

Math 2660 Topics in Linear Algebra, Quiz 4, Fall 2009 **Key**

Name:

For full credit, show all steps in details

1. True or False (1 point each)

(a) Let $A = [A_1 \ A_2]$ and $B = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}$ where A_1 is $m \times s$, A_2 is $m \times (n - s)$ and B_1 is $s \times r$ and B_2 is $(n - s) \times r$, then $AB = A_1B_1 + A_2B_2$. **True**

(b) Given any $x, y \in \mathbb{R}^n$, $x^T y = y^T x$. **True.** $x^T y = (x^T y)^T = y^T x$ since it is a scalar.

2. Find the block multiplications

(a) $[A \ I][A \ 0]^T$. (2 points)

(b) $\begin{bmatrix} A^{-1} \\ I \end{bmatrix} [A \ I]$. (2 points)

(a) $[A \ I][A \ 0]^T = [A \ I] \begin{bmatrix} A^T \\ 0 \end{bmatrix} = AA^T$.

(b) $\begin{bmatrix} A^{-1} \\ I \end{bmatrix} [A \ I] = \begin{bmatrix} A^{-1}A & A^{-1} \\ A & I \end{bmatrix} = \begin{bmatrix} I & A^{-1} \\ A & I \end{bmatrix}$.

3. Given $X = \begin{bmatrix} 1 & 2 \\ 2 & 0 \\ -1 & 0 \end{bmatrix}$, $Y = \begin{bmatrix} 1 & 0 \\ 0 & 3 \\ 2 & 1 \end{bmatrix}$. Compute the outer product expansion of XY^T .

(4 points)

$$XY^T = \begin{bmatrix} 1 & 2 \\ 2 & 0 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 3 & 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} (1, 0, 2) + \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix} (0, 3, 1) =$$

$$\begin{bmatrix} 1 & 0 & 2 \\ 2 & 0 & 4 \\ -1 & 0 & -2 \end{bmatrix} + \begin{bmatrix} 0 & 6 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 6 & 2 \\ 2 & 0 & 4 \\ -1 & 0 & -2 \end{bmatrix}$$