Math 2660 Topics in Linear Algebra, Key

2.2

1,2a,3a,c,5,6,7,8,9,14

1 (a) Switch the first and last rows to have upper triangular form. So $\det A = -2(3)(4) = -24$.

(b) Add 1st row to 3rd row to have upper triangular form. So $\det A = 1(3)(2)(5) = 30$.

(c) Expansion along the first row to have $\det A = -1$.

2 (a) $$\begin{vmatrix} 0 & 1 & 2 & 3 \\ 1 & 1 & 1 & 1 \\ -2 & -2 & 3 & 3 \\ 1 & 2 & -2 & -3 \end{vmatrix} \begin{matrix} \\ \\ R_3+2R_2 \\ R_4-R_2 \end{matrix} = \begin{vmatrix} 0 & 1 & 2 & 3 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 5 & 5 \\ 0 & 1 & -3 & -4 \end{vmatrix} = (-1)\begin{vmatrix} 1 & 2 & 3 \\ 0 & 5 & 5 \\ 1 & -3 & -4 \end{vmatrix} \; R_3 - R_1$$

$$= (-1)\begin{vmatrix} 1 & 2 & 3 \\ 0 & 5 & 5 \\ 0 & -5 & -7 \end{vmatrix} = (-1)(-35+25) = 10.$$

3 (a) $\begin{vmatrix} 3 & 1 \\ 6 & 2 \end{vmatrix} = 6 - 6 = 0$. So the matrix is singular by Theorem 2.2.2.

(c) $\begin{vmatrix} 3 & 3 & 1 \\ 0 & 1 & 2 \\ 0 & 2 & 3 \end{vmatrix} = 3\begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} = -1 \neq 0$. So the matrix is nonsingular by Theorem 2.2.2.

5 $\det(\alpha A) = \alpha^n \det A$ since αA means applying type 2 row operation on A n times, where A is $n \times n$.

6 Use Theorem 2.2.3 $\det A \det(A^{-1}) = \det(AA^{-1}) = 1$. Hence $\det(A^{-1}) = \frac{1}{\det A}$.

7 (a) $\det(AB) = \det A \det B = 4(5) = 20$.

(b) $\det(3A) = 3^3 \det A = 27(4) = 108$.

(c) $\det(2AB) = 2^3 \det A \det B = 8(4)(5) = 160$.

(d) $\det(A^{-1}B) = \det(A^{-1}) \det B = \frac{1}{\det A} \det B = \frac{5}{4}$.

8 E^T is the transpose of E. So I. $E_{ij}^T = E_{ji} = E_{ij}$, II. $E_{ii}(c)^T = E_{ii}(c)$, III $E_{ij}(c)^T = E_{ji}(c)$. So they are of the corresponding type.

9 Let A, B be 3×3 matrices with $\det A = 4$ and $\det B = 5$.

(a) $\det(E_1 A) = -\det A = -6$.

(b) $\det(E_2 A) = 3 \det A = 18$.

(c) $\det(E_3 A) = \det A = 6$,

(d) $\det(AE_1) = -\det A = -6$,

(e) $\det E_1^2 = \det I = 1$.

(f) $\det(E_1 E_2 D_3) = \det E_1 \det E_2 \det E_3 = (-1)2(1) = -2$.

14 By Theorem 2.2.3 $\det(AB) = \det A \det B$. So $\det(AB) \neq 0$ if and only if $\det A$ and $\det B$ are both nonzero. Then by Theorem 2.2.2 we have the desired result.