Math 2660 Topics in Linear Algebra, Quiz 8 Key, Fall 2008

Name:

For full credit, show all steps in details

1. True or False (1 point each)
   Let V be a vector space.
   (a) If $\mathbf{v}_1, \ldots, \mathbf{v}_k \in V$, then $\mathbf{v}_1, \ldots, \mathbf{v}_k, \mathbf{0}$ are linearly dependent. True
   (b) A subset of a linearly independent set $\{\mathbf{v}_1, \ldots, \mathbf{v}_k\}$ is linearly dependent. False
   (c) Two vectors are linearly dependent if and only if one is a scalar multiple of the other. True
   (d) Adding a vector to a linearly independent set must result a linearly dependent set. False

2. Define linear dependence of the vectors $\mathbf{v}_1, \ldots, \mathbf{v}_n$ of a vector space V. (2 points)

   $\mathbf{v}_1, \ldots, \mathbf{v}_n$ are linearly dependent if there are scalars $c_1, \ldots, c_n$, not all zero, such that $c_1\mathbf{v}_1 + \cdots + c_n\mathbf{v}_n = \mathbf{0}$.

3. Determine if the following vectors are linearly independent. Give reasons.
   (a) $\begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\1\\1\end{bmatrix}, \begin{bmatrix}2\\2\\2\end{bmatrix}$. (2 points)
   (b) $\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}, \begin{bmatrix}0 & 1\\0 & 0\end{bmatrix}, \begin{bmatrix}0 & 0\\1 & 0\end{bmatrix}$. (2 points)

   (a) $2\begin{bmatrix}1\\0\\0\end{bmatrix} + 2\begin{bmatrix}0\\1\\1\end{bmatrix} - \begin{bmatrix}2\\2\\2\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$. Solving the system So they are linearly dependent.
   (b) Set $c_1\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} + c_2\begin{bmatrix}0 & 1\\0 & 0\end{bmatrix} + c_3\begin{bmatrix}0 & 0\\1 & 0\end{bmatrix} = \begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$. So $\begin{bmatrix}c_1 & c_2\\c_3 & c_1\end{bmatrix} = \begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$. Hence $c_1 = c_2 = c_3 = 0$. Thus they are linearly independent.