6th European Congress of Mathematics
Kraków, July 1-7, 2012

Geometry in Dynamics - Satellite Thematic Session, July 1, 12:30 - 17:30 - Medium Hall
Matchbox Dynamics - Mini-symposium, July 2, 17:15 - 19:15 - Small Hall

Program for the STS and the mini-symposium

Geometry in Dynamics, STS, July 1, 12:30 - 17:30 - Medium Hall
Organizers: Alex Clark and Krystyna Kuperberg

12:30 - 12:55	Aleksander Ćwiszewski, Nicolaus Copernicus University, Poland *Positive stationary solutions of equations with p-Laplace operator*
13:00 - 13:25	Piotr Kokocki, Nicolaus Copernicus University, Poland *Periodic solutions and connecting orbits for nonlinear evolution equations at resonance*
13:30 - 13:55	Piotr Zgliczyński, Jagiellonian University, Poland *Geometric methods in the dynamics of dissipative PDEs*
14:00 - 14:25	Robbert Fokkink, Delf University, Netherlands *On paperfolding and Knaster continua*
14:30 - 14:55	Álvaro Lozano Rojo, University of Zaragoza, Spain *A universal space for Cantor expansive dynamics*
15:00 - 15:25	Pablo Gonzalez Sequeiros, University of Santiago de Compostela, Spain *Affability of laminations defined by repetitive planar tilings*
15:30 - 15:55	Klaudiusz Wójcik, Jagiellonian University, Poland *Lefschetz sequences and chaotic dynamics*
16:00 - 16:25	Paweł G. Walczak, University of Łódź, Poland *Godbillon-Vey class in codimension* > 1*: a Riemannian approach*
16:30 - 16:55	Szymon M. Walczak, University of Łódź, Poland *Metric diffusion along compact foliations*
17:00 - 17:25	Takashi Tsuboi, University of Tokyo, Japan *Homeomorphism groups of commutator width one*

Matchbox Dynamics - Mini-symposium, July 2, 17:15 - 19:15 - Small Hall
Organizer: Krystyna Kuperberg

17:15 - 17:40	Alex Clark, Leicester University, UK *Spongy matchbox manifolds*
17:45 - 18:10	Ana Rechtman, University of Strasbourg, France *Topological entropy of the dynamics of the Kuperberg minimal set*
18:15 - 18:40	Steve Hurder, University of Illinois at Chicago, USA *Cohomology and smooth embeddings for matchbox manifolds*
18:45 - 19:10	(program change) Krystyna Kuperberg, Auburn University, USA *UV-property of invariant matchbox manifolds*

Abstracts

Alex Clark, Leicester University, UK
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Co-author: Krystyna Kuperberg

Title: Spongy matchbox manifolds.

Abstract: We shall examine the homogeneity properties of continua based on matchbox manifolds that locally are homeomorphic to the product of a Cantor set and a Menger manifold.

Aleksander Ćwiszewski, Nicolaus Copernicus University, Toruń, Poland
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Co-author: Mateusz Maciejewski

Title: Positive stationary solutions of equations with p-Laplace operator

Abstract: We study the existence of solutions for nonlinear equations of the form

$$\begin{cases} -\mathrm{div}(|\nabla u(x)|^{p-2}\nabla u(x)) = f(x,u(x)), \quad x \in \Omega \\ \quad u(x) \geq 0, \quad x \in \Omega \\ \quad u(x) = 0, \quad x \in \partial\Omega, \end{cases}$$

where $\Omega \subset \mathrm{R}^N$ is an open bounded domain and $f : \Omega \times [0, +\infty) \to \mathrm{R}$ is a nonlinear continuous perturbation. To this end we construct a topological degree that, in a given set of constraints, detects coincidence points of a maximal monotone operator and a continuous perturbation. The construction is based on the tangency condition of the perturbation with respect to the constraint set. In this way we do not need the usual assumption that f should be nonnegative.

Robbert Fokkink, Delf University, Netherlands
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Title: On paperfolding and Knaster continua.

Abstract: Paperfolding curves were introduced by Heighway some forty years ago. They have been extensively studied by physicists, mathematicians and computer scientists. It turns out that paperfolding curves correspond to arc components of Knaster continua, which explains some of the ergodic properties of paperfolding curves that have been observed by Mendes-France and Van der Poorten.

Steven Hurder, University of Illinois at Chicago, USA
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Title: Cohomology and smooth embeddings for matchbox manifolds.

Abstract: We discuss the role of cohomology invariants for solenoids, and more generally minimal matchbox manifolds, for understanding their foliated embeddings into smooth (or possibly C^r for $r > 0$) foliations. In particular, we discuss relations between cohomology invariants associated to such embeddings, and the foliation dynamics, where the higher dimensional cases yield a much richer theory of invariants than for flows.

Piotr Kokocki, Nicolaus Copernicus University, Poland
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Title: Periodic solutions and connecting orbits for nonlinear evolution equations at resonance

Abstract: Assume that $A : X \supset D(A) \to X$ is a positive definite sectorial operator on a Banach space X and let X^α, where $\alpha \in [0,1)$, be a fractional power space given by $X^\alpha := D(A^\alpha)$. We shall consider differential equations of the form

$$\begin{aligned} \dot{u}(t) &= -Au(t) + \lambda u(t) + F(t, u(t)), && t > 0 && (1) \\ \ddot{u}(t) &= -Au(t) - cA\dot{u}(t) + \lambda u(t) + F(t, u(t)), && t > 0 && (2) \end{aligned}$$

where $c > 0$, λ is a real number and $F : [0, +\infty) \times X^\alpha \to X$ is a continuous map. Our goal is study the existence of T-periodic solutions ($T > 0$) and orbits connecting stationary points for the above equations being at *the resonance at infinity*, that is, $\ker(\lambda I - A) \neq \{0\}$ and F is bounded. The main difficulty lies in the fact that, due to the presence of resonance, the equations (1) and (2) may not have periodic solutions and bounded orbits for general perturbation F. Therefore we formulate geometrical conditions characterizing nonlinearity F and use them to prove theorems determining the existence of T-periodic solutions and orbits connecting stationary points for equations (1) and (2). The methods that we will use involve the application of homotopy invariants such as topological degree and Conley index to the semiflows associated with these equations. Finally, we provide applications of the obtained results in the case when A is second order differential operator on $X := L^p(\Omega)$, where $\Omega \subset \mathbb{R}^n$ is a bounded set and F is a Niemycki operator associated with continuous map $f : [0, +\infty) \times \Omega \times \mathbb{R} \to \mathbb{R}$. In particular, we prove that introduced geometrical assumptions generalize well known *Landesman–Lazer* and *strong resonance* type conditions.

References

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[4] Krzysztof P. Rybakowski, *Trajectories joining critical points of nonlinear parabolic and hyperbolic partial differential equations*, J. Differential Equations **51** (1984), no. 2, 182–212.

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Krystyna Kuperberg, Auburn University, USA
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Title: UV-property of invariant matchbox manifolds.

Abstract: A compact subset F of an ANR X is *movable in the sense of Borsuk* (or it has the UV-property) if for every neighborhood U of F there exists a neighborhood V of F such that for every neighborhood W of F there is a deformation of V into W within U. Non-trivial van Dantzig as well as McCord solenoids are not movable. On the other hand, the Denjoy sets are movable. The notion of movability is closely related to stability and thus of importance in dynamics. We shall address special cases of the following open question: Is every compact invariant set in flow on a manifold contained in a movable invariant set? This question has been also considered by Petra Šindelářová in her dissertation in 2006.

Ana Rechtman, IRMA, Université de Strasbourg, France
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Co-author: Steven Hurder

Title: Topological entropy of the dynamics of the Kuperberg minimal set

Abstract: In 1993 K. Kuperberg constructed examples of C^∞ and real analytic flows without periodic orbits on any closed 3-manifold. In the talk, I will present part of a study of the minimal set of Kuperberg's examples. In particular, I will explain that these examples are at the boundary of the set of flows with positive topological entropy in the C^1 topology.

Álvaro Lozano Rojo, Centro Universitario de la Defensa, Zaragoza, Spain
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Title: A universal space for Cantor expansive dynamics.

Abstract: Given a semigroup G and finite set of symbols S the set of maps

$$\Sigma = S^G = \{ \sigma : G \to S \}$$

with the action of G on Σ

$$\sigma(g') \cdot g \mapsto \sigma(g'g), \qquad g \in G$$

is a Bernoulli shift. A saturated subset of Σ is a subshift. Subshifts are universal spaces for expansive group actions: given an action of a group G on a Cantor set X then there exists a conjugation $\Phi : X \to \Sigma$ (for some finite set S) of the two actions of G.

Here we present the spaces of rooted subtrees of a Cayley graph constructed by Ghys and Kenyon as universal space for dynamics in a similar fashion, but allowing more dynamical objects as pseudogroups.

Pablo González Sequeiros, University of Santiago de Compostela, Spain
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Title: Affability of laminations defined by repetitive planar tilings

Abstract: A *planar tiling* is a partition of $\mathbb{R}^2$ into *tiles*, which are polygons touching face-to-face obtained by translation from a finite set of *prototiles*. If we consider $\mathbb{T}(\mathcal{P})$ the set of tilings $\mathcal{T}$ constructed from a finite set of prototiles $\mathcal{P}$, it is possible to endow it with a natural topology, the *Gromov-Hausdorff topology* [2, 3], which turns it into a compact metrizable space laminated by the orbits $L_{\mathcal{T}}$ of the natural $\mathbb{R}^2$-action. If $\mathcal{T} \in \mathbb{T}(\mathcal{P})$ is a *repetitive* tiling (i.e. for any patch M, there exists a constant $R > 0$ such that any ball of radius R contains a translated copy of M), then the closure of its orbit $\mathbb{X} = \overline{L}_{\mathcal{T}}$ is a minimal closed subset of $\mathbb{T}(\mathcal{P})$, called the *continuous hull of* $\mathcal{T}$. If $\mathcal{T}$ is also *aperiodic* (i.e. $\mathcal{T}$ has no translation symmetries), then $\mathbb{X}$ is transversally modeled by a Cantor set. Dynamical and ergodic properties of these laminations are important for the theoretical study of *quasicrystals*.

Affable equivalence relations are orbit equivalent to inductive limits of finite equivalence relations on the Cantor set. This notion has been introduced by J. Renault [7] and T. Giordano, I.F. Puntnam and C.F. Skau [6]. One can think of affability as the topological version of hyperfiniteness. A transversally Cantor lamination will be said to be *affable* if the equivalence relation induced on any total transversal is affable.

In [4], T. Giordano, H. Matui, I. Putnam and C. Skau have proved that any free minimal $\mathbb{Z}^2$-action on the Cantor set is affable. In order to demonstrate it, they combine strong convexity arguments with an important result about extension of minimal affable equivalence relations, called Absorption Theorem, given in [5]. In [1], we show that, equivalently, the continuous hull of any repetitive and aperiodic planar tiling is affable. Our proof is based on a special inflation process, which is similar to that used to construct Robinson tilings. Though we still use the Absorption Theorem, it has the advantage that no convexity argument is needed. Here we want to present the main concepts referenced and illustrate this proof.

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Takashi Tsuboi, University of Tokyo, Japan
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Title: Homeomorphism groups of commutator width one.

Abstract: We talk about the group Homeo(X) of homeomorphisms of a topological space X. The question is whether every element of Homeo(X) can be written as one commutator. We give several topological spaces which satisfy this property.

Paweł G. Walczak, Uniwersytet Łódzki, Poland
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Title: Godbillon-Vey class in codimension > 1: a Riemannian approach

Abstract: A Riemannian geometry formula defining a form representing the Godbillon-Vey class of a codimension-one foliation has been known since 1973. Here, we will provide such a formula for foliations of codimension q ($q \geq 1$) on $(2q+1)$-dimensional Riemannian manifolds. Our formula involves algebraic invariants for pairs of matrices (or, rather, endomorphisms of either linear spaces or vector bundles) used already in our recent paper joint with V. Rovenski and provides some topological obstructions for non-vanishing of its Godbillon-Vey class.

This gives a partial answer to one of the questions formulated during the problem session of the conference FOLIATIONS 2005 (held in Łódź, Poland, in June of 2005): Given a codimension $q > 1$ foliation $\mathcal{F}$ with $\mathrm{gv}(\mathcal{F}) \neq 0$, what can be said about the geometry and dynamics of $\mathcal{F}$?

Szymon M. Walczak, Uniwersytet Łódzki, Poland
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Title: Metric diffusion along compact foliations

Abstract: Let $(M, \mathcal{F}, g)$ be a smooth compact foliated manifold equipped with a Riemannian metric g carrying foliation $\mathcal{F}$. Let $t > 0$ be a real number. Using the Wasserstein distance $d_{\mathcal{W}}$, one can define *the diffused metric along the foliation* $\mathcal{F}$ by the formula

$$d_t(x, y) = d_{\mathcal{W}}(D_t\delta_x, D_t\delta_y), \quad x, y \in M,$$

where δ_z denotes the Dirac measure at point z, and D_t denotes the foliated heat diffusion operator. Since $(\delta_x, \delta_y) = d(x, y)$ for any two points $x, y \in M$ and $D_0 = \mathrm{id}$, the original metric d is the same metric as d_0. The metric space (M, d_t) will be denoted by M_t.

In the talk, the topology of M_t will be studied and some results on Gromov Hausdorff limits of diffused metrics along compact foliations, i.e., foliations with all leaves compact, will be presented.

Klaudiusz Wójcik, Jagiellonian University, Poland
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Title: Lefschetz sequences and chaotic dynamics.

Abstract: We present the arithmetical properties of the Lefschetz sequence and its dual sequence. We show some applications for detecting chaotic dynamics generated non-autonomous ordinary differential equations.

Piotr Zgliczyński, Jagiellonian University, Poland
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Title: Geometric methods in the dynamics of dissipative PDEs.

Abstract: We will discuss the method of self-consistent bounds for dissipative PDEs. This method allows for a direct application of tools from dynamical system theory (finite dimensional) to dissipative PDE. This includes both: abstract theorems and rigorous algorithms for integration of PDEs. As an example we will discuss a computer assisted proof of the existence of some heteroclinic connections between fixed points for Kuramoto-Sivashinski PDE on the line with odd and periodic boundary conditions.

The proof consists of the following stages:

1. the proof of the existence of two fixed points, “the source” and “the target”
2. rigorous estimates for the attracting region around the target point
3. rigorous estimates for one dimensional unstable manifold of the source point
4. rigorous integration of PDE – the propagation of the unstable manifold of the source until it enters the basin of attraction of the target point.