

Useful Formulas

Half argument

$$\sin^2 \frac{x}{2} = \frac{1 - \cos x}{2}$$

$$\cos^2 \frac{x}{2} = \frac{1 + \cos x}{2}$$

$$\operatorname{tg} \frac{x}{2} = \frac{\sin x}{1 + \cos x} = \frac{1 - \cos x}{\sin x}$$

$$\sin x \pm \sin y = 2 \sin \frac{x \pm y}{2} \cos \frac{x \mp y}{2}$$

$$\cos x + \cos y = 2 \cos \frac{x + y}{2} \cos \frac{x - y}{2}$$

$$\cos x - \cos y = -2 \sin \frac{x + y}{2} \sin \frac{x - y}{2}$$

$$\operatorname{tg} x \pm \operatorname{tg} y = \frac{\sin(x \pm y)}{\cos x \cos y}$$

f(x) figure area

$$S = \int_a^b f(x) dx$$

$$S = \left| \int_{t_1}^{t_2} \phi(t) \psi'(t) dt \right|$$

$$S = \int_{\phi_1}^{\phi_2} r^2(\phi) d\phi$$

Newton's binomial

$$(x+a)^n = \sum_{k=0}^n C_n^k x^{n-k} a^k$$

Universal substitute

$$\sin 2x = \frac{2 \operatorname{tg} x}{1 + \operatorname{tg}^2 x}$$

$$\cos 2x = \frac{1 - \operatorname{tg}^2 x}{1 + \operatorname{tg}^2 x}$$

$$\operatorname{tg} 2x = \frac{2 \operatorname{tg} x}{1 - \operatorname{tg}^2 x}$$

$$\sin x \sin y = \frac{1}{2} (\cos(x-y) - \cos(x+y))$$

$$\cos x \cos y = \frac{1}{2} (\cos(x-y) + \cos(x+y))$$

$$\sin x \cos y = \frac{1}{2} (\sin(x-y) + \sin(x+y))$$

f(x) curve length

$$L = \int_a^b \sqrt{1 + f'(x)^2} dx$$

$$L = \int_{\alpha}^{\beta} \sqrt{\phi'^2(t) + \psi'^2(t)} dt$$

$$L = \int_{\alpha}^{\beta} \sqrt{r^2(\phi) + r'^2(\phi)} d\phi$$

Binomial differential

$$\int x^m (a + bx^n)^p dx$$

1) $p \in \mathbb{Z} : x = t^N, N = \operatorname{com.div.}(m, n)$

2) $\frac{m+1}{n} \in \mathbb{Z} : a + bx^n = t^N, N = \operatorname{div.}(p)$

3) $\frac{m+1}{n} + p \in \mathbb{Z} : ax^{-n} + b = t^N, N = \operatorname{div.}(p)$

Inverse hype func

$$\operatorname{Arsh} x = \ln(x + \sqrt{x^2 + 1})$$

$$\operatorname{Arch} x = \ln(x \pm \sqrt{x^2 - 1})$$

$$\operatorname{Arth} x = \frac{1}{2} \ln \frac{1+x}{1-x}$$

Curvature radius

$$R = \frac{(\dot{x}^2 + \dot{y}^2)^{3/2}}{|\dot{x}\ddot{y} - \dot{y}\ddot{x}|}$$

$$R = \frac{(1 + y'^2)^{3/2}}{y''}$$

f(x) rotation body parameters

$$V_{Ox} = \pi \int_a^b f^2(x) dx$$

$$S_{Ox} = \int_a^b 2\pi f \sqrt{1 + f'^2} dx$$