

Course syllabus for Lie Algebra

Course Number: Under Special Topics (Math 7970)

Course Title: Lie Algebra

Credit Hours: 3

Prerequisites: A good comprehension of linear algebra and MATH 5320/6320 or equivalent, or departmental approval.

Objectives: To give a general introduction of Lie algebras and the classifications of semisimple Lie algebras.

Course Content:

- (1) Lie algebras, subalgebras, linear Lie algebras and linear groups, adjoint representations, abelian Lie algebras. [4 days]
- (2) Ideals, centers, derived algebras, simple algebras, quotient algebras, homomorphisms, representations. [3 days]
- (3) Solvable Lie algebras, radical, semisimple Lie algebras, nilpotent Lie algebras, Engel's Theorem. [3 days]
- (4) Lie's Theorem, Jordan-Chevalley decomposition, Cartan's Criterion for solvable Lie algebras. [3 days]
- (5) Killing forms, radicals, simple ideals, inner derivations, abstract Jordan decompositions. [3 days]
- (6) Lie algebra modules, module homomorphisms, irreducible modules, completely reducible modules, Schur's Lemma, Casimir element, Weyl's Theorem, preservation of Jordan decomposition. [5 days]
- (7) Representations of $\mathfrak{sl}(2, \mathbb{F})$. [2 days]
- (8) Toral subalgebras, root space decomposition (Cartan decomposition), properties of maximal toral subalgebras. [5 days]
- (9) Reflections, root system, roots, dual root system, base, Weyl chambers, Weyl group. [6 days]
- (10) Cartan matrix, Cartan integers, Coxeter graphs, Dynkin diagrams, classification theorem of simple Lie algebras, automorphisms of root systems. [5 days]
- (11) Weights, root lattices, half sum of positive roots, highest weight, saturated sets of weights. [3 days]

Possible Textbooks: [Introduction to Lie Algebras and Representation Theory](#), James E. Humphreys, GTM 9, Springer-Verlag New York, 1972.

References:

- (1) [Lie Algebras](#), Nathan Jacobson, Dover, New York, 1979.
- (2) [Complex semisimple Lie algebras](#), Jean-Pierre Serre, Springer, Berlin Heidelberg, 1987.
- (3) [Modular Lie Algebras and Their Representations](#), Helmut Strade and Rolf Farnsteiner, Pure and Applied Mathematics, Marcel Dekker, New York, 1988.

Sample Grading and Evaluation Procedures: Homeworks, quizzes and tests, at the instructors discretion. A student is expected to prove rigorous theorems as well as to compute concrete examples.

Suggested Grade Calculation: Homeworks 30%, Quizzes 30%, Tests 40%. There may be variations in these procedures depending on the individual instructors and the nature of the specific material.

Justification for Graduate Credit: The course demands considerable mathematics background and a degree of mathematical maturity traditionally found at the graduate level. The 7000-level course will inculcate the same analytical skills and depth of understanding previously demanded by the comparable 600-level quarter course. In order to successfully complete the course

the student will have to demonstrate an ability to creatively examine and apply the mathematics presented in the course.