Math 2660 Topics in Linear Algebra, Quiz 10 Key, Fall 2008

Name:

For full credit, show all steps in details

1. True or False (1 point each)

   (a) Let A, B be $n \times n$ matrices. A is similar to B if there is a nonsingular matrix S such that $B = S^{-1}AS$. True

   (b) For $\mathbf{x}, \mathbf{y} \in \mathbb{R}^2$, $\|\mathbf{x}\|\|\mathbf{y}\| < |\mathbf{x}^T\mathbf{y}|$. False

2. Let

$$\mathbf{u}_1 = \begin{bmatrix} 3 \\ 1 \end{bmatrix}, \ \mathbf{u}_2 = \begin{bmatrix} 5 \\ 2 \end{bmatrix} \qquad \mathbf{v}_1 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}, \ \mathbf{v}_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

   and let $L : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear operator whose matrix representation with respect to the ordered basis is $\{\mathbf{u}_1, \mathbf{u}_2\}$ is $A = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$

   (a) Determine the transition matrix from $\{\mathbf{v}_1, \mathbf{v}_2\}$ to $\{\mathbf{u}_1, \mathbf{u}_2\}$. (2 points)

   (b) Find the matrix representation of L with respect to $\{\mathbf{v}_1, \mathbf{v}_2\}$. (2 points)

   (a) The transition matrix from $\{\mathbf{v}_1, \mathbf{v}_2\}$ to $\{\mathbf{u}_1, \mathbf{u}_2\}$ is

$$U^{-1}V = \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 1 \\ -2 & -1 \end{bmatrix} = \begin{bmatrix} 2 & -5 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ -2 & -1 \end{bmatrix} = \begin{bmatrix} 12 & 7 \\ -7 & -4 \end{bmatrix}.$$

   (b) The matrix representation of L with respect to $\{\mathbf{v}_1, \mathbf{v}_2\}$ is given by

$$\begin{aligned} S^{-1}AS &= \begin{bmatrix} 12 & 7 \\ -7 & -4 \end{bmatrix}^{-1} \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 12 & 7 \\ -7 & -4 \end{bmatrix} \\ &= \begin{bmatrix} -4 & -7 \\ 7 & 12 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 12 & 7 \\ -7 & -4 \end{bmatrix} = \begin{bmatrix} -222 & -131 \\ 383 & 226 \end{bmatrix}. \end{aligned}$$

3. Let $\mathbf{u} = (1, 1, 0)^T$ and $\mathbf{v} = (1, 0, 0)^T$. Find

   (a) the lengths of the vectors $\mathbf{u}$ and $\mathbf{v}$. (1 points)

   (b) the angle between the vectors u and $\mathbf{v}$. (2 points)

   (c) the distance between the vectors $\mathbf{u}$ and $\mathbf{v}$. (1 points)

   (a) $\|\mathbf{u}\| = \sqrt{2}$, $\|\mathbf{v}\| = 1$.
   (b) $\cos\theta = \frac{\mathbf{u}^T\mathbf{v}}{\|\mathbf{u}\|\|\mathbf{v}\|} = \frac{1}{\sqrt{2}}$. So $\theta = \pi/4$.
   (c) $\|\mathbf{u} - \mathbf{v}\| = \|(0, 1, 0)^T\| = 1$ is the distance between u and v.