Math 2660 Topics in Linear Algebra, Quiz 9, Fall 2009 Key

Name:

For full credit, show all steps in details

1. True or False (1 point each)

   (a) If S is the transition matrix from basis B to basis C of the vector space V, then S^T is the transition matrix from C to B. False

   (b) If the columns of $V = [\mathbf{v}_1 \cdots \mathbf{v}_n]$ and $U = [\mathbf{u}_1 \cdots \mathbf{u}_n]$ form a basis of $\mathbb{R}^n$ respectively, then $U^{-1}V$ is the transition matrix from $\{\mathbf{v}_1, \ldots, \mathbf{v}_n\}$ to $\{\mathbf{u}_1, \ldots, \mathbf{u}_n\}$. True

2. Define the coordinate vector $[\mathbf{v}]_B$ of $\mathbf{v} \in V$ with respect to the basis $B = \{\mathbf{v}_1, \ldots, \mathbf{v}_n\}$. (2 points)

   Since B is a basis for V, each $v = c_1\mathbf{v}_1 + \cdots + c_n\mathbf{v}_n$ for some scalars $c_1, \ldots, c_n$. Then the vector $[\mathbf{v}]_B = (c_1, \ldots, c_n)^T$ is called the coordinate vector of $\mathbf{v}$.

3. (a) Find the transition matrix S from $\{\mathbf{v}_1, \mathbf{v}_2\} = \left\{ \begin{bmatrix} 5 \\ 2 \end{bmatrix}, \begin{bmatrix} 7 \\ 3 \end{bmatrix} \right\}$ to $\{\mathbf{u}_1, \mathbf{u}_2\} = \left\{ \begin{bmatrix} 3 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \end{bmatrix} \right\}$. (2 points)

   (b) Find the coordinate vector $[\mathbf{x}]_U$ of $\mathbf{x} = 2\mathbf{v}_1 - \mathbf{v}_2$ with respect to $\{\mathbf{u}_1, \mathbf{u}_2\}$. (2 point)

   (c) What is the transition matrix R from $\{\mathbf{u}_1, \mathbf{u}_2\}$ to $\{\mathbf{v}_1, \mathbf{v}_2\}$. (2 points)

(a) $S = U^{-1}V = \begin{bmatrix} 3 & 2 \\ 2 & 2 \end{bmatrix}^{-1} \begin{bmatrix} 5 & 7 \\ 2 & 3 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 2 & -2 \\ -2 & 3 \end{bmatrix} \begin{bmatrix} 5 & 7 \\ 2 & 3 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 6 & 8 \\ -4 & -5 \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ -2 & -\frac{5}{2} \end{bmatrix}$.

(b) $[\mathbf{x}]_U = S[\mathbf{x}]_V = \begin{bmatrix} 3 & 4 \\ -2 & -\frac{5}{2} \end{bmatrix} \begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 2 \\ -\frac{3}{2} \end{bmatrix}$

(c) $R = S^{-1} = \begin{bmatrix} 3 & 4 \\ -2 & -\frac{5}{2} \end{bmatrix}^{-1} = 2 \begin{bmatrix} -\frac{5}{2} & -4 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} -5 & -8 \\ 4 & 6 \end{bmatrix}$.