Math 2660 Topics in Linear Algebra, Quiz 4 Key, Fall 2008

Name:

For full credit, show all steps in details

1. True or False (1 point each)

   (a) If A_1 and A_2 is nonsingular then $\begin{bmatrix} A_1 & A_3 \\ 0 & A_2 \end{bmatrix}$ is also nonsingular. True

   (b) Given any two $n \times n$ matrices X and Y, the outer products XY^T and YX^T are the same. False

2. Find the block multiplications

   (a) $[A \;\; I][A \;\; I]^T$. (2 points)

   (b) $\begin{bmatrix} A^{-1} \\ I \end{bmatrix} [A \;\; I]$. (2 points)

   (a) $[A \;\; I][A \;\; I]^T = [A \;\; I] \begin{bmatrix} A^T \\ I \end{bmatrix} = AA^T + I.$

   (b) $\begin{bmatrix} A^{-1} \\ I \end{bmatrix} [A \;\; I] = \begin{bmatrix} A^{-1}A & A^{-1} \\ A & I \end{bmatrix} = \begin{bmatrix} I & A^{-1} \\ A & I \end{bmatrix}.$

3. Given $A = \begin{bmatrix} 0 & I \\ B & 0 \end{bmatrix}$. Find A^2, A^3, A^4, and a general expression of A^{2n} and A^{2n+1}. (4 points)

   $A = \begin{bmatrix} 0 & I \\ B & 0 \end{bmatrix}$, $A^2 = \begin{bmatrix} 0 & I \\ B & 0 \end{bmatrix} \begin{bmatrix} 0 & I \\ B & 0 \end{bmatrix} = \begin{bmatrix} B & 0 \\ 0 & B \end{bmatrix}.$

   $A^3 = A^2 A = \begin{bmatrix} B & 0 \\ 0 & B \end{bmatrix} \begin{bmatrix} 0 & I \\ B & 0 \end{bmatrix} = \begin{bmatrix} 0 & B \\ B^2 & 0 \end{bmatrix}.$

   $A^4 = A^2 A^2 = \begin{bmatrix} B & 0 \\ 0 & B \end{bmatrix} \begin{bmatrix} B & 0 \\ 0 & B \end{bmatrix} = \begin{bmatrix} B^2 & 0 \\ 0 & B^2 \end{bmatrix}.$

   In general $A^{2n} = \begin{bmatrix} B^n & 0 \\ 0 & B^n \end{bmatrix}$ and $A^{2n} = \begin{bmatrix} B^n & 0 \\ 0 & B^n \end{bmatrix} \begin{bmatrix} 0 & I \\ B & 0 \end{bmatrix} = \begin{bmatrix} 0 & B^n \\ B^{n+1} & 0 \end{bmatrix}$, where n is a positive integer