Math 2660 Topics in Linear Algebra, Quiz 2 Key, Fall 2008

Name:

For full credit, show all steps in details

1. True or False (1 point each)

   (a) If A is $m \times n$ and B is $n \times r$, then $(AB)^T = B^T A^T$. True

   (b) All $n \times n$ matrices are nonsingular. False. Zero matrix is singular.

   (c) For nonsingular $n \times n$ matrices A and B, $(AB)^{-1} = A^{-1}B^{-1}$. False. $(AB)^{-1} = B^{-1}A^{-1}$.

   (d) Let A be an $m \times n$ matrix and x be an $n \times 1$ vector, then $Ax = x_1 a_1 + \cdots + x_n a_n$, where $a_1, \ldots, a_n$ are the columns of A. True

2. Define a symmetric $n \times n$ matrix A (1 point)

   A symmetric $n \times n$ matrix A is a matrix satisfying $A^T = A$.

3. Given $A = \begin{bmatrix} -1 & 4 \\ 0 & 1 \\ -3 & -1 \end{bmatrix}$, find B if $A^T + B = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -1 & 0 \end{bmatrix}$. (2 points)

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -1 & 0 \end{bmatrix} - A^T = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -1 & 0 \end{bmatrix} - \begin{bmatrix} -1 & 0 & -3 \\ 4 & 1 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 2 & 6 \\ -1 & -2 & 1 \end{bmatrix}$$

4. Let $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$. Find A^2, A^3 and then A^k where $k \geq 4$. (3 points)

$$A^2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \; A^3 = AA^2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0.$$

   Thus $A^k = A^{k-3}A^3 = 0$ for $k \geq 4$.