Math 2660 Topics in Linear Algebra, Quiz 6 Key, Fall 2008

Name:

For full credit, show all steps in details

1. True or False (1 point each)
   Let V be a vector space.
   (a) $0\mathbf{x} = \mathbf{0}$ for all $\mathbf{x} \in V$. True
   (b) $\alpha\mathbf{0} = \mathbf{0}$ for all scalars α. True
   (c) $(-1)\mathbf{x} = -\mathbf{x}$ for all $\mathbf{x} \in V$. True
   (d) If $\alpha\mathbf{x} = \mathbf{0}$ where $\mathbf{x} \in V$ and α is a scalar, then either $\alpha = 0$ or $\mathbf{x} = \mathbf{0}$. True

2. Give the definition of a vector space including the eight axioms. (3 points)
   See p.120-121

3. Show that the identity element $\mathbf{0} \in V$ of a vector space V is unique. (3 points)
   Suppose $\mathbf{0}$ and $\mathbf{0}'$ are zero elements. Then by A3, $\mathbf{0} + \mathbf{0}' = \mathbf{0}'$ (view $\mathbf{0}$ as the zero element) on one hand and $\mathbf{0} + \mathbf{0}' = \mathbf{0}$ (view $\mathbf{0}'$ as the zero element) on the other hand. So $\mathbf{0} = \mathbf{0}'$.