

Math 2660 Topics in Linear Algebra, Quiz 4 **Key**, Fall 2008

Name:

For full credit, show all steps in details

1. True or False (1 point each)

(a) If A_1 and A_2 is nonsingular then $\begin{bmatrix} A_1 & A_3 \\ 0 & A_2 \end{bmatrix}$ is also nonsingular. **True**

(b) Given any two $n \times n$ matrices X and Y , the outer products XY^T and YX^T are the same. **False**

2. Find the block multiplications

(a) $[A \ I][A \ I]^T$. (2 points)

(b) $\begin{bmatrix} A^{-1} \\ I \end{bmatrix} [A \ I]$. (2 points)

(a) $[A \ I][A \ I]^T = [A \ I] \begin{bmatrix} A^T \\ I \end{bmatrix} = AA^T + I$.

(b) $\begin{bmatrix} A^{-1} \\ I \end{bmatrix} [A \ I] = \begin{bmatrix} A^{-1}A & A^{-1} \\ A & I \end{bmatrix} = \begin{bmatrix} I & A^{-1} \\ A & I \end{bmatrix}$.

3. Given $A = \begin{bmatrix} 0 & I \\ B & 0 \end{bmatrix}$. Find A^2 , A^3 , A^4 , and a general expression of A^{2n} and A^{2n+1} . (4 points)

$$A = \begin{bmatrix} 0 & I \\ B & 0 \end{bmatrix}, A^2 = \begin{bmatrix} 0 & I \\ B & 0 \end{bmatrix} \begin{bmatrix} 0 & I \\ B & 0 \end{bmatrix} = \begin{bmatrix} B & 0 \\ 0 & B \end{bmatrix}.$$

$$A^3 = A^2A = \begin{bmatrix} B & 0 \\ 0 & B \end{bmatrix} \begin{bmatrix} 0 & I \\ B & 0 \end{bmatrix} = \begin{bmatrix} 0 & B \\ B^2 & 0 \end{bmatrix}.$$

$$A^4 = A^2A^2 = \begin{bmatrix} B & 0 \\ 0 & B \end{bmatrix} \begin{bmatrix} B & 0 \\ 0 & B \end{bmatrix} = \begin{bmatrix} B^2 & 0 \\ 0 & B^2 \end{bmatrix}.$$

In general $A^{2n} = \begin{bmatrix} B^n & 0 \\ 0 & B^n \end{bmatrix}$ and $A^{2n+1} = \begin{bmatrix} B^n & 0 \\ 0 & B^n \end{bmatrix} \begin{bmatrix} 0 & I \\ B & 0 \end{bmatrix} = \begin{bmatrix} 0 & B^n \\ B^{n+1} & 0 \end{bmatrix}$, where n is a positive integer