Math 2660 Topics in Linear Algebra, Quiz 6, Fall 2009 Key

Name:

For full credit, show all steps in details

1. True or False (1 point each)
   Let V be a vector space.
   (a) The null space $N(A)$ of $A \in \mathbb{R}^{m \times n}$ is a subspace of $\mathbb{R}^m$. False. $N(A)$ is a subspace of $\mathbb{R}^n$.
   (b) $\alpha \mathbf{0} = \mathbf{0}$ for all scalars α. True. Theorem 3.3.1
   (c) $(-1)\mathbf{x} = -\mathbf{x}$ for all $\mathbf{x} \in V$. True. Theorem 3.1.1
   (d) The set of $m \times n$ matrices with usual addition and scalar multiplication is a vector space. True. p.118

2. Give the definition of subspace S of a vector space V. (2 points)

   S is a subspace of V if
   (1) S is nonempty,
   (2) $\alpha \mathbf{x} \in S$ if $\mathbf{x} \in S$,
   (3) $\mathbf{x} + \mathbf{y} \in S$ if $\mathbf{x}, \mathbf{y} \in S$.

3. Determine if the following is a subspace of $\mathbb{R}^2$ or not.
   (a) $\{(x_1, x_2)^T : x_1 + x_2 = 0\}$. (2 points)
   (b) $\{(x_1, x_2)^T : x_1^2 = x_2\}$. (2 points)

   (a) Let $S = \{(x_1, x_2)^T : x_1 + x_2 = 0\}$.
   (1) $\mathbf{0} = (0, 0)^T \in S$ since $0 + 0 = 0$
   (2) Let $\mathbf{x} = (x_1, x_2)^T \in S$ and $\alpha \in \mathbb{R}$. Then $x_1 + x_2 = 0$. Consider $\alpha \mathbf{x} = (\alpha x_1, \alpha x_2)^T \in S$ since $\alpha x_1 + \alpha x_2 = \alpha(x_1 + x_2) = 0$.
   (3) Let $\mathbf{x} = (x_1, x_2)^T, \mathbf{y} = (y_1, y_2)^T \in S$. Then $x_1 + x_2 = y_1 + y_2 = 0$. Then $\mathbf{x} + \mathbf{y} = (x_1 + y_1, x_2 + y_2) \in S$ since $x_1 + y_1 + x_2 + y_2 = x_1 + x_2 + y_1 + y_2 = 0$.
   So S is a subspace.

   (b) It is not a subspace since $(1, 1)^T \in S$ $(1^2 = 1)$ but $(-1, -1)^T \notin S$ $((-1)^2 \neq -1)$.