Math 2660 Topics in Linear Algebra, Fall 2009 Quiz 7 Key

Name:

For full credit, show all steps in details

1. True or False (1 point each)

   (a) Two vectors $\mathbf{x}$ and $\mathbf{y}$ in V linear dependent means that one is a scalar multiple of the other. True

   (b) Let $x_1, \ldots, x_n$ be n vectors in $\mathbb{R}^n$ and $X = [x_1 \ldots x_n]$. The vectors $x_1, \ldots, x_n$ are linearly dependent if and only if X is nonsingular. False

   (c) A subset of a linearly independent set of vectors is also linearly independent. True

2. Determine whether the following sets are linearly independent. Explain.

   (a) $(0, 1, -1)^T, (0, 1, 0)^T$ in $\mathbb{R}^3$. (2 points)

   (b) $(4, 2, 3)^T$, $(2, 3, 1)^T$, $(0, 1, 0)^T$ in $\mathbb{R}^3$. (2 points)

   (c) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$ in $\mathbb{R}^{2\times 2}$. (3 points)

   (a) They are not scalar multiple to each other. So they are linearly independent or dependent.

   (b) $\det \begin{bmatrix} 4 & 2 & 0 \\ 2 & 3 & 1 \\ 3 & 1 & 0 \end{bmatrix} = -\det \begin{bmatrix} 4 & 2 \\ 3 & 1 \end{bmatrix} = -2 \neq 0$. So by Theorem 3.3.1, the vectors are linearly independent.

   (c) Set $a_1 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + a_2 \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} + a_3 \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$.

   So $\begin{bmatrix} a_1 + 2a_3 & a_2 + a_3 \\ 0 & a_1 + a_2 + 3a_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$.

   So we have to solve the system of equations

   $$a_1 + 2a_3 = 0, \ a_2 + a_3 = 0, \ a_1 + a_2 + 3a_3 = 0$$

   Solving the system:

   $$\left[\begin{array}{ccc|c} 1 & 0 & 2 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 3 & 0 \end{array}\right] R_3 - R_1 \left[\begin{array}{ccc|c} 1 & 0 & 2 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right] R_3 - R_2 \left[\begin{array}{ccc|c} 1 & 0 & 2 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

   So it has infinitely many solution and thus the three matrices are linearly dependent.