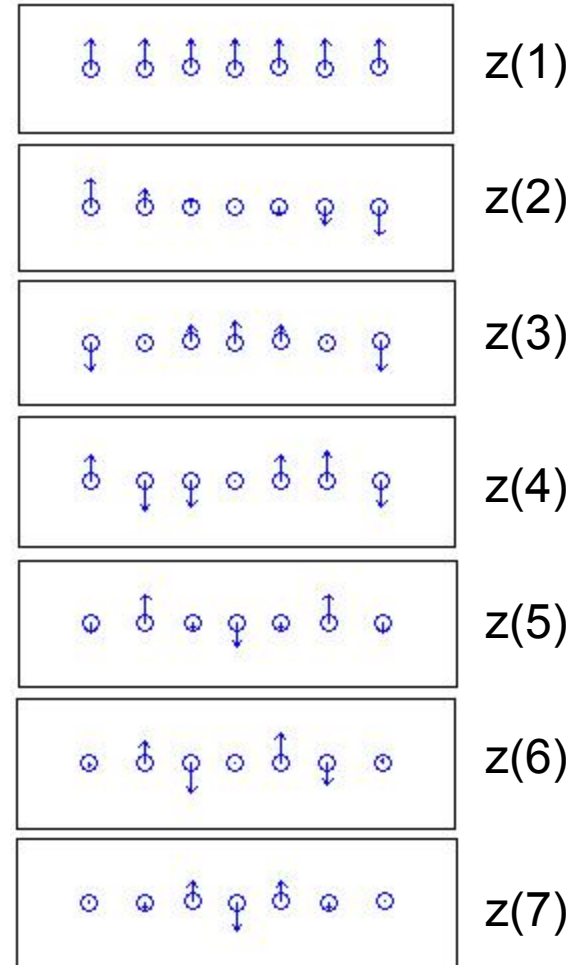
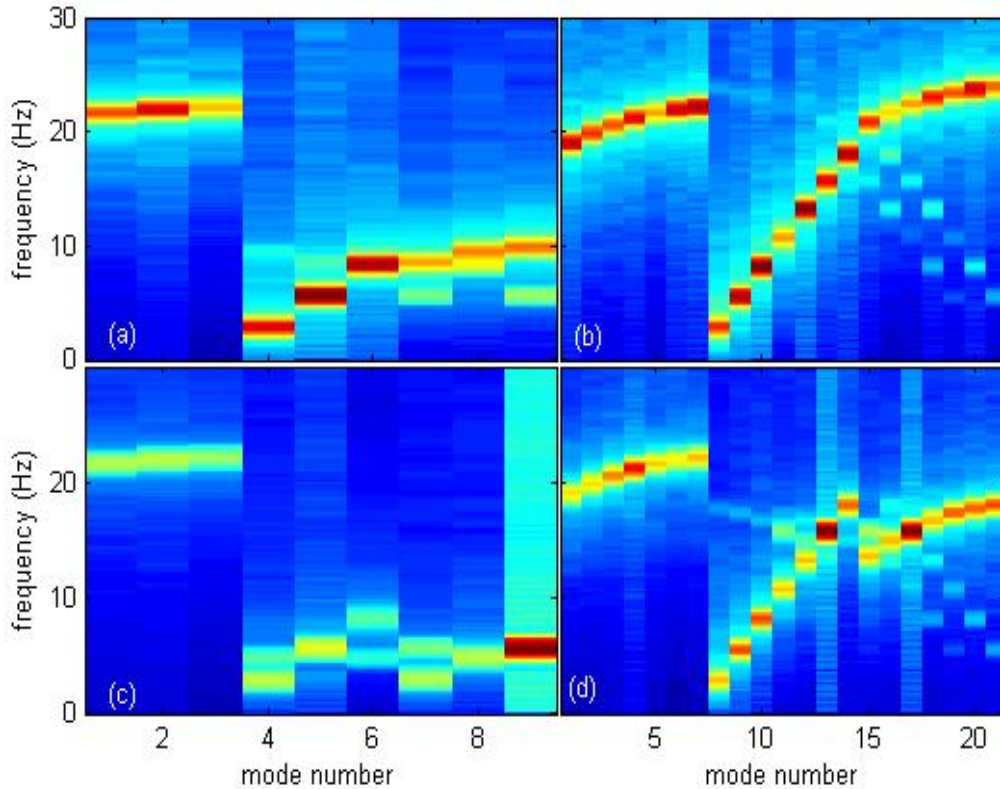
Finite Horizontal Chain

- 1D system
- Simpler rules of mode coupling
- Transition to dispersion relation in large systems
- Chain structure similar to that observed in poly-atomic molecules

- $N = 3\sim 50$ (box_tree N-body code)
- $Q = 22000 e$, $m = 5.5e-13$ kg, $\lambda = 500$ μm
- $q = Q/4$ ($Q/8$ for $N > 6$), $l_q = \lambda/2$
- Parabolic confinement: $f_x \approx 3.5$ Hz, $f_z \approx 4\sim 30$ Hz



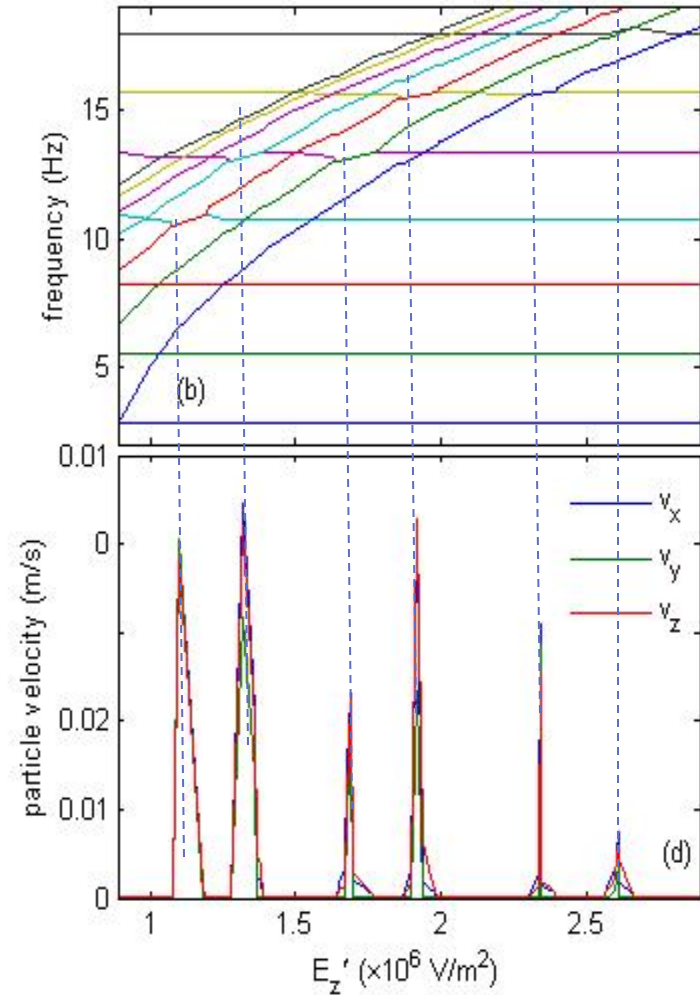
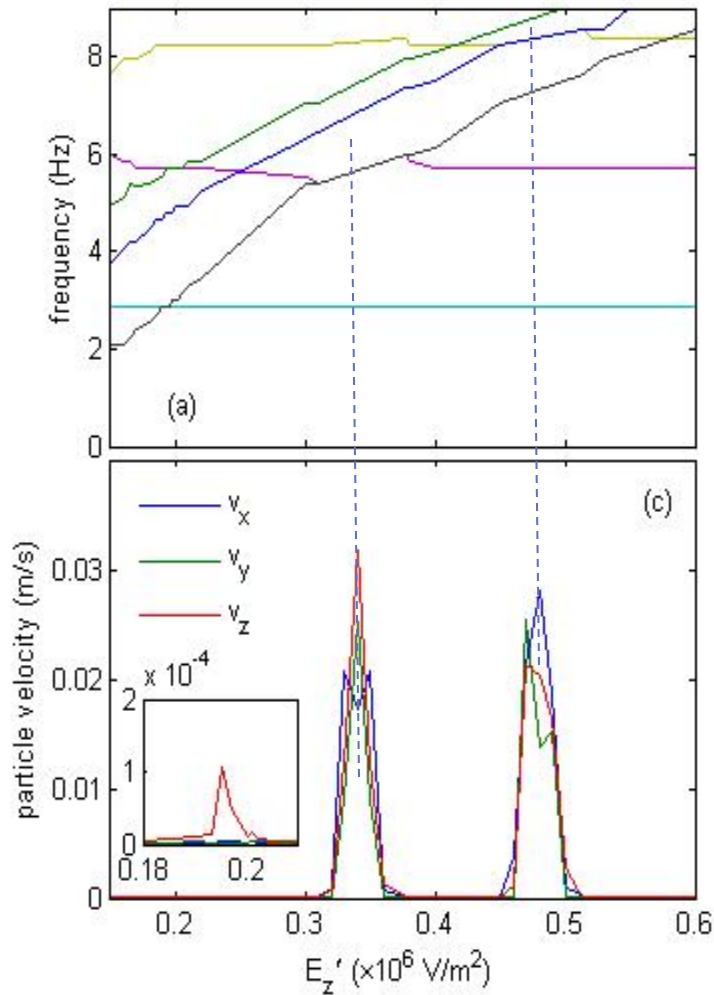
Mode coupling



Rule of coupling: coupled with modes of adjacent indexes.

Three particles: $x(2)z(1)$ resonance, Seven particles: $x(6)z(5)$ resonance.

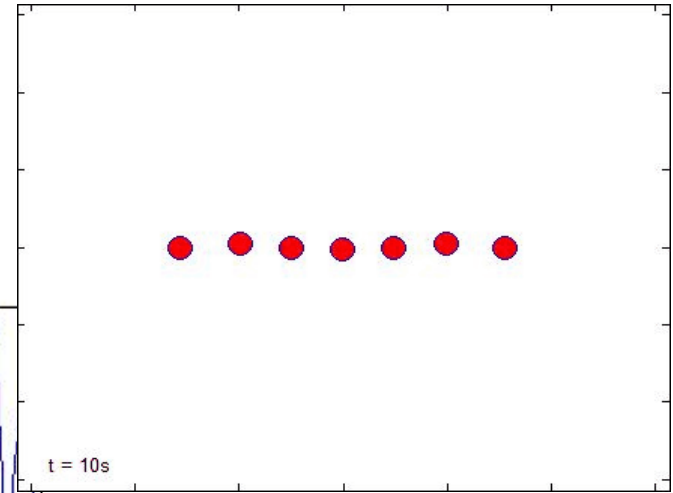
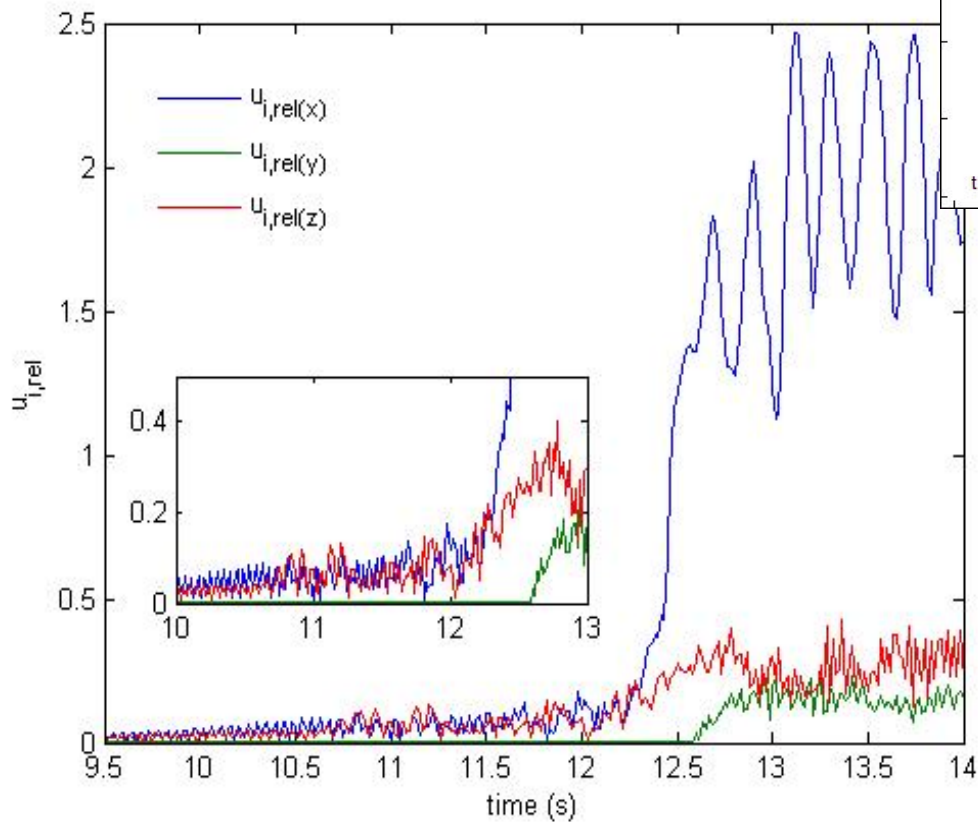
Resonance instabilities



Type I and III resonance

Melting

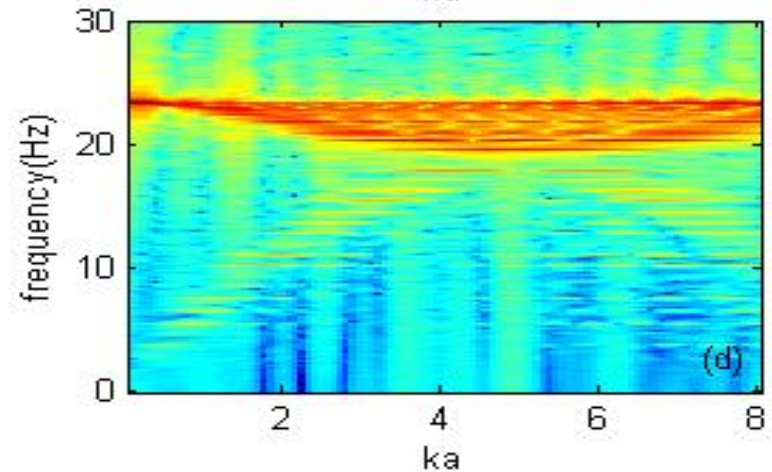
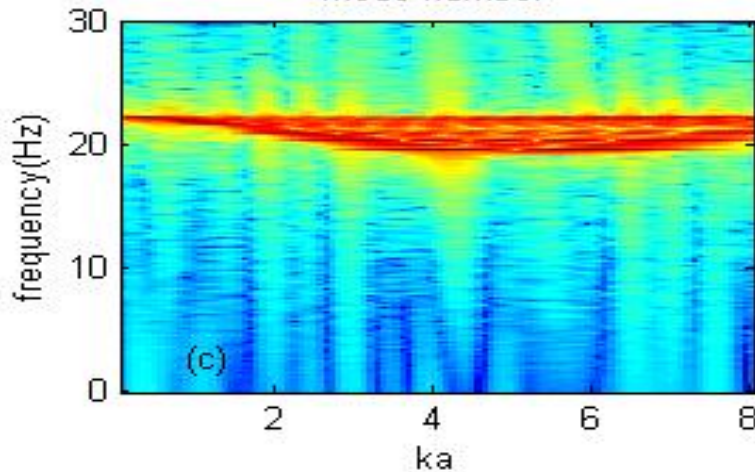
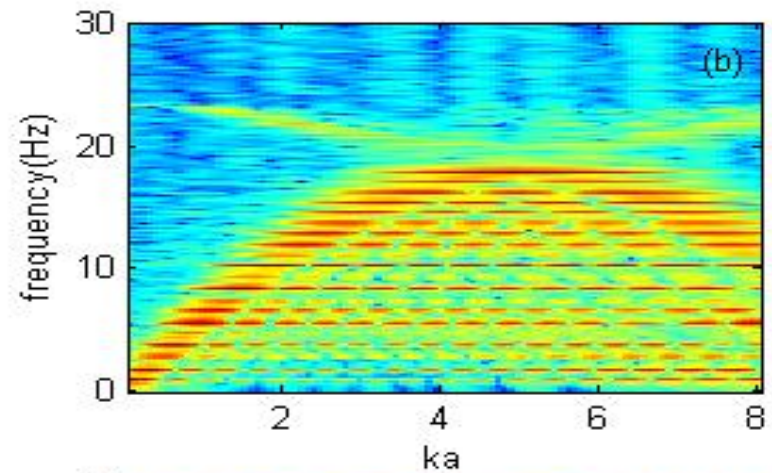
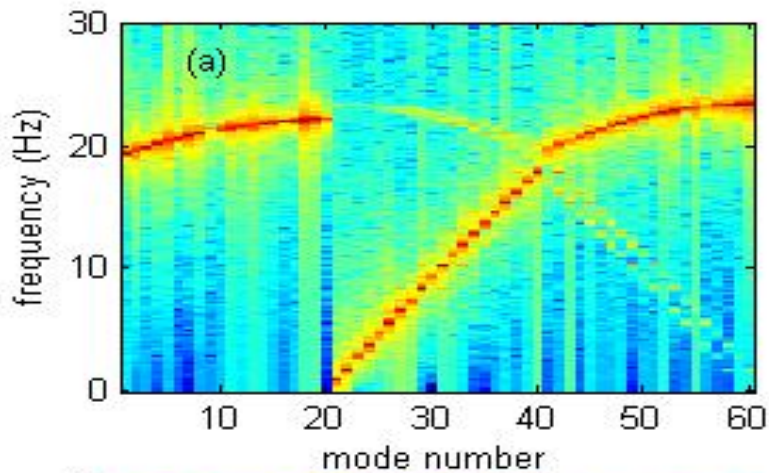
Two-step melting



Lindemann criterion:

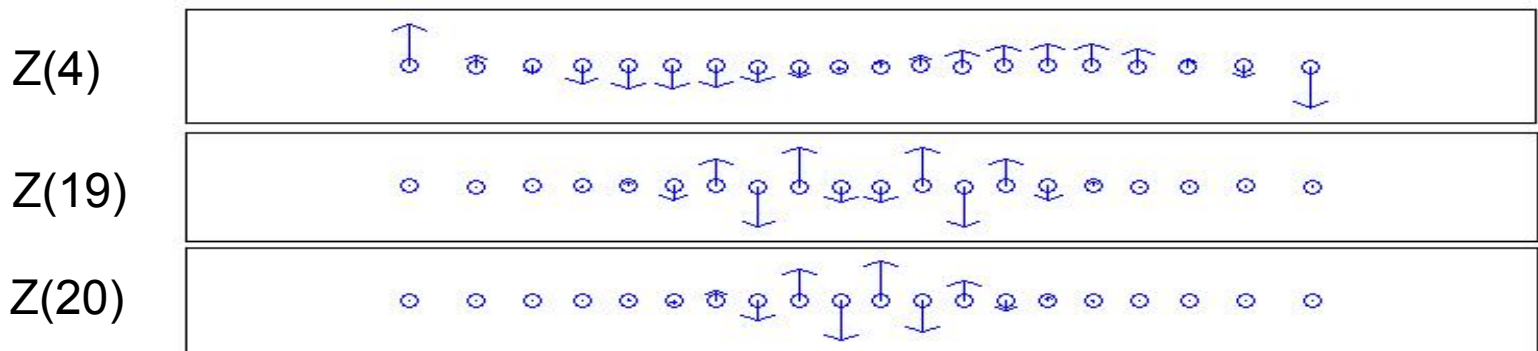
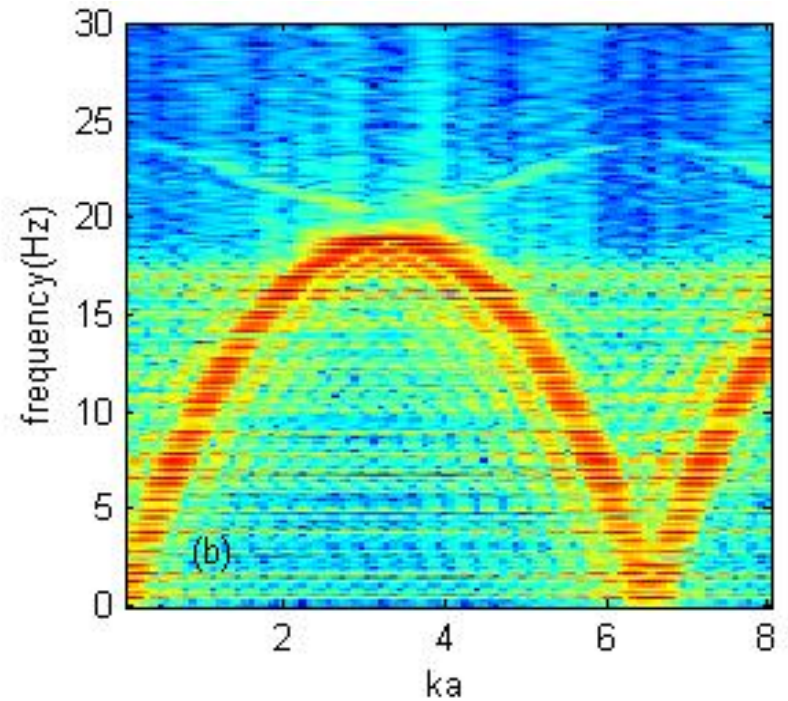
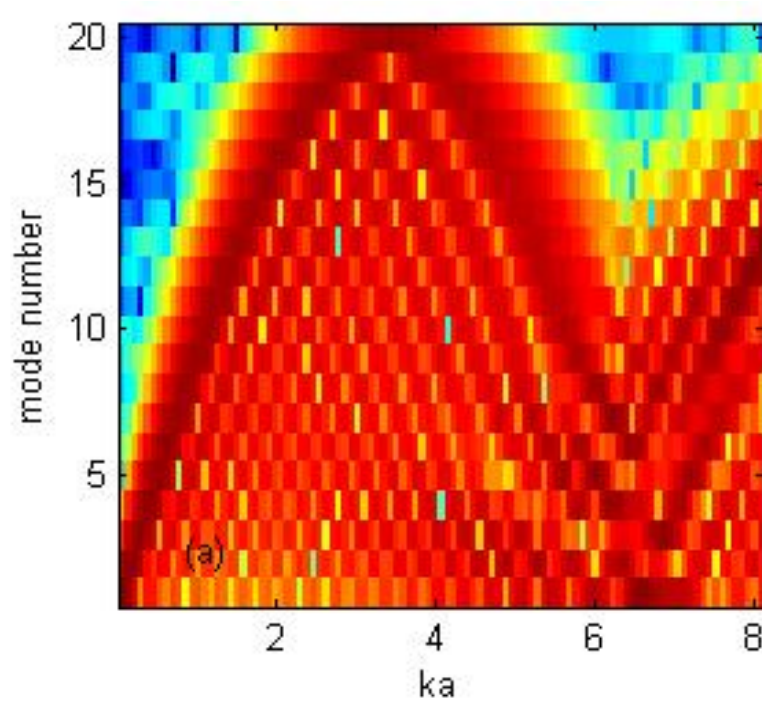
$$u_{i,rel}(x) \approx u_{i,rel}(z) \approx 0.1$$

Mode spectra and dispersion relations (twenty particles)

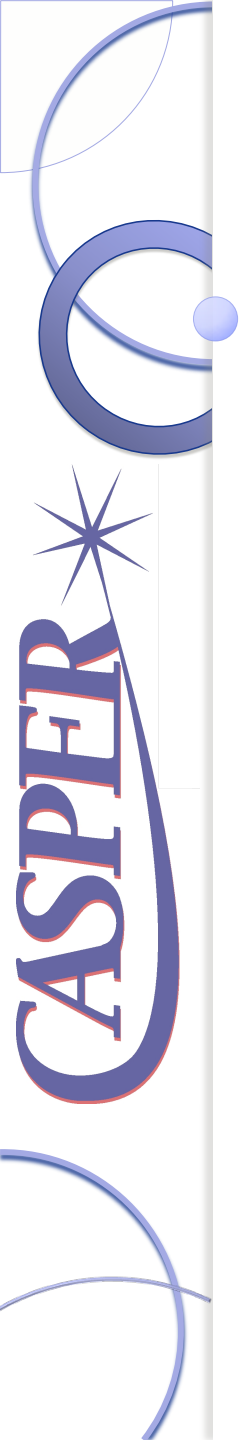
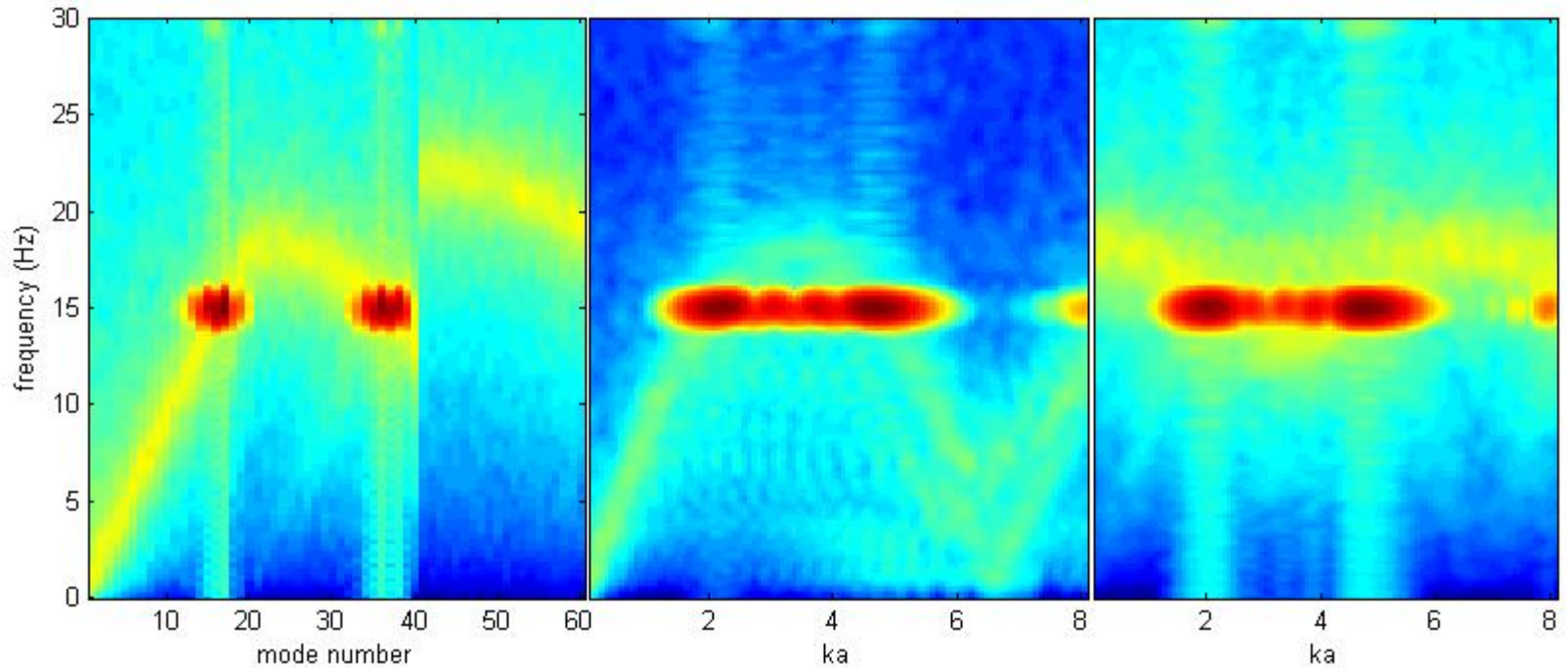


Transformation between two bases, the normal mode eigenvectors and the Fourier series on k space $U_{\downarrow n, k}(\sigma) = \sum_{i=1}^N e_{\downarrow n, k}(\sigma) e^{-ikx_{\downarrow i}}$

Transition matrix and effect of finiteness

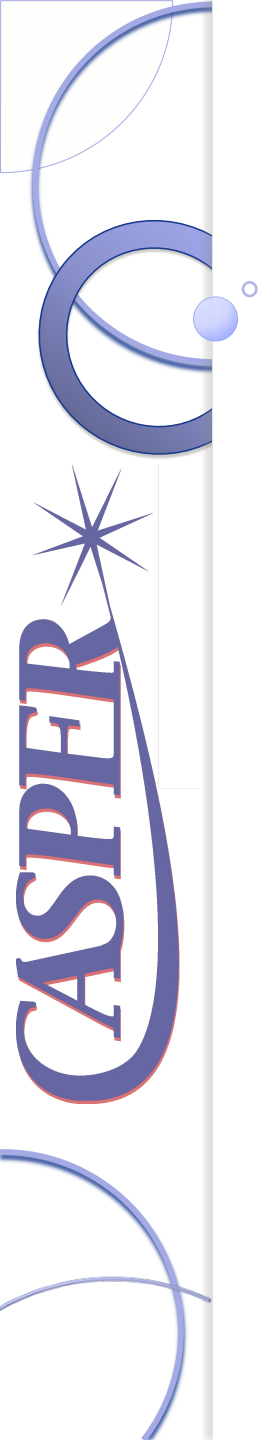


crossing of the x and z branches



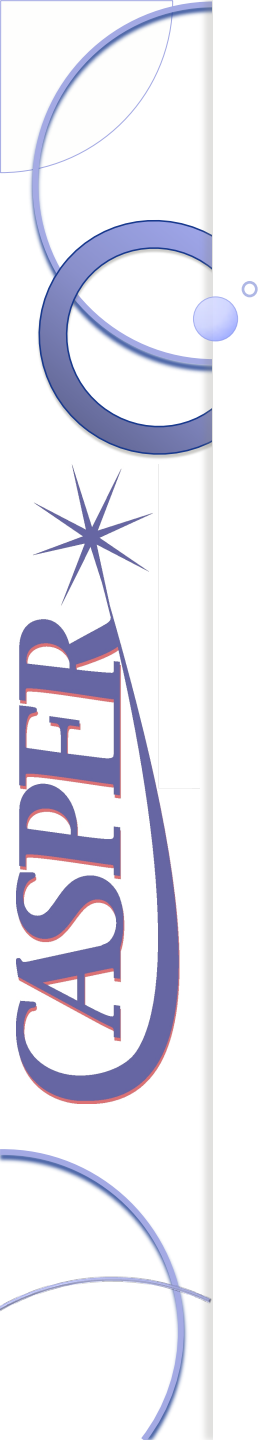
Conclusions

- Coupling pattern between individual modes
- Different coupling rules between circular clusters and chains
- Three type of resonance and discrete instabilities
- Instability induced melting obeying Lindemann criterion
- Transition between mode spectra and dispersion relations



Future Research

- Experimental verification
- Vertical chains
- Other phenomena caused by mode coupling
 - Energy transfer in polyatomic molecules
 - Glass transition



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