Math 2660 Topics in Linear Algebra, Key

1.1

1c, 3, 6a,d, 10, 11

1 (c) Back substitution: $x_4 = \frac{4}{4} = 1$,
$-x_3 + 2x_4 = -1 \Rightarrow x_3 = 1 + 2x_4 = 1 + 2 = 3$,
$3x_2 + x_3 - 2x_4 = 1 \Rightarrow x_2 = \frac{1}{3}(1 - x_3 + 2x_4) = \frac{1}{3}(1 - 3 + 2) = 0$
$x_1 + 2x_2 + 2x_3 + x_4 = 5 \Rightarrow x_1 = 5 - 2x_2 - 2x_3 - x_4 = 5 - 0 - 2(3) - 1 = -2$.
So $x = (-2, 0, 3, 1)$.

3 (a) Only one solution, $(3, 1)$.
(b) No solution, two parallel lines.
(c) Infinitely many solutions because they represent the same line.
(d) No solution.

6 (a)

$$\left[\begin{array}{cc|c} 1 & -2 & 5 \\ 3 & 1 & 1 \end{array}\right] \begin{array}{c} \\ R_2 - 3R_1 \end{array} \left[\begin{array}{cc|c} 1 & -2 & 5 \\ 0 & 7 & -14 \end{array}\right] \begin{array}{c} \\ \frac{1}{7}R_2 \end{array} \left[\begin{array}{cc|c} 1 & -2 & 5 \\ 0 & 1 & -2 \end{array}\right]$$

So $x_2 = -2$, $x_1 = 5 + 2x_2 = 5 - 4 = 1$. Hence $x = (1, -2)$.

(d)

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 2 & -1 & 1 & 3 \\ -1 & 2 & 3 & 7 \end{array}\right] \begin{array}{c} \\ R_2 - 2R_1 \\ R_3 + R_1 \end{array} \left[\begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 0 & -5 & 3 & 1 \\ 0 & 4 & 2 & 8 \end{array}\right] \begin{array}{c} \\ -\frac{1}{5}R_2 \\ \\ \end{array} \left[\begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 0 & 1 & -\frac{3}{5} & -\frac{1}{5} \\ 0 & 4 & 2 & 8 \end{array}\right]$$

$$\begin{array}{c} \\ \\ R_3 - 4R_2 \end{array} \left[\begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 0 & 1 & -\frac{3}{5} & -\frac{1}{5} \\ 0 & 0 & \frac{22}{5} & \frac{44}{5} \end{array}\right].$$

Back substitution gives $x = (1, 1, 2)$.

10 It is always consistent because the trivial solution $x = (0, 0)$ is always a solution to the homogenous system.

11 A system of three equations in three unknowns represents three planes. If the planes are parallel or one plan is parallel to the line of intersection of the other two, then the solution set will be empty. The three equations could represent the same plane or the three planes could all intersect in a line. In either case the solution set will contain infinitely many points. If the three planes intersect in a point, then the solution set will contain only that point. Hence, either no solution, exactly one solution, or infinitely many solutions.