Math 2660 Topics in Linear Algebra, Quiz 9 Key, Fall 2008

Name:

For full credit, show all steps in details

1. True or False (1 point each)

   (a) If $L : V \to W$ is a linear transformation, then $L(\mathbf{0}) = \mathbf{0}$. True

   (b) Given a nonzero $\mathbf{a} \in \mathbb{R}^2$, the translation $L : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $L(\mathbf{x}) = \mathbf{x} + \mathbf{a}$ is a linear transformation. False

   (c) Given $x \in \mathbb{R}$, the map $L : \mathbb{R}^n \to \mathbb{R}^n$, $L(\mathbf{x}) = c\,\mathbf{x}$ is a linear transformation. True

2. What is a linear transformation $L : V \to W$ between the vector spaces V and W? (2 points)

   $L : V \to W$ is a linear transformation if $L(\alpha\mathbf{x} + \beta\mathbf{y}) = \alpha L(\mathbf{x}) + \beta L(\mathbf{y})$ for all $\mathbf{x}, \mathbf{y} \in V$ and $\alpha, \beta \in \mathbb{R}$.

3. Find a matrix A that represents the linear transformation $L : \mathbb{R}^3 \to \mathbb{R}^2$ such that $L(\mathbf{x}) = A\mathbf{x}$ for each $\mathbf{x} \in \mathbb{R}^3$ where $L((x_, x_2, x_3)^T) = (x_2 - x_1, x_3 - x_2)^T$. Then use it to compute $L(\mathbf{x})$ where $\mathbf{x} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$. (5 points)

   $L(\mathbf{e}_1) = L(1, 0, 0)^T = (-1, 0)^T$,
   $L(\mathbf{e}_2) = (L(0, 1, 0)^T = (1, -1)^T$,
   $L(\mathbf{e}_3) = L(0, 0, 1)^T = (0, 1)^T$.
   So the matrix representation with respect to $\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$ of $\mathbb{R}^3$ and $\{\mathbf{e}_1, \mathbf{e}_2\}$ of $\mathbb{R}^2$ is

   $$[L(\mathbf{e}_1)\ L(\mathbf{e}_2)\ L(\mathbf{e}_3)] = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}.$$

   Then

   $$L\left(\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$