**Math 1610a Final Spring 2004**

Put your name at the top of the page. Work each problem according to instructions. Problems marked with "pc" are partial credit problems where your work will be graded. all work must be shown on "pc" problems to earn credit. Problems marked with "ao" are answer only problems. On answer only problems, you will get full credit or zero depending only on the answer shown.

1. (5, pc) Find equation of the line tangent to the graph of $y = \sqrt{x}$ at the point $(25, 5)$

2. (5, pc) Find the derivative of the given function using the definition. $f(x) = \sqrt{x}$

3. (2, ao, each part) Use derivative rules to find derivatives

   (a) $f(x) = \int_0^{2x} e^{t^2} dt$

   (b) $f(x) = \frac{x^2 - 2x}{x}$

   (c) $f(x) = \ln(e^{x^2})$

(d) $f(x) = \sin(x) - x\cos(x)$

(e) $f(x) = e^{\sin(x)}$

4. (5, pc)Use logarithmic differentiation to find the derivative of $f(x) = x^{\cos(x)}$

5. (5, pc) Find $\frac{dy}{dx}$ using implicit differentiation $x^2y = 1 + xy^2$

6. (4, pc, each part) Find the following higher derivatives

   (a) find $f^{(3)}$ for $f(x) = \sqrt{2x+1}$

   (b) find $\frac{d^2}{dx^2}\arctan(x)$

7. (5, pc) Sand is being dumped at a rate of 2 cu ft per second onto a conical pile whose base diameter is always equal to its height. At what rate is the pile getting taller when it is 5 ft high?

8. (5, pc) Find the max and min of $f(x) = \sin(x) + \cos(x)$ on $[0, \frac{\pi}{2}]$

9. Find the limits, using L'Hopitals rule when appropriate.

   (a) (2, ao) $\lim_{x\to\infty} xe^{-x}$

   (b) (5, pc) $\lim_{x\to\infty} x^{\frac{1}{x}}$

10. (1, ao each feature) Sketch the graph of a function satisfying all of the following

    - intervals of increase: $[-\infty, -1], [1, \infty]$
    - interval of decrease: $[-1, 1]$
    - interval of concave up: $[0, \infty]$
    - interval of concave down: $[-\infty, 0]$

11. (5, pc) Find the area and dimensions of the largest rectangle that can be drawn inside a half-circle of radius 1

12. (4, pc, each part) Find the most general (ie include free constants if possible) $f(x)$ based on the given information:

    (a) $f''(x) = x$

    (b) $f'(x) = \sin(x), f(0) = 0$

13. (2, ao) Write the Riemann Sum approximation for $\int_0^4 x^2 dx$ integrals using $n = 4$ terms and left endpoints

14. (3, ao, each part) State both parts of the fundamental Theorem of Calculus

15. (5, pc, each part) Evaluate the definite integrals

(a) $\int_{-1}^{2} x^3 dx$

(b) $\int_{1}^{2} x\sqrt{x-1}dx$

16. (5, pc, each part) Evaluate the integrals

(a) $\int x\cos(x^2)dx$

(b) $\int \frac{\ln(x)}{x}dx$