

# Math 2660 Topics in Linear Algebra, Quiz 1, Fall 2009 **Key**

Name:

**For full credit, show all steps in details**

1. True or False (1 point each)
  - (a) Some system of linear equations may have exactly 2 solutions. **False**
  - (b) A homogenous system, i.e.,  $Ax = 0$ , always has solution. **True**

2. Describe the elementary row operations I, II, and III. (2 points)

**I. Interchange two rows.**

**II. Multiply a row by a nonzero real number**

**III. Replace a row by its sum with a multiple of another row.**

3. Consider the linear system:

$$\begin{array}{cccccc} x_1 & + & 3x_2 & - & 2x_3 & & = & 0 \\ 2x_1 & + & 6x_2 & - & 6x_3 & - & 2x_4 & = & -2 \\ 2x_1 & + & 6x_2 & + & & + & 8x_4 & = & 6 \end{array}$$

- (a) Write down the corresponding augmented matrix. (1 point)
- (b) Change the system to row echelon form. (2 point)
- (c) List the leading variable(s) and the free variable(s). (1 point)
- (d) Find all solutions of the system. (2 points)

(a) 
$$\left[ \begin{array}{cccc|c} 1 & 3 & -2 & 0 & 0 \\ 2 & 6 & -6 & -2 & -2 \\ 2 & 6 & 0 & 8 & 6 \end{array} \right]$$

(b)

$$\left[ \begin{array}{cccc|c} 1 & 3 & -2 & 0 & 0 \\ 2 & 6 & -6 & -2 & -2 \\ 2 & 6 & 0 & 8 & 6 \end{array} \right] \begin{array}{l} R_2 - 2R_1 \\ R_3 - 2R_1 \end{array} \left[ \begin{array}{cccc|c} 1 & 3 & -2 & 0 & 0 \\ 0 & 0 & -2 & -2 & -2 \\ 0 & 0 & 4 & 8 & 6 \end{array} \right] \begin{array}{l} \\ -\frac{1}{2}R_2 \end{array} \left[ \begin{array}{cccc|c} 1 & 3 & -2 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 4 & 8 & 6 \end{array} \right]$$

$$R_3 - 4R_2 \left[ \begin{array}{cccc|c} 1 & 3 & -2 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 4 & 2 \end{array} \right] \begin{array}{l} \\ \\ \frac{1}{4}R_2 \end{array} \left[ \begin{array}{cccc|c} 1 & 3 & -2 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & \frac{1}{2} \end{array} \right]$$

(c) The lead variables are  $x_1$ ,  $x_3$  and  $x_4$ . The free variable is  $x_2$ .

(d) Set  $x_2 = \alpha$ . Now  $x_4 = \frac{1}{2}$ ,  $x_3 = 1 - \frac{1}{2} = \frac{1}{2}$ ,  $x_2 = \alpha$ ,  $x_1 = 2x_3 - 3x_2 = 1 - 3\alpha$  by back substitution. So the solution set is  $\{(1 - 3\alpha, \alpha, \frac{1}{2}, \frac{1}{2}) : \alpha, \beta \in \mathbb{R}\}$ .