Math 2660 Topics in Linear Algebra, Quiz 2, Fall 2009 Key

Name:

For full credit, show all steps in details

1. True or False (1 point each)
   (a) If A is an $m \times n$ matrix, then $(\alpha A)^T = \alpha A^T$, $\alpha \in \mathbb{R}$. True
   (b) For nonsingular $n \times n$ matrices A and B, $(A+B)^{-1}$ exists and $(A+B)^{-1} = A^{-1} + B^{-1}$. False. $1 = 2^{-1} + 2^{-1} \neq (2+2)^{-1} = \frac{1}{4}$.
   (c) For nonsingular $n \times n$ matrices A and B, $(AB)^{-1} = B^{-1}A^{-1}$. True.
   (d) The inverse of an $n \times n$ nonsingular matrix A is unique. True

2. Define a nonsingular $n \times n$ matrix A (1 point)

   An $n \times n$ matrix A is nonsingular if there is an $n \times n$ matrix B such that $AB = I = BA$.

3. Given $A = \begin{bmatrix} -1 & 4 \\ 0 & 1 \\ -3 & -1 \end{bmatrix}$, find B if $A^T - B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix}$. (2 points)

   $$B = A^T - \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix} = \begin{bmatrix} -1 & 0 & -3 \\ 4 & 1 & -1 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix} = \begin{bmatrix} -2 & 0 & -3 \\ 4 & 2 & -1 \end{bmatrix}$$

4. Let $A = \begin{bmatrix} 1/2 & -1/2 \\ -1/2 & 1/2 \end{bmatrix}$. Find A^2 and then A^n where $n \geq 3$. (3 points)

   $$A^2 = \begin{bmatrix} 1/2 & -1/2 \\ -1/2 & 1/2 \end{bmatrix} \begin{bmatrix} 1/2 & -1/2 \\ -1/2 & 1/2 \end{bmatrix} = \begin{bmatrix} 1/2 & -1/2 \\ -1/2 & 1/2 \end{bmatrix} = A.$$

   So $A^3 = A^2 A = A^2 = A$. In general $A^n = A$ for all positive integer n.