Experimental study of global synchronization of the dust acoustic wave in a weakly-coupled dusty plasma system

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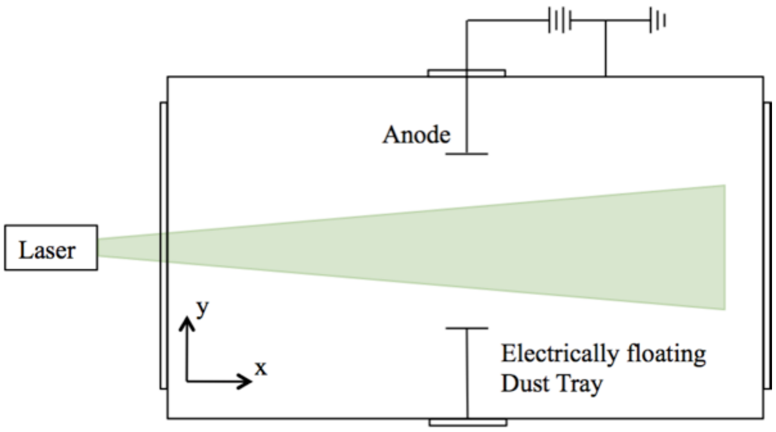
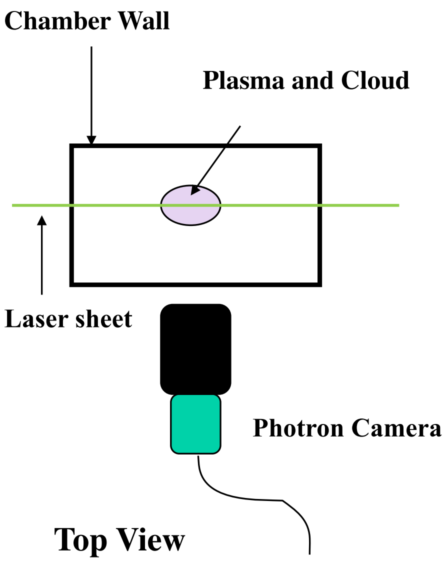
Wittenberg University DUsty Plasma Experiment

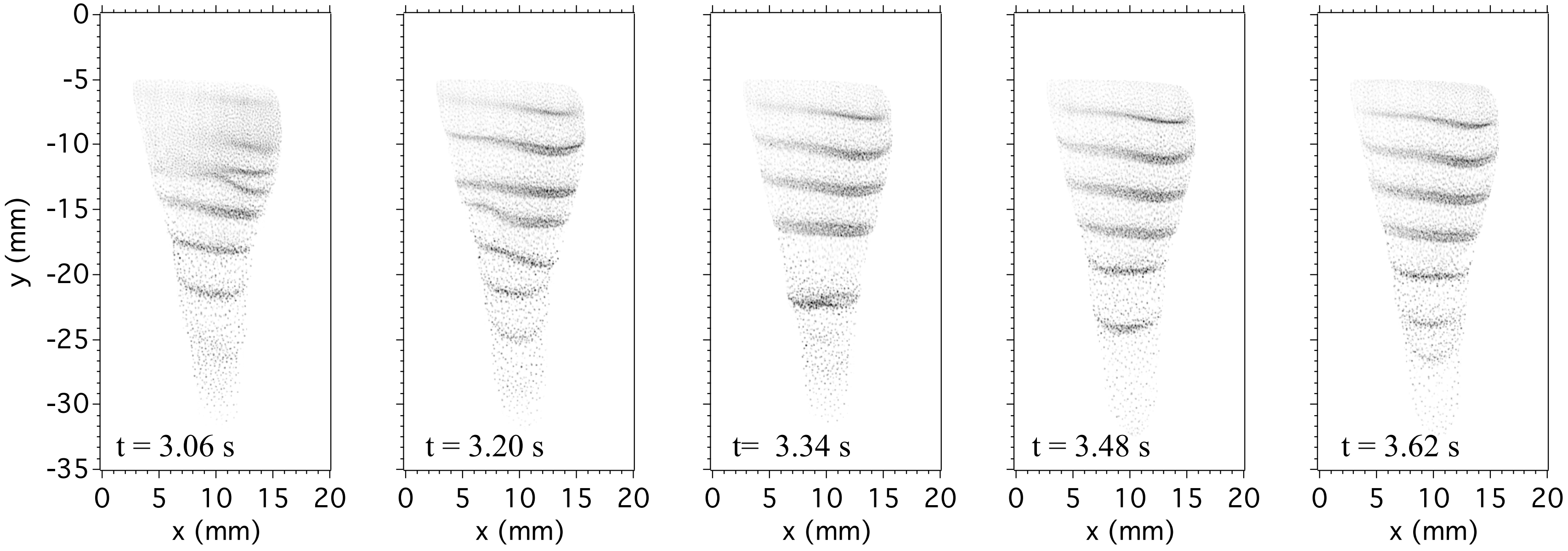




- An argon DC glow discharge plasma is generated using a constant current supply.
- Dust clouds composed of monodisperse melamine microspheres (d = 1.98 μm, ρ_d = 1510 kg/m^3) were suspended below the anode.
- A ~1 mm thick slice of the dust cloud was illuminated and imaged at 500 fps

Experimental Procedure

- Superimpose a sinusoidal modulation on the discharge current

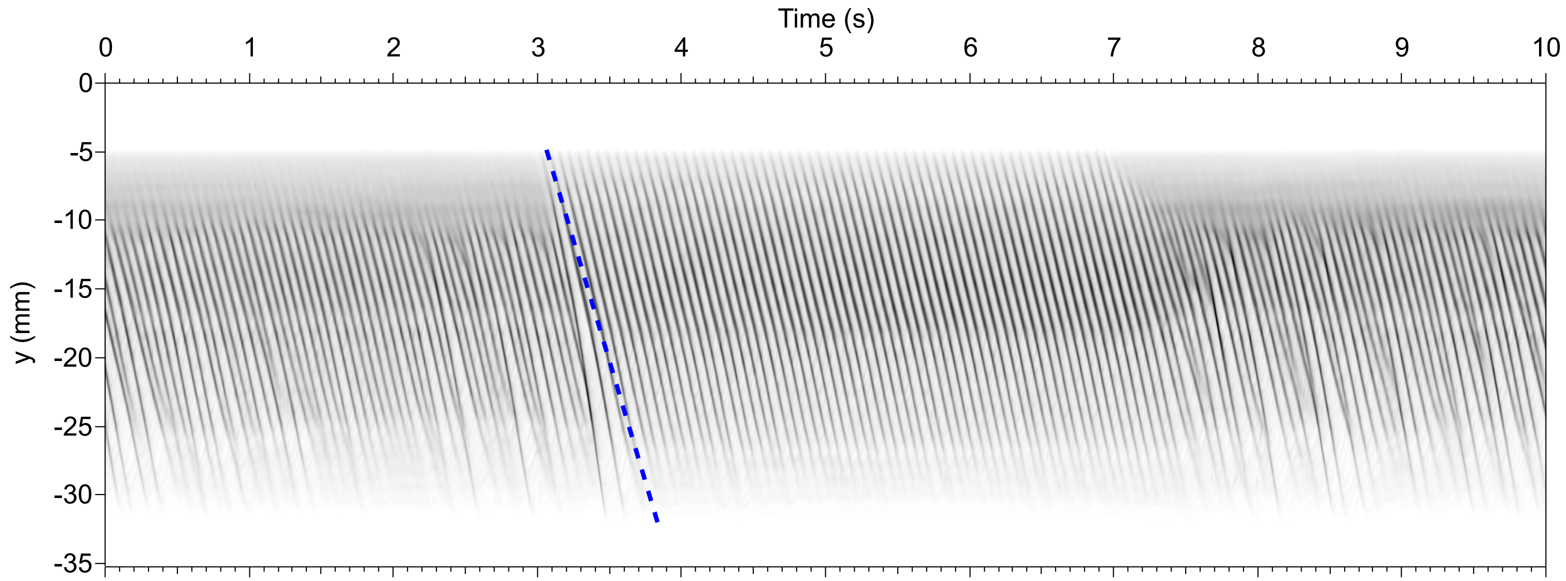


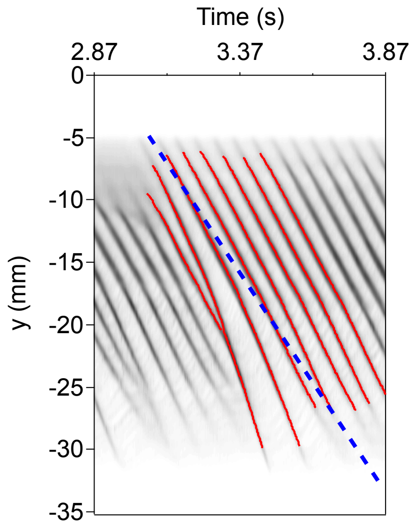
$p = 90$ mTorr
$V_{anode} = 347$ V
$I_{dis} = 0.35$ mA

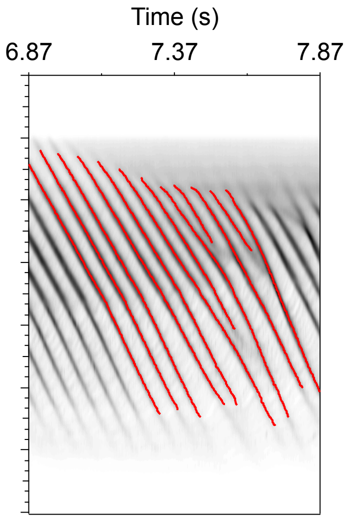
$n_e = 6.7\times10^{16}$ m^{-3}
$T_e = 5.5$ eV
$Q_d = 6500\ q_e$

J. Williams, Phys. Rev. E **90**, 043103 (2014).

Results I


Time (s)
0
1
2
3
4
5
6
7
8
9
10
y (mm)
0
-5
-10
-15
-20
-25
-30
-35


Time (s)
2.87
3.37
3.87
y (mm)
0
-5
-10
-15
-20
-25
-30
-35


Time (s)
6.87
7.37
7.87

Hilbert Transform

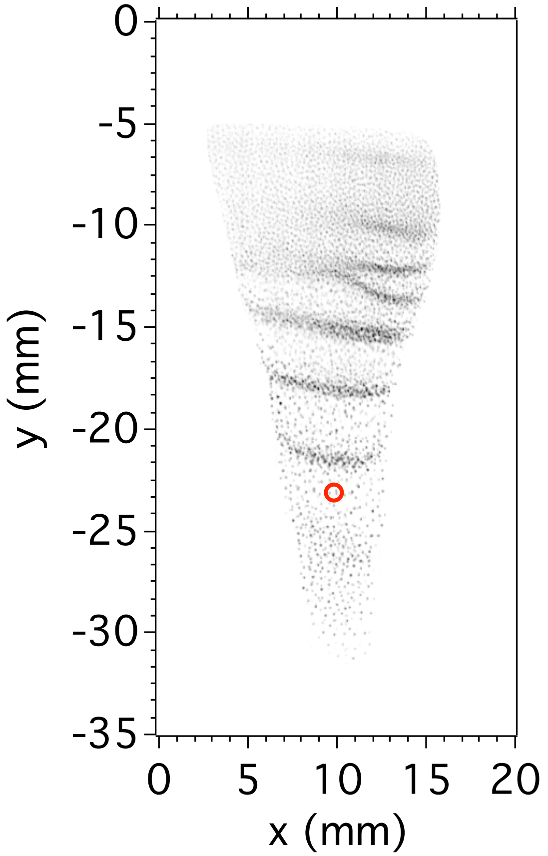
- Extract the wave structure, $n_d(x, y, t)$ at a single pixel and construct the analytic signal by expanding the wave structure into the complex plane.

$$\begin{aligned} A(x,y,t) &= n(x,y,t) + i\hat{n}(x,y,t) \\ &= E(x,y,t)\exp\left[i\cdot\phi(x,y,t)\right] \end{aligned}$$

where

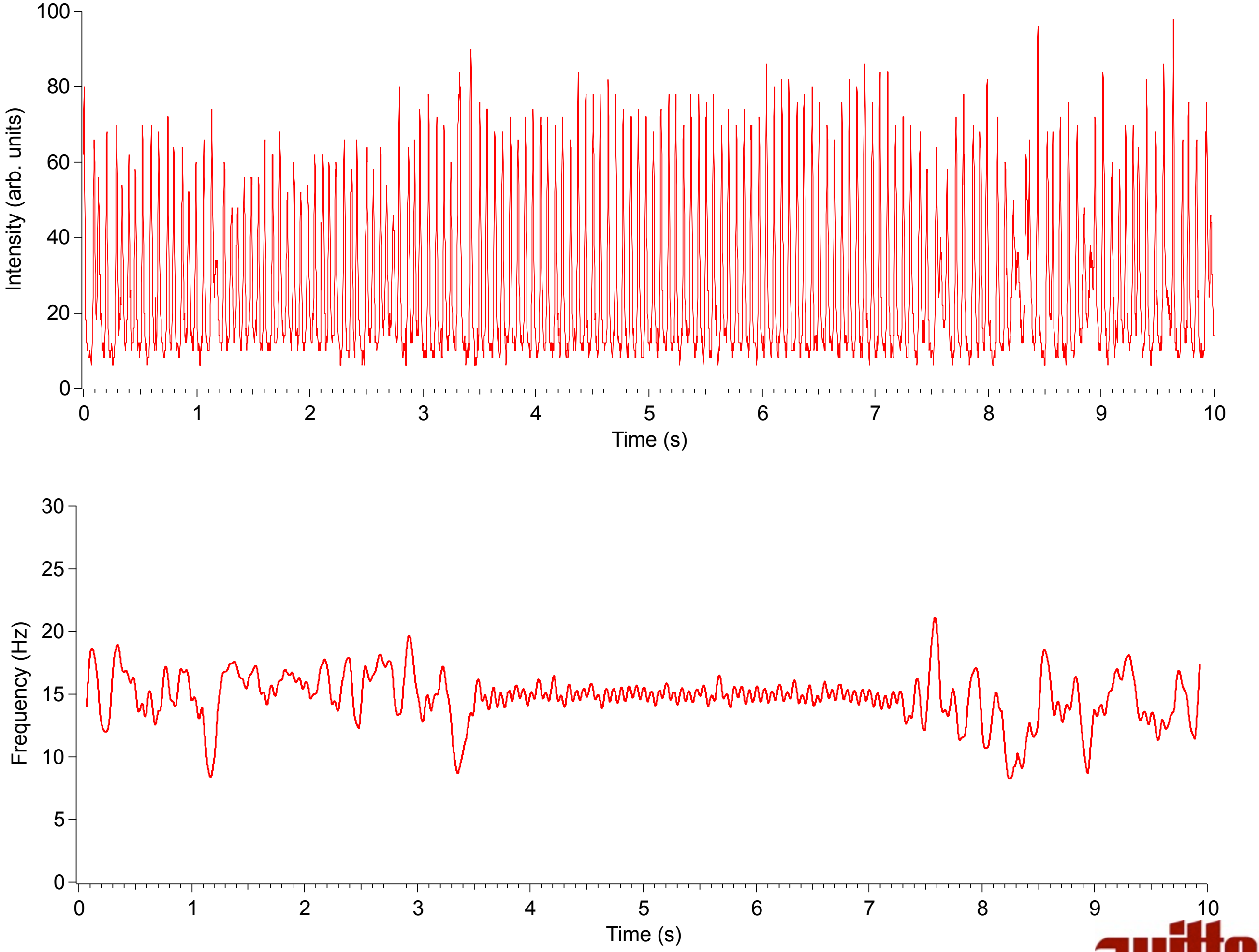
$$\hat{n}_d(x,y,t) = \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{n_d(x,y,\tau)}{t-\tau}\mathrm{d}\tau$$

- Dominant frequency mode is found by taking the temporal derivative of the unwrapped phase.

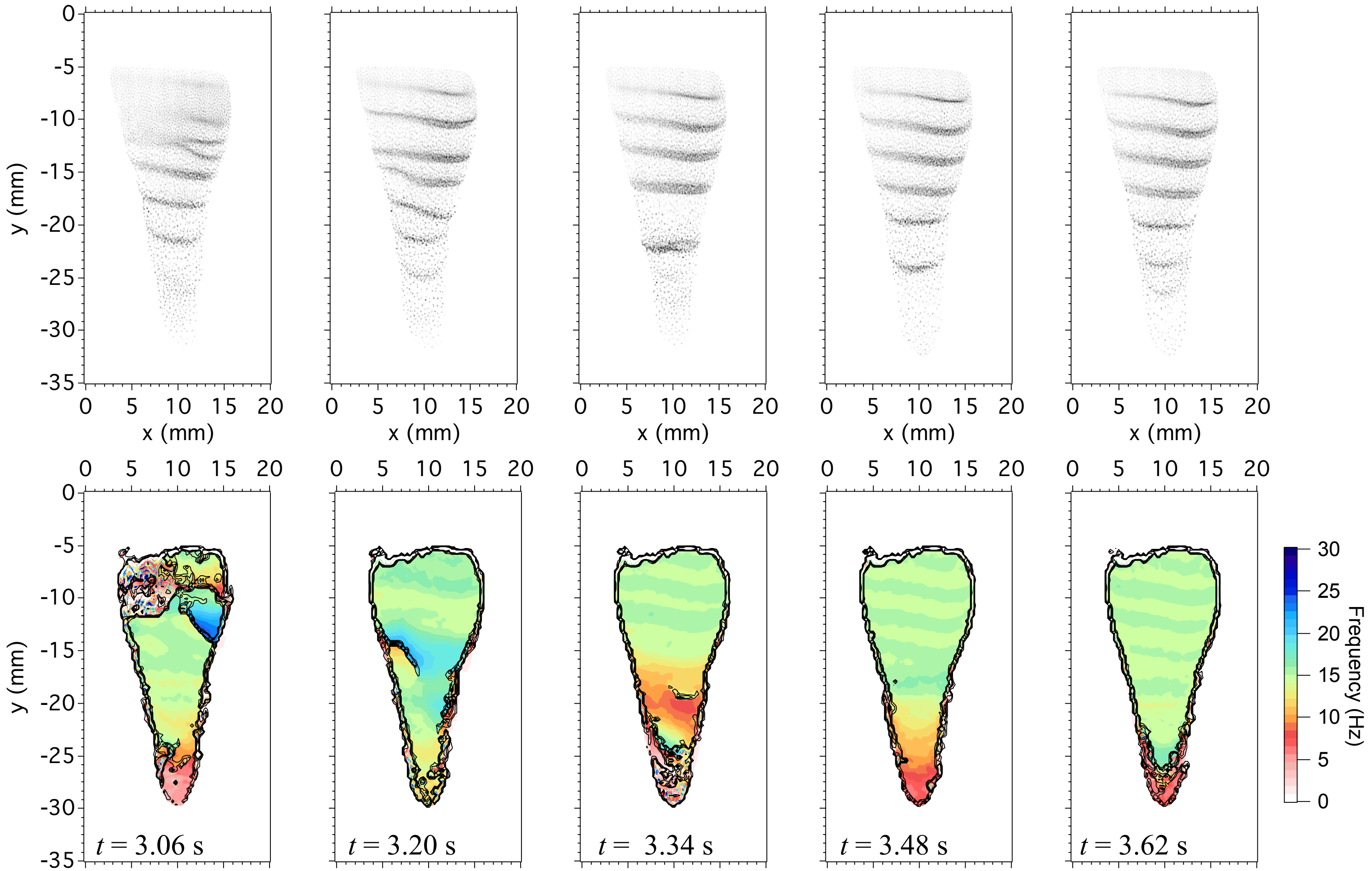


K.O. Menzel, O. Arp and A. Piel, Phys. Rev. E **83**, 016402 (2011).
J. Williams, Phys. Rev. E **89**, 023105 (2014).

Hilbert Transform II

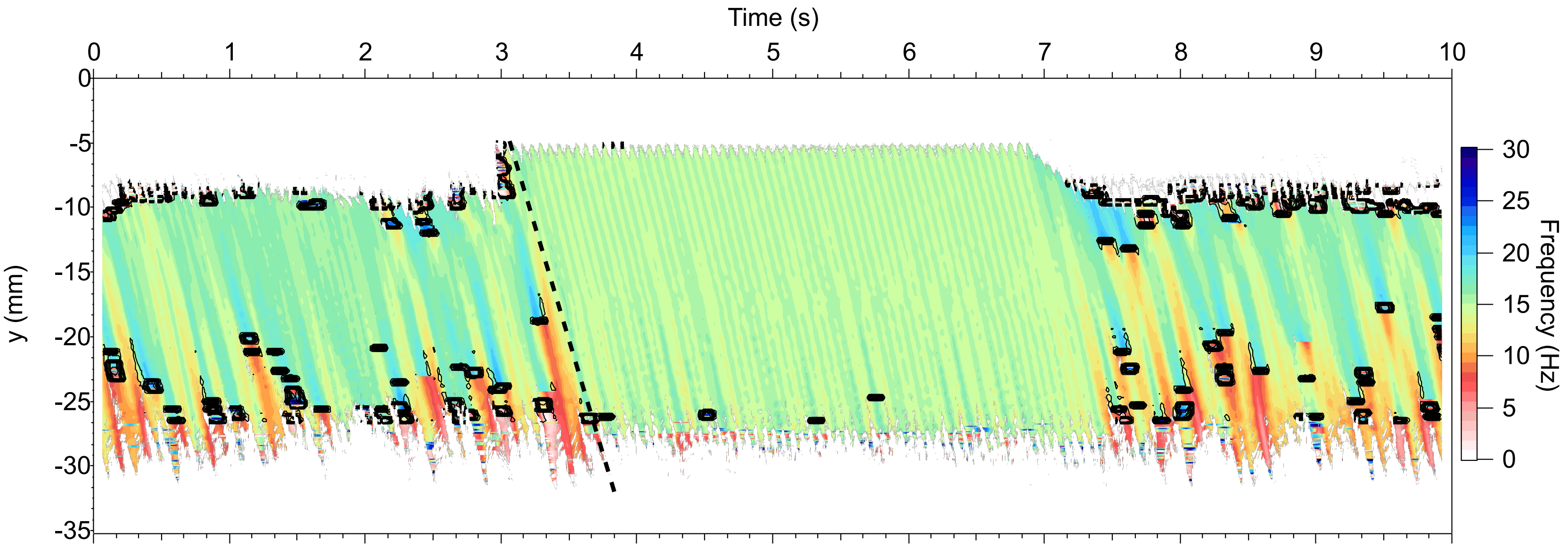


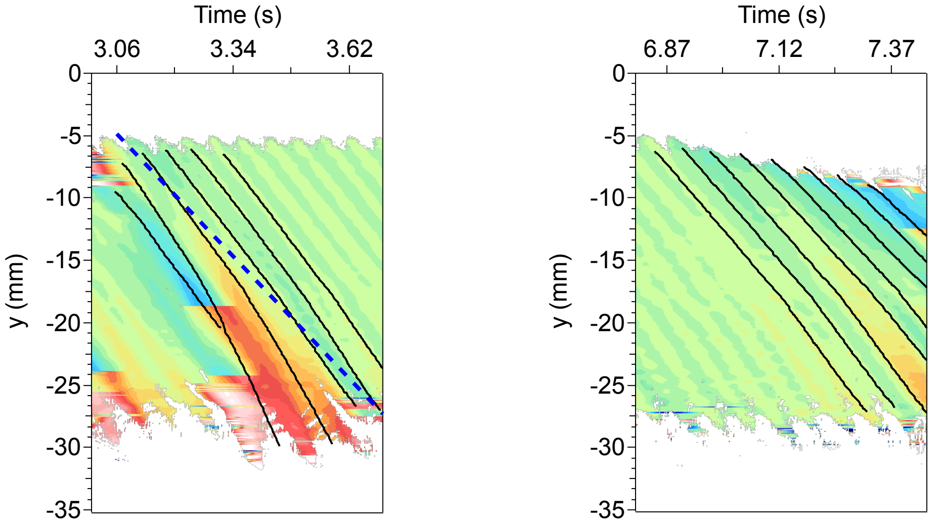
Hilbert Transform III



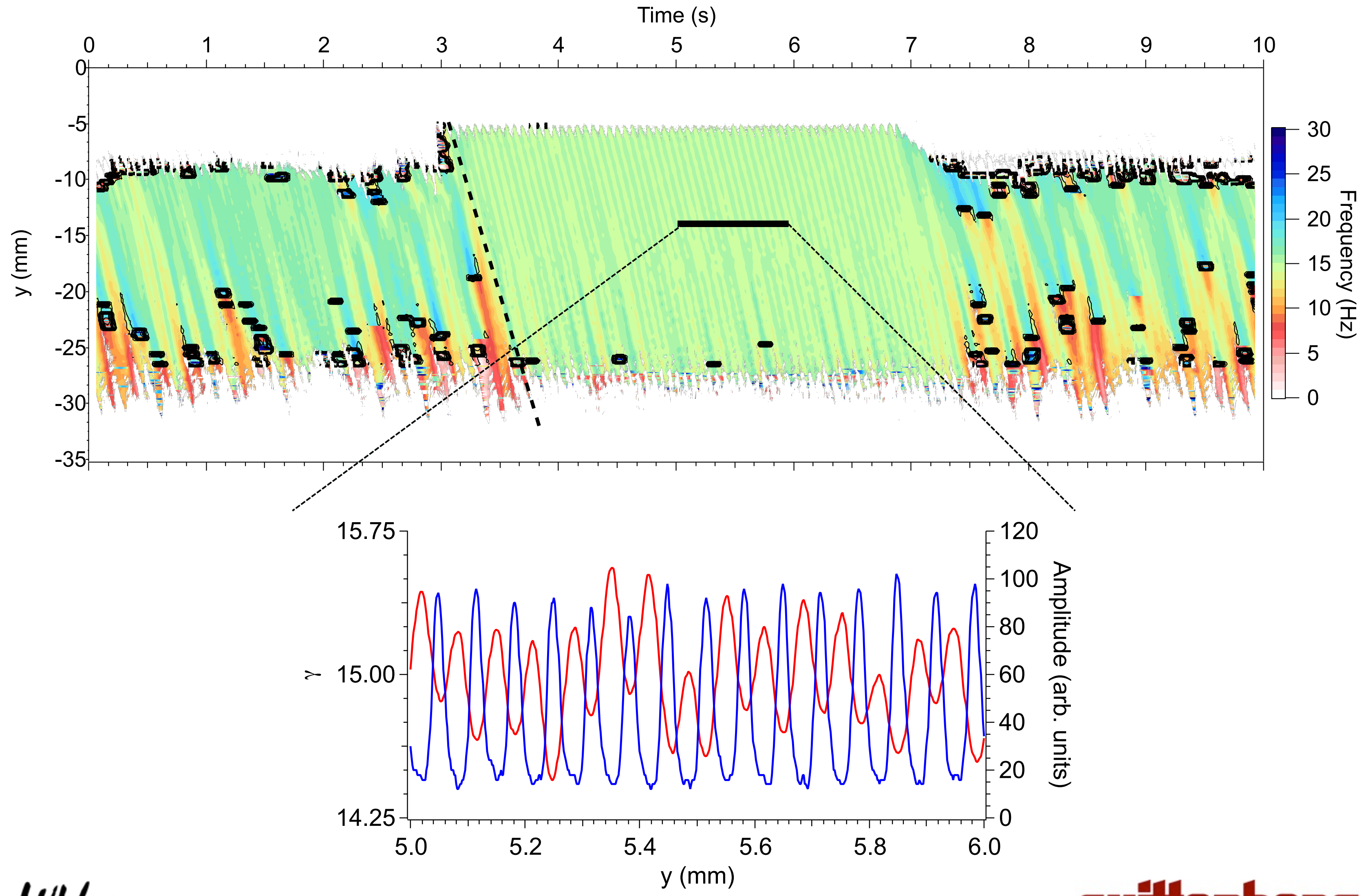
J. Williams, Phys. Rev. E **89**, 023105 (2014).
J. Williams, Phys. Rev. E **90**, 043103 (2014).

Hilbert Transform IV





Hilbert Transform IV



Synchronization Index I

- Compute the Shannon entropy by constructing a distribution of cumulative difference in the unwrapped phase (mod 2p), $\Delta\phi$, found from the external modulation $I_{mod,pp}(t)$ and the wave structure at a given location using a rolling window of 101 frames.

$$H = \sum_{k=1}^{N} p_k ln(p_k)$$

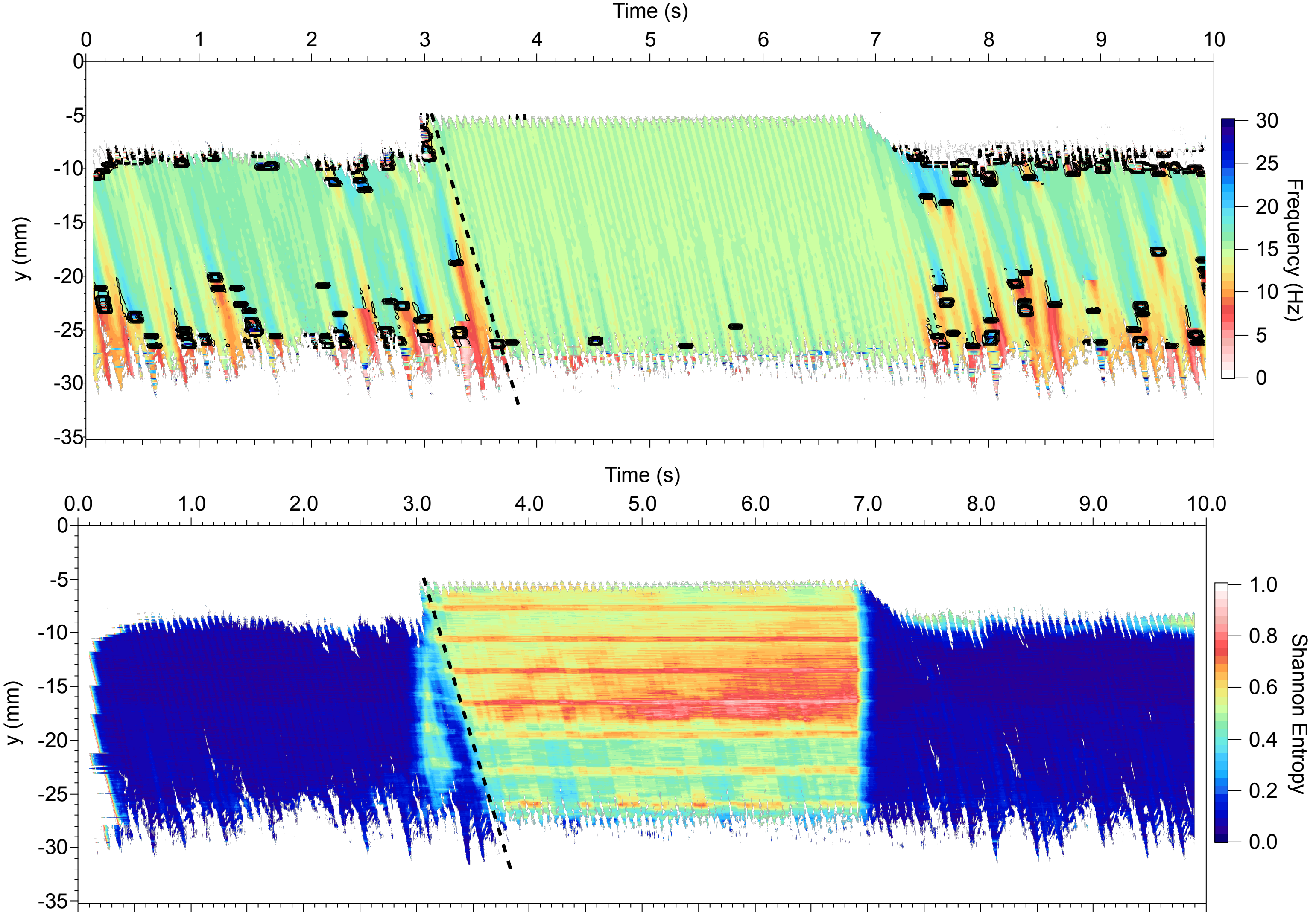
- From the Shannon Entropy, we calculate the synchronization index

$$\gamma = \frac{H_{max} - H}{H_{max}}$$

where $H_{max} = ln(N)$ is the maximum entropy corresponding to a uniform distribution.

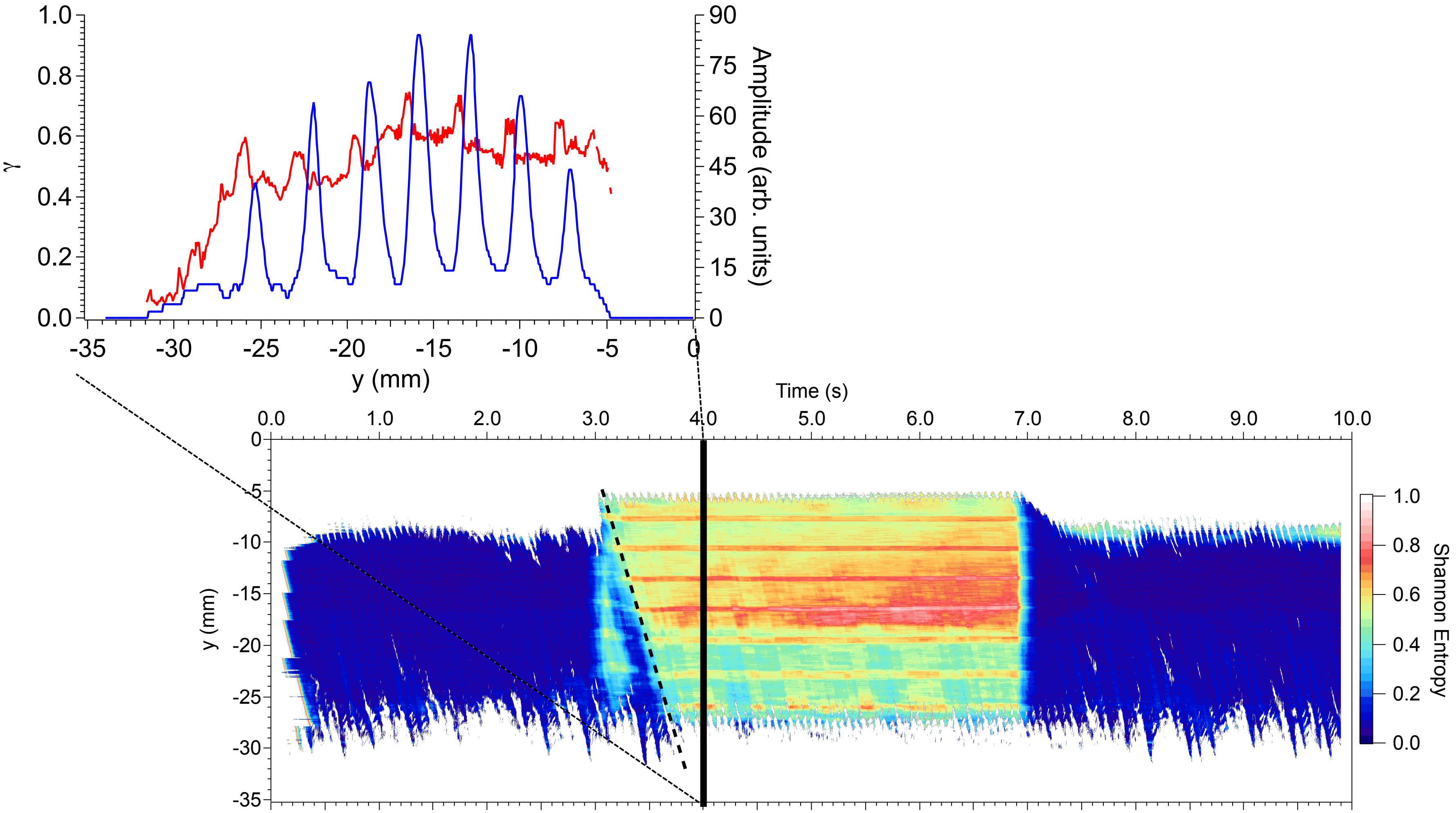
L. Couëdel, *et. al.*, Phys. Rev. E **89**, 053108 (2014).

Synchronization Index II



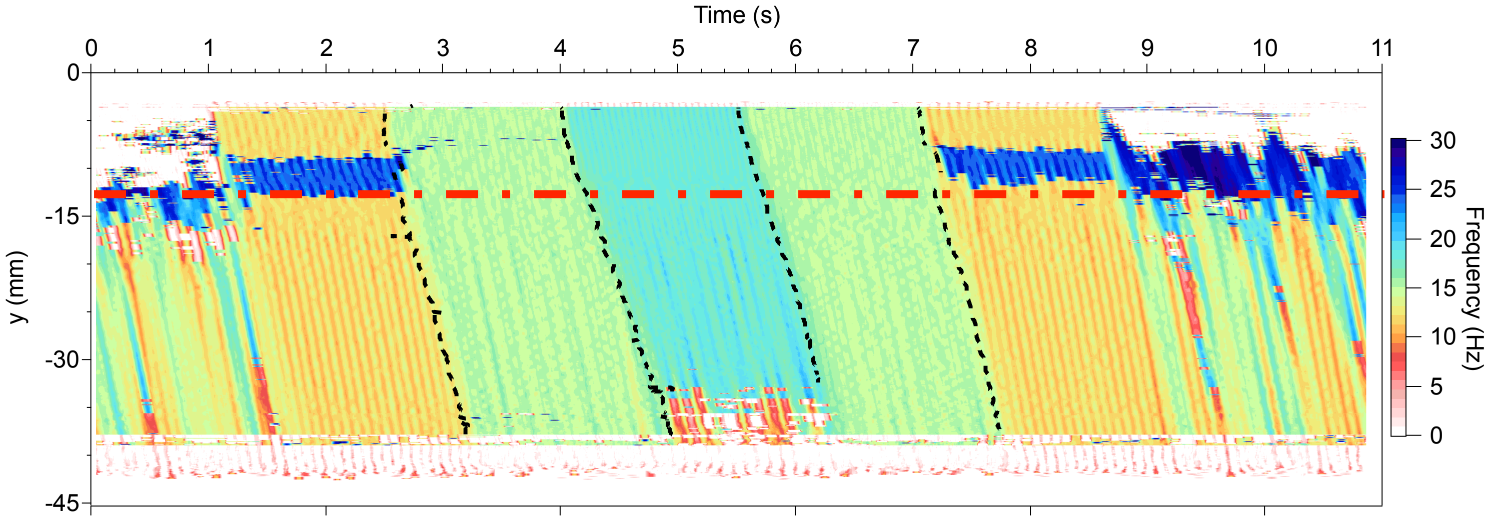
J. Williams, Phys. Rev. E **90**, 043103 (2014).

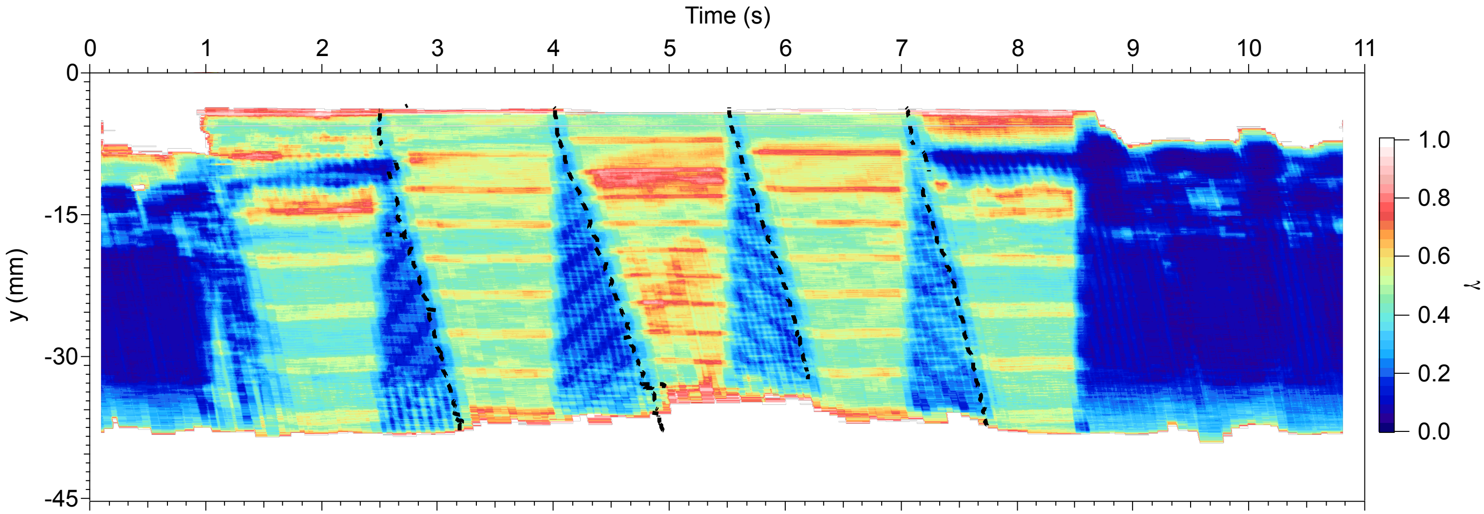
Synchronization Index II



J. Williams, Phys. Rev. E **90**, 043103 (2014).

Results I

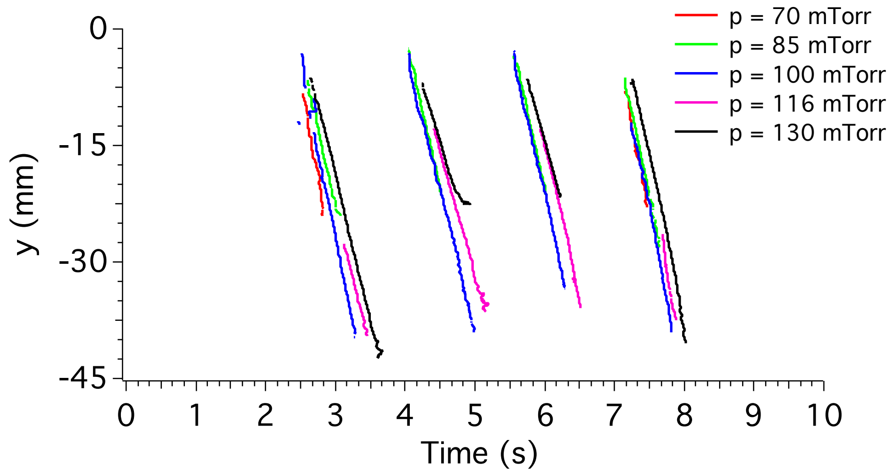


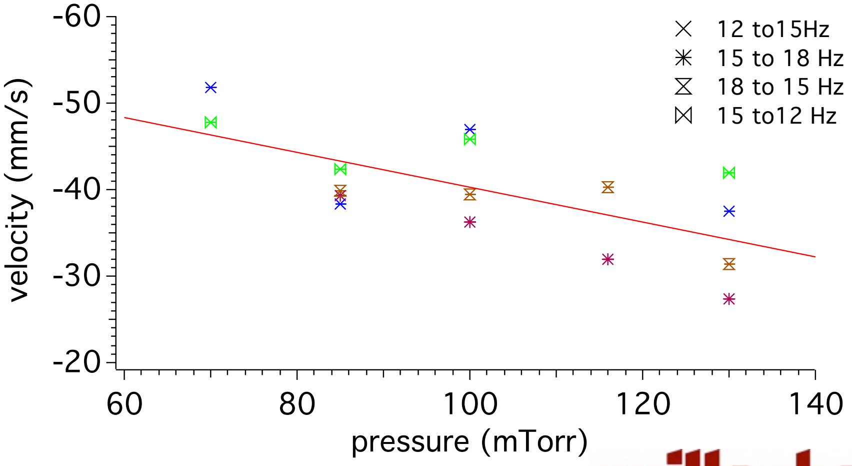


$p = 100$ mTorr
$V_{anode} = 323.5$ V
$I_{dis} = 0.37$ mA

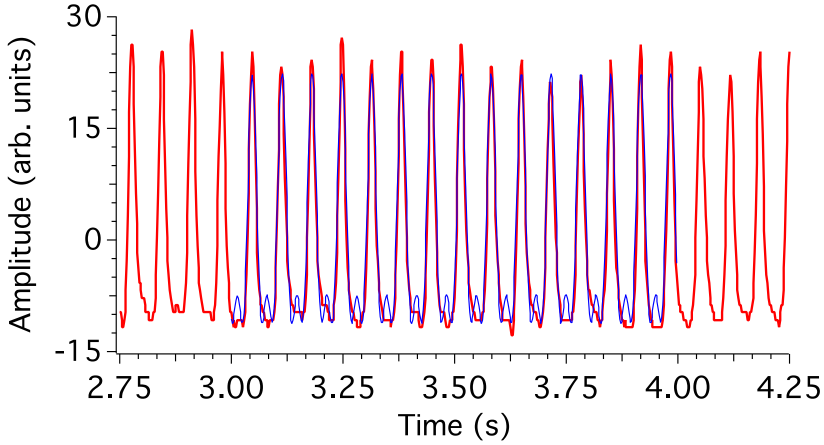
$n_e = 1.75 \times 10^{16}$ m^{-3}
$T_e = 3.9$ eV
$Q_d = 5400\, q_e$

Results – Synchronization front



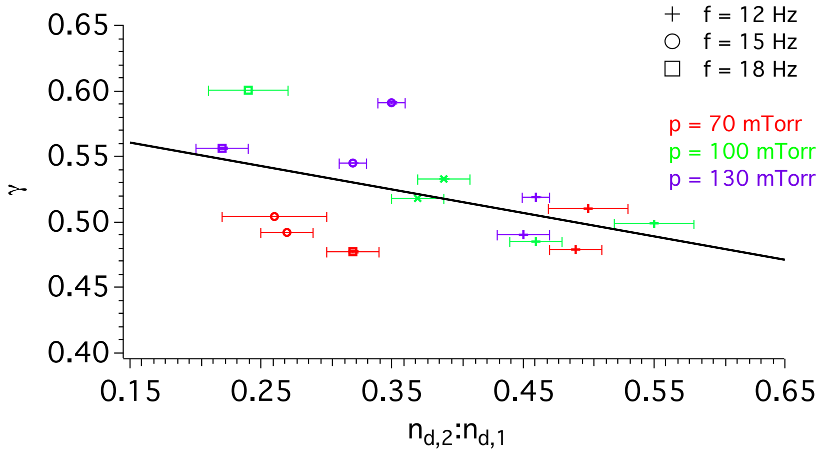


Results – Degree of Synchronization



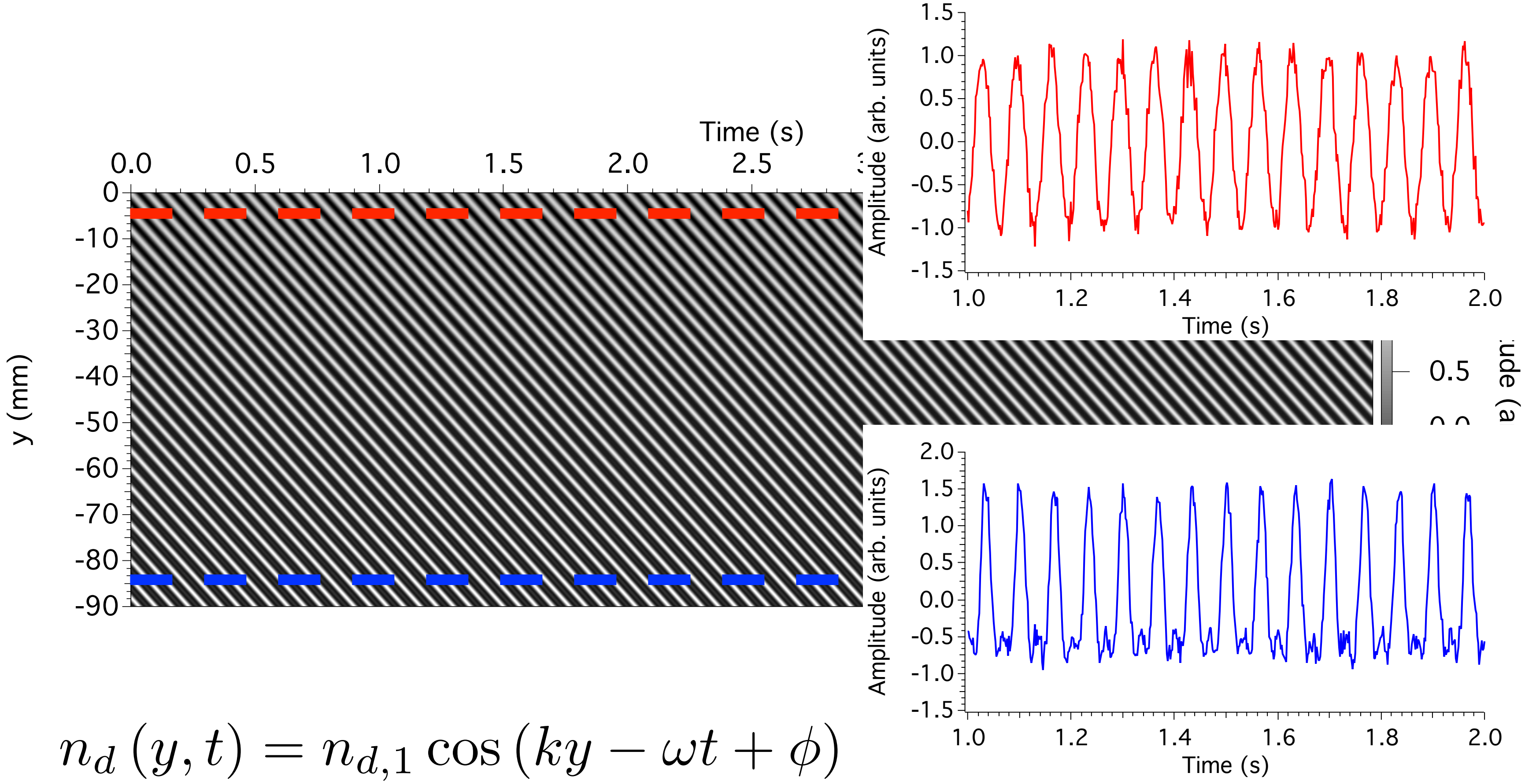
$$n_d\left(y,t\right) = n_{d,1}\cos\left(ky - \omega t + \phi\right) + n_{d,2}\cos\left(2\left[ky - \omega t + \phi\right]\right)$$

$$\left(\frac{n_{d,2}}{n_{d,1}}\right)_{wave} = 0.50 \pm 0.01$$



R. L. Merlino, Physica Scripta **85**, 035506 (2012).

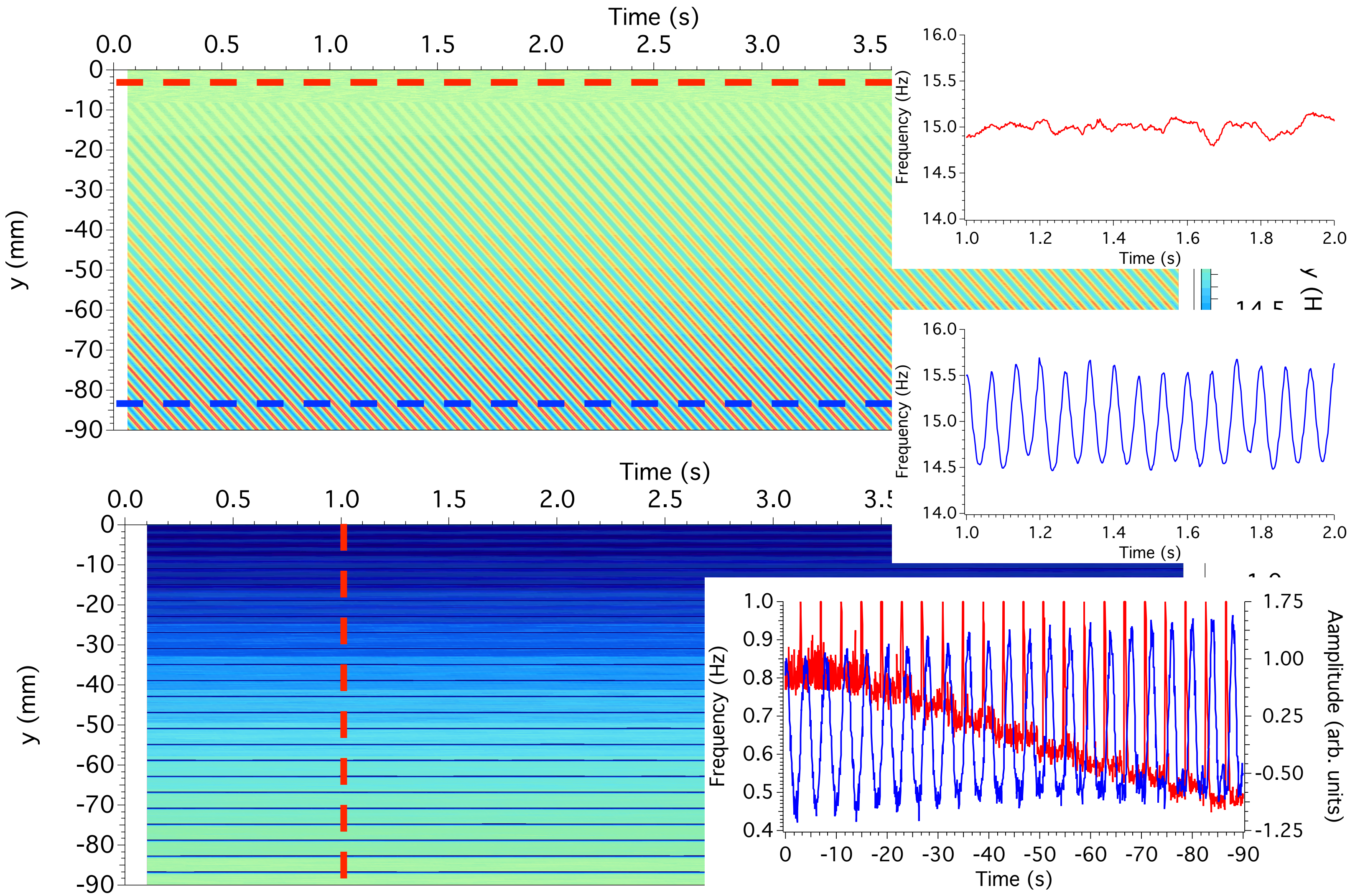
Nonlinearity I – Surrogate data



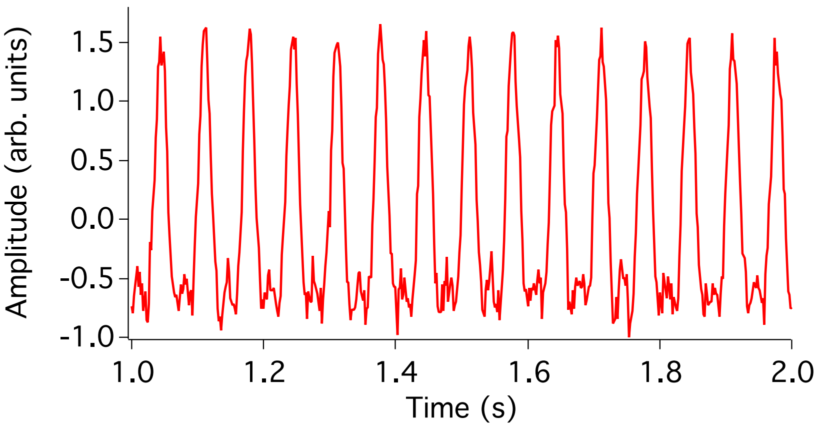
$$n_d\left(y,t\right) = n_{d,1}\cos\left(ky-\omega t+\phi\right) + n_{d,2}\cos\left(2\left[ky-\omega t+\phi\right]\right) + gnoise$$

R. L. Merlino, Physica Scripta **85**, 035506 (2012).

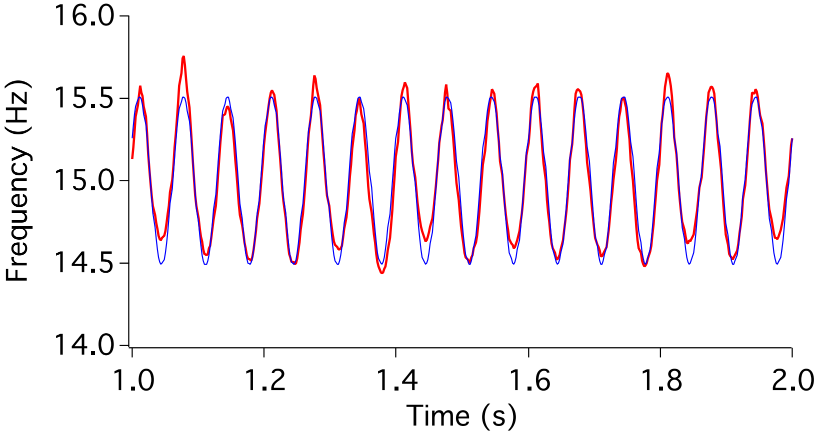
Nonlinearity II – Surrogate data



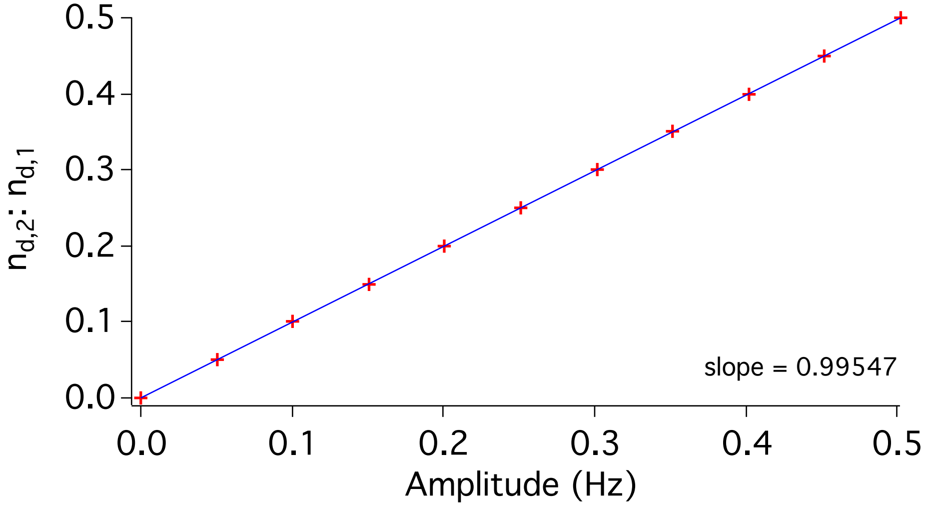
Nonlinearity III – Surrogate data



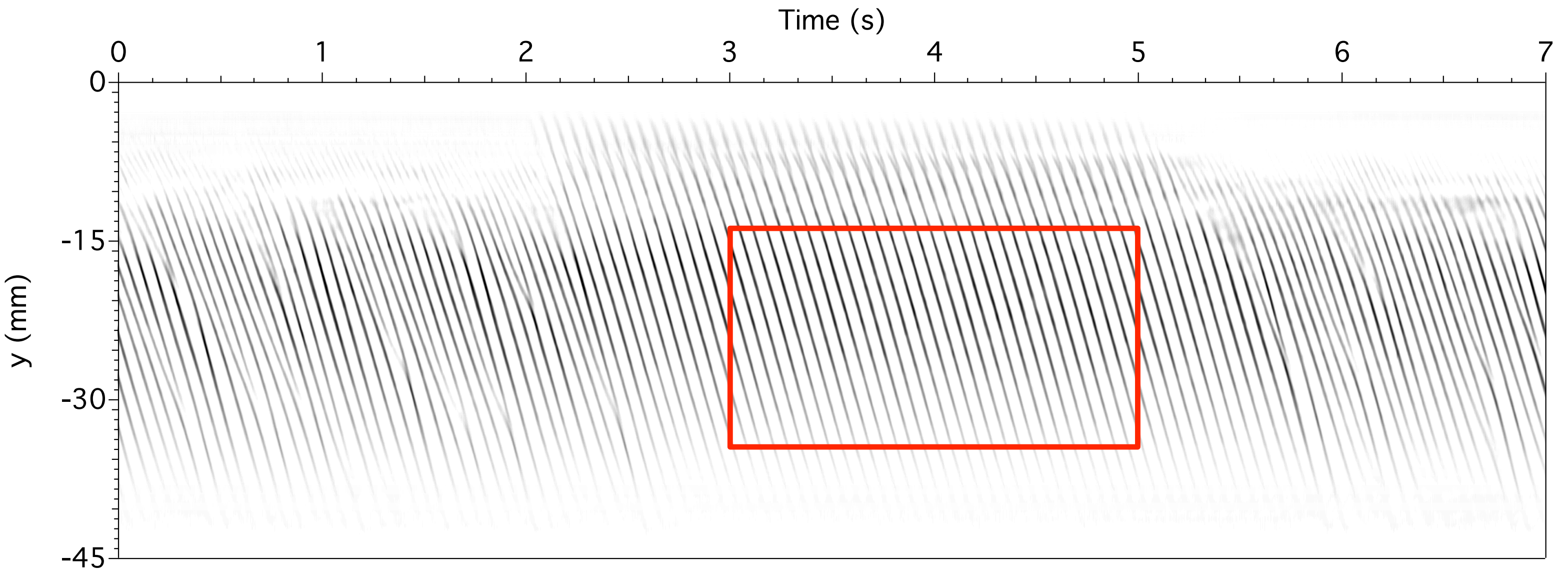
$$\left(\frac{n_{d,2}}{n_{d,1}}\right)_{actual} = 0.5$$



$$\left(\frac{n_{d,2}}{n_{d,1}}\right)_{Hilbert} = 0.508 \pm 0.002$$



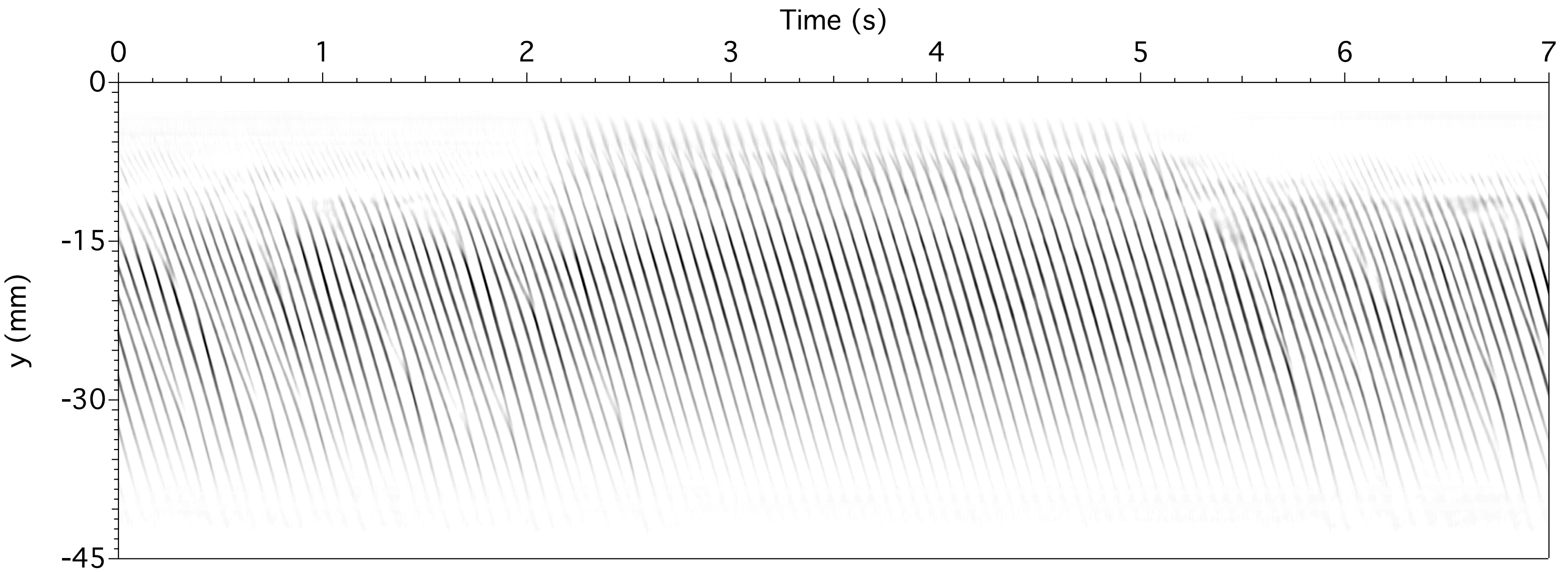
Nonlinearity IV – Real data



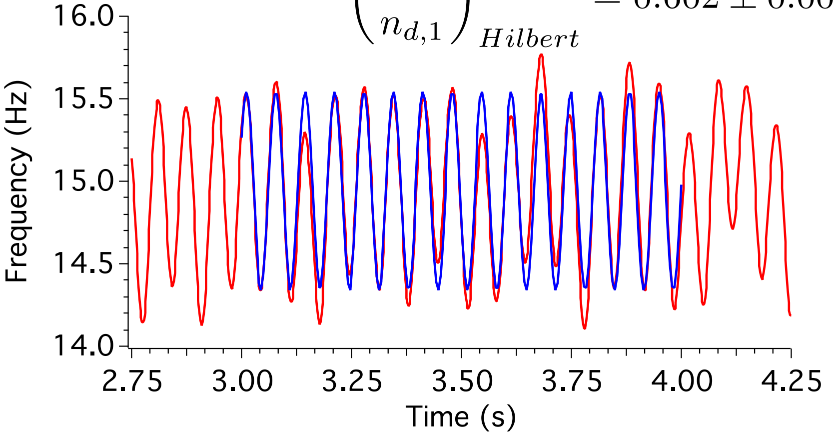
$p = 87$ mTorr
$V_{anode} = 323$V
$I_{dis} = 0.3$ mA

$n_e = 5.97\times10^{16}$ m^{-3}
$T_e = 2.5$ eV
$Q_d = 3400\ q_e$

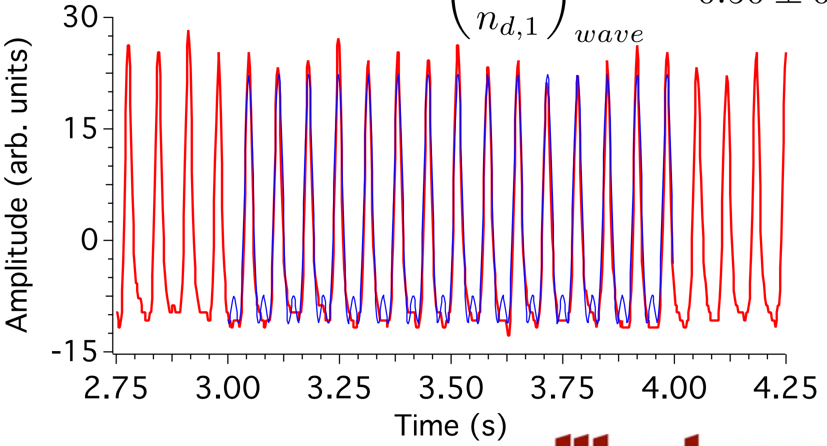
Nonlinearity V – Real data



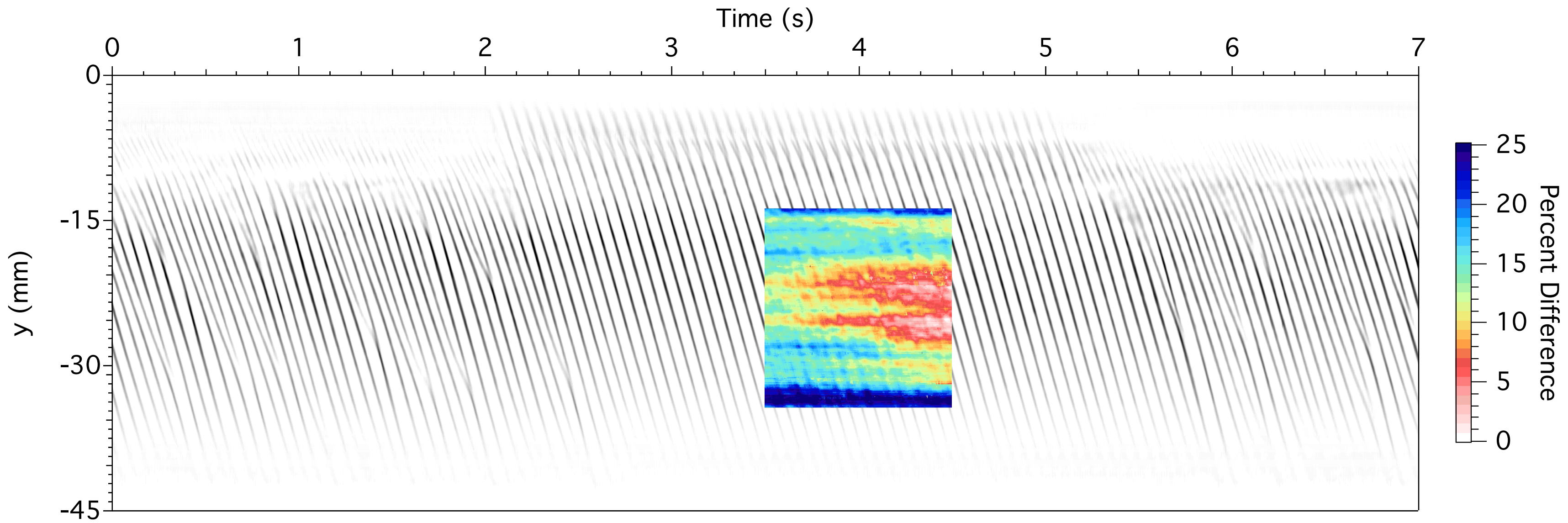
$$\left(\frac{n_{d,2}}{n_{d,1}}\right)_{Hilbert} = 0.602 \pm 0.008$$



$$\left(\frac{n_{d,2}}{n_{d,1}}\right)_{wave} = 0.50 \pm 0.01$$

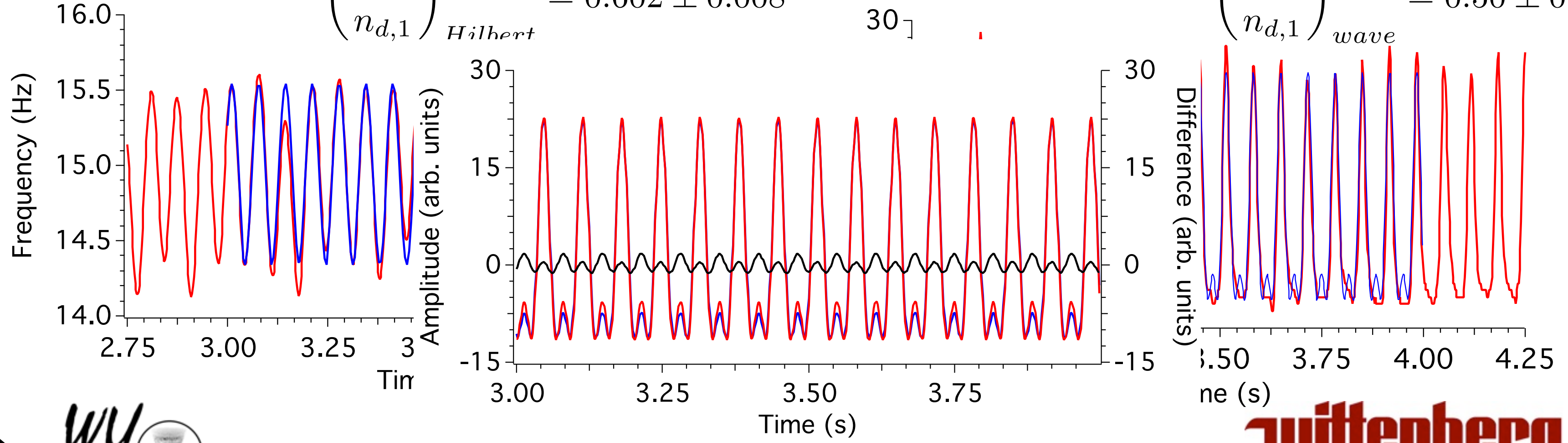


Nonlinearity V – Real data



$$\left(\frac{n_{d,2}}{n_{d,1}}\right)_{Hilbert} = 0.602 \pm 0.008$$

$$\left(\frac{n_{d,2}}{n_{d,1}}\right)_{wave} = 0.50 \pm 0.01$$



Summary

- We have made time-resolved measurements of a naturally-occuring dust acoustic wave synchronizing to an external modulation using a Hilbert Transform in a weakly-coupled dc glow discharge dusty plasma system over a range of neutral gas pressures.

- We observed that the
  - synchronization occurs behind a propagating "synchronization" front that travels at a different speed than the phase velocity of the wave mode and the speed depends on the neutral gas pressure.
  - degree of synchronization depends on the nonlinearity of the wave
  - Hilbert Transform may provide a means to measure the nonlinearity in the driven wave mode.

Thank you for your kind attention