practice1610a.tex

instructions here

1. sketch the graph of a function that satisfies all of the following: $\lim_{x \to 3^+} f(x) = 4, \lim_{x \to 3^-} f(x) = 2, \lim_{x \to 2} f(x) = 2, f(3) = 3, f(-2) = 1$
2. determine the infinite limit $\lim_{x \to 5^+} \frac{4}{x-5}$
3. compute the limit, if it exists $\lim_{h \to 0} \frac{(1+h)^3 - 1}{h}$
4. let $f(x) = |x-1|$ compute or state "does not exist": $\lim_{x \to 1^+} f(x), \lim_{x \to 1^-} f(x), \lim_{x \to 1} f(x)$
5. sketch the graph of a function that satisfies all of the following: $\lim_{x \to +\infty} f(x) = -2, \lim_{x \to -\infty} f(x) = 2, \lim_{x \to 2^+} f(x) = -\infty, \lim_{x \to 2^-} f(x) = +\infty f(0) = 0, f(4) = 0.$
6. sketch the graph of a function satisfying all of the following $g(0) = 0, g'(0) = 1, g(1) = 1, g'(1) = 0, g(2) = 0, g'(2) = -1$
7. find the derivative of the given function using the definition. $f(x) = \frac{1}{x^2}$
8. find the derivative of the given function using the definition. $f(x) = x^2 + x$
9. find the derivative of the given function using the definition. $f(x) = \sqrt{x}$
10. use derivative rules to find derivatives
    (a) $f(x) = (2x)^3$
    (b) $f(x) = \sin(\sqrt{x}(x-1))$
    (c) $f(x) = \frac{x^2 - 2x}{x}$
    (d) $f(x) = 3x^{\frac{1}{2}} + e^x$
    (e) $f(x) = x\sqrt{x} + \frac{1}{x\sqrt{x}}$
    (f) $f(x) = \frac{1+x^2}{1-x^2}$
    (g) $f(x) = \sin(x) - x\cos(x)$
    (h) $f(x) = e^x \sec(x)$
    (i) $f(x) = e^{\sin(x)}$
11. Find the derivative of $f(x) = x^3 - x^2 - x + 1$ and use it to find the points on the graph of $f(x)$ where the tangent is horizontal.
12. Find the two points on the curve $y = x^2$ where there is a tangent line passing through the point $(0, -4)$

13. a particle moves along the y axis, its position at time t is given by $y = t^3 - 9t^2 + 15t$. Answer the following.

    (a) at what times t is the particle at the origin
    (b) at what times t is the particle not moving?
    (c) at what times t is the acceleration zero?

14. Find equation of the line tangent to the graph of $y = \sin(x) + \cos(2x)$ at the point $(\frac{\pi}{6}, 1)$

15. Suppose that $f(x) = h(g(x)), g(3) = 6, g'(3) = 4 f'(3) = 2$ and $f'(6) = 7$ Find $f'(3)$

16. Find equation of the line tangent to the graph of $y = \sin(x) + \cos(2x)$ at the point $(\frac{\pi}{6}, 1)$

17. Find $\frac{dy}{dx}$ using implicit differentiation

    (a) $\sqrt{xy} = 1 + xy^2$
    (b) $x\cos(y) = y\sin(x)$

18. Find equation of the line tangent to the graph of $\frac{x^2}{16} - \frac{y^2}{9} = 1$ at the point $(-5, \frac{9}{4})$

19. find the following higher derivatives

    (a) find $f^{(3)}$ for $f(x) = \sqrt{2x+1}$
    (b) find $\frac{d^2}{dx^2}\arctan(x)$

20. (3.8) Compute the derivative of each of the following

    (a) $f(x) = \ln(e^{x^2})$
    (b) $f(x) = \ln(\sqrt{\frac{x+1}{x^2+1}})$
    (c) $f(x) = \ln(x + \sqrt{x})$
    (d) $f(x) = x^x$
    (e) $f(x) = \log_{10}(x^2 + 2)$

21. (3.8) Compute y' using logarithmic differentiation

    (a) $y = \sqrt{\frac{(x^2+1)^3(x-1)^5}{(2x-1)(1-x^{-1})}}$
    (b) $y = (x-2)(x-1)x(x+1)(x+2)$
    (c) $y = x^{\ln(x)}$

22. (3.10) Related rates

    (a) Two cars start from the same point. One travels south at a rate of 50 mph and the other travels west at a rate of 60 mph. At what rate is the distance between the cars increasing 1 hr later?

(b) A lader 10 ft long rests against a vertical wall. If the top of the ladder is lowered at a rate of 1 ft per sec, how fast is the bottom end moving away from the wall when it is 6 ft away?

(c) Sand is being dumped at a rate of 2 cu ft per second onto a conical pile whose base diameter is always equal to its height. At what rate is the pile getting taller when it is 5 ft high?

23. (4.1) Find the critical points of

(a) $f(x) = 2x^3 + 3x^2 + 6x + 4$

(b) $f(x) = \frac{x}{x^2+1}$

(c) $f(x) = x\ln(x)$

(d) $f(x) = \sin(x)$

24. (4.1) find the max and min of the given function on the given interval

(a) $f(x) = 2x^3 + 3x^2 + 4; [-2, 1]$

(b) $f(x) = \sin(x) + \cos(x); [0, \frac{\pi}{2}]$

(c) $f(x) = xe^{-x}; [0, 5]$

25. (4.4) Find the limits, using L'Hopitals rule when appropriate.

(a) $\lim_{x\to 0} \frac{\sin(x)}{x}$

(b) $\lim_{x\to\infty} e^{-x}x^2$

(c) $\lim_{x\to 0} \frac{1}{x^2} - \frac{1}{x^4}$

(d) $\lim_{x\to\infty} x^{\frac{1}{x}}$

(e) $\lim_{x\to\infty} \sqrt{\frac{3x^3+2x^2-1}{2x^3+4x^2+x}}$

26. (4.3) For each of the following functions, find

- intervals of increase and decrease
- local max and min
- intervals of concave up and concave down
- points of inflection

(a) $f(x) = x^4 - 6x^2$

(b) $f(x) = \sin(x) + x$ on interval $[-2\pi, 2\pi]$

(c) $f(x) = x\sqrt{x^2+1}$

27. (4.3) For each of the following functions, find horizontal and vertical asymptotes

(a) $f(x) = \frac{1+x^2}{1-x^2}$

(b) $f(x) = \frac{e^x}{1+e^x}$

(c) $f(x) = x\tan(x)$ on the interval $[0, \pi]$

28. (4.3) Sketch the graph of a function satisfying all of the following

- intervals of increase: $[-\infty, -1], [1, \infty]$
- interval of decrease: $[-1, 1]$
- interval of concave up: $[0, \infty]$
- interval of concave down: $[-\infty, 0]$

29. (4.7) Word problems involving max and min

(a) Find the dimensions and volume of the largest open topped box that can be made with 600 square inches of material.

(b) Find a positive number such that the sum of the number and its reciprocal is as small as possible

(c) Find the area and dimensions of the largest rectangle that can be drawn inside a half-circle of radius 1

30. (4.10) Find the most general (ie include free constants if possible) $f(x)$ based on the given information:

(a) $f''(x) = x$

(b) $f'(x) = 1 + \frac{1}{x^2}, f(1) = 0$

(c) $f''(x) = x + \sqrt{x}, f(1) = 1, f'(1) = 2$

(d) $f'(x) = \sin(x) f(0) = 0$

31. (5.2) Write the Riemann Sum approximation for $\int_0^4 x^2 dx$ integrals using

(a) $n = 4$ terms and right endpoints

(b) $n = 4$ terms and left endpoints

(c) $n = 2$ terms and midpoints

32. (5.3) State both parts of the fundamental Theorem of Calculus

33. (5.3) Compute derivatives of the following functions $f(x)$

(a) $f(x) = \int_0^{2x} t\sin(t)dt$

(b) $f(x) = \int_0^x e^{t^2} dt$

34. (5.3, 5.5) Evaluate the definite integrals

(a) $\int_{-1}^{2} x^3 dx$

(b) $\int_{e}^{e^2} \frac{1}{x} dx$

(c) $\int_{0}^{\pi} \sin(x) dx$

(d) $\int_{1}^{2} x\sqrt{x-1} dx$

(e) $\int_{0}^{3} \frac{dx}{2x+3}$

35. (5.4,5.5) Evaluate the integrals

(a) $\int x\cos(x^2) dx$

(b) $\int \frac{x}{\sqrt{x^2+1}} dx$

(c) $\int \frac{\ln(x)}{x} dx$

(d) $\int \cos(2x) dx$