Math 2660 Topics in Linear Algebra, Quiz 7 Key, Fall 2008

Name:

For full credit, show all steps in details

1. True or False (1 point each)

   (a) If S is a subspace of a vector space V, then $\mathbf{0} \in S$. True

   (b) The null space of an $m \times n$ matrix A is the solution set to the homogeneous system $A\mathbf{x} = \mathbf{0}$. True

2. Determine whether the following sets form a subspace of $\mathbb{R}^3$. Give reasons.

   (a) $S = \{(x_1, x_2, x_3)^T : x_2 + x_3 = 0\}$. (2 points)

   (b) $S = \{(x_1, x_2, x_3)^T : x_2 = 1\}$. (2 points)

   (a) (i) $\mathbf{0} = (0, 0, 0)^T \in S$ since the sum of the last two coordinates of $\mathbf{0}$ is zero.
   (ii) Let $\mathbf{x}, \mathbf{y} \in S$, i.e., $x_2 + x_3 = y_2 + y_3 = 0$. Now $\mathbf{x} + \mathbf{y} = (x_1 + y_1, x_2 + y_2, x_3 + y_3)^T$ and $x_2 + y_2 + x_3 + y_3 = 0$. Thus $\mathbf{x} + \mathbf{y} \in S$.
   (iii) Let $\mathbf{x} \in S$, i.e., $x_2 + x_3 = 0$, and $\alpha \in \mathbb{R}$. Now $\alpha\mathbf{x} = (\alpha x_1, \alpha x_2, \alpha x_3)^T$ so that $\alpha x_2 + \alpha x_3 = \alpha(x_2 + x_3) = 0$. Thus $\alpha\mathbf{x} \in V$.

   (b) $\mathbf{0} \notin S$. So it is not a subspace of V.

3. Is the set $\{\mathbf{e}_1, \mathbf{e}_2 + \mathbf{e}_3, \mathbf{e}_1 + \mathbf{e}_3\}$ a spanning set for $\mathbb{R}^3$? Give reason. (4 points)

   Consider the augmented matrix

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & a \\ 0 & 1 & 0 & b \\ 0 & 1 & 1 & c \end{array}\right] \xrightarrow{R_3 - R_2} \left[\begin{array}{ccc|c} 1 & 0 & 1 & a \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & c-b \end{array}\right] \xrightarrow{R_1 - R_3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & a-c+b \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & c-b \end{array}\right].$$

   In other words, the system is consistent so that it is a spanning set for $\mathbb{R}^3$. Indeed $(a, b, c)^T = (a - c + b)(1, 0, 0)^T + b(0, 1, 1)^T + (c - b)(1, 0, 1)^T$.