Math 2660 Topics in Linear Algebra, Key

3.1

1a,b,7,8,9

1 (a) The length of $\mathbf{x_1}$ is $\sqrt{8^2+6^2}=10$ and the length of $\mathbf{x_2}$ is $\sqrt{4^2+(-1)^2}=\sqrt{15}$.

(b) $\mathbf{x_3}=\mathbf{x_1}+\mathbf{x_2}=(8,6)^T+(4,-1)^T=(12,5)^T$ so that the length of $\mathbf{x_3}$ is $\sqrt{12^2+5^2}=\sqrt{169}=13$.

7. Suppose $\mathbf{0}$ and $\mathbf{0}'$ are zero elements. Then by A3, $\mathbf{0}+\mathbf{0}'=\mathbf{0}'$ (view $\mathbf{0}$ as the zero element) on one hand and $\mathbf{0}+\mathbf{0}'=\mathbf{0}$ (view $\mathbf{0}'$ as the zero element) on the other hand. So $\mathbf{0}=\mathbf{0}'$

8 Suppose $\mathbf{x}+\mathbf{y}=\mathbf{x}+\mathbf{z}$. Add $-\mathbf{x}$ (the additive inverse of $\mathbf{x}$) on both sides to have $-\mathbf{x}+\mathbf{x}+\mathbf{y}=-\mathbf{x}+\mathbf{x}+\mathbf{z}$ so that $\mathbf{y}=\mathbf{z}$ (see A4 on p.119).

9 (a) $\beta\mathbf{0}=\beta(\mathbf{0}+\mathbf{0})=\beta\mathbf{0}+\beta\mathbf{0}$ by A3 and A5. Then add $-(\beta\mathbf{0})$ on both sides to have $\beta\mathbf{0}=\mathbf{0}$ (see A4 on p.119).

(b) Suppose $\alpha\mathbf{x}=\mathbf{0}$. If $\alpha\neq 0$, then divide both sides by $\frac{1}{\alpha}$ to have $\mathbf{x}=\mathbf{0}$ by A7 and (a).