

# Math 2660 Topics in Linear Algebra, Quiz 10, Fall 2009 **Key**

Name:

**For full credit, show all steps in details**

1. True or False (1 point each)

- (a) If  $A \in \mathbb{R}^{m \times n}$ , then the rank of  $A$  plus the nullity of  $A$  equals  $m$ . **False. It should be  $n$ .**
- (b) The dimension of the row space is less than the dimension of the column space. **False. They are the same.**
- (c) The rank of  $A$  is the dimension of the column space of  $A$ . **True**
- (d) The system of linear equations  $Ax = b$  is consistent if and only if  $b$  is in the row space of  $A$ . **False. Theorem 3.6.2.**

2. Let  $A = \begin{bmatrix} 1 & 3 & -2 & 0 \\ 2 & 6 & -6 & -2 \\ 2 & 6 & 0 & 8 \end{bmatrix}$ .

- (a) Find a basis for the row space of  $A$ . (2 points)
- (b) Find a basis for the column space of  $A$ . (2 points)
- (c) Find rank  $A$ . What is the nullity of  $A$ ? (2 points)

$$\begin{bmatrix} 1 & 3 & -2 & 0 \\ 2 & 6 & -6 & -2 \\ 2 & 6 & 0 & 8 \end{bmatrix} \begin{array}{l} R_2 - 2R_1 \\ R_3 - 2R_1 \end{array} \begin{bmatrix} 1 & 3 & -2 & 0 \\ 0 & 0 & -2 & -2 \\ 0 & 0 & 4 & 8 \end{bmatrix} \begin{array}{l} \\ -\frac{1}{2}R_2 \end{array} \begin{bmatrix} 1 & 3 & -2 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 4 & 8 \end{bmatrix}$$

$$R_3 - 4R_2 \begin{bmatrix} 1 & 3 & -2 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 4 \end{bmatrix} \begin{array}{l} \\ \\ \frac{1}{4}R_2 \end{array} \begin{bmatrix} 1 & 3 & -2 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- (a) So  $\{(1, 3, -2, 0), (0, 0, 1, 1), (0, 0, 0, 1)\}$  is a basis for the row space of  $A$
- (b) From the leading ones we know that the first, third and the four columns of  $A$  form a basis for the column space of  $A$ . Namely  $\left\{ \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} -2 \\ -6 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -2 \\ 8 \end{bmatrix} \right\}$
- (c) Since the rank is the dimension of the column space (or row space) of  $A$ , rank  $A = 3$ . Nullity of  $A$  is  $4 - 3 = 1$ .