Math 2660 Topics in Linear Algebra, Key

2.3

1a,c,2a,c,10

1 (a) $A = \begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix}$.

(i) $\det A = -1 - 6 = -7$.

(ii) $\text{adj } A = \begin{bmatrix} -1 & -2 \\ -3 & 1 \end{bmatrix}$.

(iii) $A^{-1} = \frac{1}{\det A}\text{adj } A = -\frac{1}{7}\begin{bmatrix} -1 & -2 \\ -3 & 1 \end{bmatrix}$.

(c) $A = \begin{bmatrix} 1 & 3 & 1 \\ 2 & 1 & 1 \\ -2 & 2 & -1 \end{bmatrix}$

(i) $\det A = \begin{vmatrix} 1 & 3 & 1 \\ 2 & 1 & 1 \\ -2 & 2 & -1 \end{vmatrix} \begin{matrix} \\ R_2 - 2R_1 \\ R_2 + 2R_1 \end{matrix} \begin{vmatrix} 1 & 3 & 1 \\ 0 & -5 & -1 \\ 0 & 8 & 1 \end{vmatrix} = -5 + 8 = 3.$

(ii) $A_{11} = \begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix} = -3,\ A_{12} = -\begin{vmatrix} 2 & 1 \\ -2 & -1 \end{vmatrix} = 0,\ A_{13} = \begin{vmatrix} 2 & 1 \\ -2 & 2 \end{vmatrix} = 6,\ A_{21} = -\begin{vmatrix} 3 & 1 \\ 2 & -1 \end{vmatrix} = 5,\ A_{22} = \begin{vmatrix} 1 & 1 \\ -2 & -1 \end{vmatrix} = 1,\ A_{23} = -\begin{vmatrix} 1 & 3 \\ -2 & 2 \end{vmatrix} = 8,\ A_{31} = \begin{vmatrix} 3 & 1 \\ 1 & 1 \end{vmatrix} = 2,$ $A_{32} = -\begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} = 1,\ A_{33} = \begin{vmatrix} 1 & 3 \\ 2 & 1 \end{vmatrix} = -5.$

(iii) So $A^{-1} = \frac{1}{\det A}\text{adj } A = \frac{1}{3}\begin{bmatrix} -3 & 5 & 2 \\ 0 & 1 & 1 \\ 6 & 8 & -5 \end{bmatrix}$

2 (a) $A \begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix}$, $b = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$, $\det A = -1 - 6 = -7$.

$$x_1 = \frac{\det A_1}{\det A} = \frac{\begin{vmatrix} 3 & 2 \\ 1 & -1 \end{vmatrix}}{-7} = \frac{5}{7}, \quad x_2 = \frac{\det A_2}{\det A} = \frac{\begin{vmatrix} 1 & 3 \\ 3 & 1 \end{vmatrix}}{-7} = \frac{8}{7}.$$

(c) $A = \begin{bmatrix} 2 & 1 & -3 \\ 4 & 5 & 1 \\ -2 & -1 & 4 \end{bmatrix}$, $b = \begin{bmatrix} 0 \\ 8 \\ 2 \end{bmatrix}$, $\det A = 6$ by direct computation.

$$x_1 = \frac{\det A_1}{\det A} = \frac{\begin{vmatrix} 0 & 1 & -3 \\ 8 & 5 & 1 \\ 2 & -1 & 4 \end{vmatrix}}{6} = \frac{24}{6} = 4, \quad x_2 = \frac{\det A_2}{\det A} = \frac{\begin{vmatrix} 2 & 0 & -3 \\ 4 & 8 & 1 \\ -2 & 2 & 4 \end{vmatrix}}{6} = \frac{-12}{6} = -2,$$

and

$$x_3 = \frac{\det A_3}{\det A} = \frac{\begin{vmatrix} 2 & 1 & 0 \\ 4 & 5 & 8 \\ -2 & -1 & 2 \end{vmatrix}}{6} = \frac{12}{6} = 2.$$

10 If A is nonsingular, then $\det A \neq 0$. So by the formula on p.106, $A(\frac{1}{\det A}\text{adj } A) = I$. Apply the formula on A^{-1} i.e., $(A^{-1})^{-1} = \frac{1}{\det(A^{-1})}\text{adj }(A^{-1})$. Then

$$(\text{adj } A)^{-1} = \det(A^{-1})A = \text{adj }(A^{-1}).$$