Math 2660 Topics in Linear Algebra, Key

2.1

2,3c,e,4,6

2 (a) $\det\begin{bmatrix}3 & 5\\ 2 & 4\end{bmatrix} = 3\cdot 5 - 5\cdot 2 = 2$. So the matrix is nonsingular by Theorem 2.2.2.

(b) $\det\begin{bmatrix}3 & 6\\ 2 & 4\end{bmatrix} = 3\cdot 4 - 6\cdot 2 = 0$. So the matrix is singular by Theorem 2.2.2.

(c) $\det\begin{bmatrix}3 & 5\\ 2 & 4\end{bmatrix} = 3\cdot 5 - 5\cdot 2 = 2$. So the matrix is nonsingular by Theorem 2.2.2.

3 (c) $\begin{vmatrix}3 & 1 & 2\\ 2 & 4 & 5\\ 2 & 4 & 5\end{vmatrix} = 3\begin{vmatrix}4 & 5\\ 4 & 5\end{vmatrix} - 1\begin{vmatrix}2 & 5\\ 2 & 5\end{vmatrix} + 2\begin{vmatrix}2 & 4\\ 2 & 4\end{vmatrix} = 0$. Indeed from Theorem 2.1.4 we know the answer is zero since the matrix has two identical rows.

(e) $\begin{vmatrix}1 & 3 & 2\\ 4 & 1 & -2\\ 2 & 1 & 3\end{vmatrix} = 1\begin{vmatrix}1 & -2\\ 1 & 3\end{vmatrix} - 3\begin{vmatrix}4 & -2\\ 2 & 3\end{vmatrix} + 2\begin{vmatrix}4 & 1\\ 2 & 1\end{vmatrix} = 5 - 3(16) + 4 = -39.$

4 (a) $\begin{vmatrix}3 & 5\\ 2 & 4\end{vmatrix} = 2.$

(b) $\begin{vmatrix}2 & 0 & 0\\ 4 & 1 & 0\\ 7 & 3 & -2\end{vmatrix} = 2(1)(-2) = -4.$

(c) $\begin{vmatrix}2 & 0 & 0\\ 2 & 1 & 1\\ 1 & 2 & 2\end{vmatrix} = 3\begin{vmatrix}1 & 1\\ 2 & 2\end{vmatrix} = 0.$

(d) By Theorem 2.1.4, the determinant is zero since the second column is zero.

6 $\begin{vmatrix}2-\lambda & 4\\ 3 & 3-\lambda\end{vmatrix} = (2-\lambda)(3-\lambda) - 12 = \lambda^2 - 5\lambda - 6 = (\lambda - 6)(\lambda + 1)$. So $\lambda = 6$ or -1 to have determinant zero.