Math 2660 Topics in Linear Algebra, Quiz 3, Fall 2009 Key

Name:

For full credit, show all steps in details

1. True or False (1 point each)

   (a) The $n \times n$ matrix A is nonsingular if A is row equivalent to I. True

   (b) Each elementary matrix E is nonsingular and its inverses is of the same type of E. True.

2. Given $A = \begin{bmatrix} 2 & 1 & 1 \\ 6 & 4 & 5 \\ 4 & 1 & 3 \end{bmatrix}$.

   (a) Find elementary matrices E_1, E_2, E_3 such that $E_3E_2E_1A = U$ where U is an upper triangular matrix. (2 points)

   (b) Determine the inverses of E_1, E_2, E_3. What is the lower triangular matrix L such that $A = LU$? (3 points)

(a) $\begin{bmatrix} 2 & 1 & 1 \\ 6 & 4 & 5 \\ 4 & 1 & 3 \end{bmatrix} \begin{matrix} \\ R_2 - 3R_1 \\ R_3 - 2R_1 \end{matrix} \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & -1 & 1 \end{bmatrix} \begin{matrix} \\ \\ R_3 + R_2 \end{matrix} \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 3 \end{bmatrix} = U$. So

$$E_1 = \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, E_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}, E_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}.$$

(b) $E_1^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, E_2^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}, E_3^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$

$L = E_1^{-1}E_2^{-1}E_3^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 2 & -1 & 1 \end{bmatrix}$.

3. Find the inverse of $A = \begin{bmatrix} 2 & 1 \\ 6 & 4 \end{bmatrix}$. (3 points)

$[A|I] = \left[\begin{array}{cc|cc} 2 & 1 & 1 & 0 \\ 6 & 4 & 0 & 1 \end{array}\right] \begin{matrix} \\ R_2 - 3R_1 \end{matrix} \left[\begin{array}{cc|cc} 2 & 1 & 1 & 0 \\ 0 & 1 & -3 & 1 \end{array}\right] \begin{matrix} \frac{1}{2}R_1 \\ \\ \end{matrix} \left[\begin{array}{cc|cc} 1 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 1 & -3 & 1 \end{array}\right] \begin{matrix} R_1 - \frac{1}{2}R_2 \\ \\ \end{matrix}$

$\left[\begin{array}{cc|cc} 1 & 0 & 2 & -\frac{1}{2} \\ 0 & 1 & -3 & 1 \end{array}\right]$. So $A^{-1} = \begin{bmatrix} 2 & -\frac{1}{2} \\ -3 & 1 \end{bmatrix}$.