

## Math 2660 Topics in Linear Algebra, Quiz 5, Fall 2009 **Key**

Name:

**For full credit, show all steps in details**

- True or False (1 point each)
  - $\det A = \det(A^T)$  for any  $A \in \mathbb{R}_{n \times n}$ . **True**
  - $\det E = 1$  if  $E$  is the elementary matrix corresponding to switching two rows. **False.  $\det E = -1$ .**
  - $A \in \mathbb{R}_{n \times n}$  is nonsingular if and only if  $\det A = 0$ . **False**
  - If  $A \in \mathbb{R}_{n \times n}$  is upper triangular, then  $\det A$  is the product of the diagonal entries. **True**
  - If  $A, B \in \mathbb{R}_{n \times n}$  are both nonsingular, then  $AB$  is also nonsingular. **True**

- Find all the cofactors and the determinant of the matrix  $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$  (5 points)

$$\begin{aligned} A_{11} &= \begin{vmatrix} 1 & 0 \\ 1 & 0 \end{vmatrix} = 0, & A_{21} &= - \begin{vmatrix} 2 & 3 \\ 1 & 0 \end{vmatrix} = 3, & A_{31} &= \begin{vmatrix} 2 & 3 \\ 1 & 0 \end{vmatrix} = -3 \\ A_{12} &= - \begin{vmatrix} 0 & 0 \\ 1 & 0 \end{vmatrix} = 0, & A_{22} &= \begin{vmatrix} 1 & 3 \\ 1 & 0 \end{vmatrix} = -3, & A_{32} &= - \begin{vmatrix} 1 & 3 \\ 0 & 0 \end{vmatrix} = 0 \\ A_{13} &= \begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} = -1, & A_{23} &= - \begin{vmatrix} 1 & 2 \\ 1 & 1 \end{vmatrix} = 1, & A_{33} &= \begin{vmatrix} 1 & 2 \\ 0 & 1 \end{vmatrix} = 1 \end{aligned}$$

Use cofactor expansion along the last column of  $A$  to have  $\det A = 3 \begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} = -3$ .