Math 2660 Topics in Linear Algebra, Quiz 5 Key, Fall 2008

Name:

For full credit, show all steps in details

1. True or False (1 point each)

   (a) $(\text{adj}\, A)A = A(\text{adj}\, A)$ for any $n \times n$ matrices. True

   (b) Cramer's rule can be used to find solution of $Ax = b$ where A is 3×4 and b is 3×1. False

2. Find the adjoint and the inverse of the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$ (5 points)

   Use cofactor expansion along the last column of A to have $\det A = 3 \begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} = -3$. Now

$$A_{11} = \begin{vmatrix} 1 & 0 \\ 1 & 0 \end{vmatrix} = 0, \quad A_{21} = -\begin{vmatrix} 2 & 3 \\ 1 & 0 \end{vmatrix} = 3, \quad A_{31} = \begin{vmatrix} 2 & 3 \\ 1 & 0 \end{vmatrix} = -3$$

$$A_{12} = -\begin{vmatrix} 0 & 0 \\ 1 & 0 \end{vmatrix} = 0, \quad A_{22} = \begin{vmatrix} 1 & 3 \\ 1 & 0 \end{vmatrix} = -3, \quad A_{32} = -\begin{vmatrix} 1 & 3 \\ 0 & 0 \end{vmatrix} = 0$$

$$A_{13} = \begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} = -1, \quad A_{23} = -\begin{vmatrix} 1 & 2 \\ 1 & 1 \end{vmatrix} = 1, \quad A_{33} = \begin{vmatrix} 1 & 2 \\ 0 & 1 \end{vmatrix} = 1$$

   So

$$A^{-1} = \frac{\text{adj}\, A}{\det A} = -\frac{1}{3} \begin{bmatrix} 0 & 3 & -3 \\ 0 & -3 & 0 \\ -1 & 1 & 1 \end{bmatrix}.$$

3. Solve the system of linear equation $Ax = b$ by using Cramer's rule where $A = \begin{bmatrix} 1 & 2 \\ 3 & 7 \end{bmatrix}$ and $b = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$. (3 points)

   $\det A = \begin{vmatrix} 1 & 2 \\ 3 & 7 \end{vmatrix} = 1$. By Cramer's rule $x_1 = \dfrac{\begin{vmatrix} 1 & 2 \\ 0 & 7 \end{vmatrix}}{\det A} = 7$ and $x_2 = \dfrac{\begin{vmatrix} 1 & 1 \\ 3 & 0 \end{vmatrix}}{\det A} = -3.$