

Math 2660 Topics in Linear Algebra, Quiz 3 Key, Fall 2008

Name:

For full credit, show all steps in details

1. True or False (1 point each)

- (a) If the $n \times n$ matrix A is nonsingular, then the system $Ax = 0$ has exactly one solution. **True**
- (b) Each elementary matrix E is nonsingular and its inverse is of the same type of E . **True.**

2. Given $A = \begin{bmatrix} 2 & 1 & 1 \\ 6 & 4 & 5 \\ 4 & 1 & 3 \end{bmatrix}$.

- (a) Find elementary matrices E_1, E_2, E_3 such that $E_3E_2E_1A = U$ where U is an upper triangular matrix. (2 points)
- (b) Determine the inverses of E_1, E_2, E_3 . What is the lower triangular matrix L such that $A = LU$. (3 points)

(a) $\begin{bmatrix} 2 & 1 & 1 \\ 6 & 4 & 5 \\ 4 & 1 & 3 \end{bmatrix} \begin{matrix} R_2 - 3R_1 \\ R_3 - 2R_1 \end{matrix} \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & -1 & 1 \end{bmatrix} \begin{matrix} R_3 + R_2 \end{matrix} \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 3 \end{bmatrix} = U$. So

$$E_1 = \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, E_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}, E_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}.$$

(b) $E_1^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, E_2^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}, E_3^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$

$$L = E_1^{-1}E_2^{-1}E_3^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 2 & -1 & 1 \end{bmatrix}.$$

3. Find the inverse of $A = \begin{bmatrix} 3 & 0 \\ 9 & 3 \end{bmatrix}$. (3 points)

$$[A|I] = \left[\begin{array}{cc|cc} 3 & 0 & 1 & 0 \\ 9 & 3 & 0 & 1 \end{array} \right] \begin{matrix} R_2 - 3R_1 \\ \frac{1}{3}R_1 \\ \frac{1}{3}R_2 \end{matrix} \left[\begin{array}{cc|cc} 3 & 0 & 1 & 0 \\ 0 & 3 & -3 & 1 \end{array} \right] \begin{matrix} \frac{1}{3}R_1 \\ \frac{1}{3}R_2 \end{matrix} \left[\begin{array}{cc|cc} 1 & 0 & \frac{1}{3} & 0 \\ 0 & 1 & -1 & \frac{1}{3} \end{array} \right].$$

So $A^{-1} = \begin{bmatrix} \frac{1}{3} & 0 \\ -1 & \frac{1}{3} \end{bmatrix}$.