On the Capacity of Secondary Users in a Cognitive Radio Network

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Abstract— **Cognitive radio networks deal with opportunistic spectrum access leading to greater utilization of the spectrum. The extent of utilization depends on the primary user's traffic and also on the way the spectrum is accessed by the primary and secondary users. In this paper Continuous-time Markov chains are used to model the spectrum access. The proposed three-dimensional model represents a more accurate cognitive system than the existing models with increased spectrum utilization and than the random and reservation based spectrum access. A non-random access method is proposed to remove the forced termination states. In addition, call dropping and blocking probabilities are reduced. It is further shown that channel utilization is higher than random access and reservation based access.**

Keywords- Cognitive Network, markov chain, channel reservation, random access.

I. INTRODUCTION

A cognitive network is an opportunistic network. *Spectrum opportunity* deals with the usage of an available (free) channel that is a part of the spectrum which is not currently used by primary users [1]. The licensed owner of a frequency band is called a *primary user* (PU) and the one who utilizes spectrum opportunities for communication is called a *secondary user* (SU). Secondary users are equipped with cognitive radios to enable them to sense the presence of primary users and tune to the spectrum band (channel) which is not in use at any point of time for its own communication. When the primary user of that channel returns the secondary user is forced to vacate the channel. This is called *forced termination* in [2]. The secondary user may then shift to another available channel and recover from that state. This is called *spectrum hand-off.* Thus, the secondary users are serviced when the channels are free resulting in higher utilization of the spectrum.

Since the availability of the spectrum depends on the primary users' traffic, the number of secondary users serviced also varies with primary users' traffic. The amount of service that can be squeezed in from the free bands in a spectrum accessed by unrestricted primary users is called the capacity of secondary users. In this paper we model capacity of secondary users using three dimensional continuous time Markov chains. Markov chains were used to model dynamic spectrum access networks in [2-6]. [3] proposes a markov model, but it does not allow for the secondary users to reoccupy another free channel once it has been forced to vacate from a channel and considers the call to be completely dropped. The spectrum handoff capability of the cognitive radio is thus not modeled in this work. [2] tries to reduce the *forced termination* of the secondary radios at the cost of blocking probability by reserving some of the channels for primary user access only. Both of these papers discuss on the optimal reservation of the channels for primary users to reduce the dropping probability and forced termination when in-fact these states can be totally avoided with spectrum hand-off capability of a cognitive radio. Analysis in [4-7], does not consider prioritized primary users. [8-9] proposes Markov models to study secondary user contention and obtain fairness among them in a resource sharing environment.

In this paper, we model a system in which the primary users are prioritized as well as the secondary users are having spectrum hand-off capability with fair access to channels. The Markov model proposed in [3] has been modified to accommodate the spectrum hand-off capability. The distinction between the forced termination, dropping and blocking is made clear. A non-random channel access method is proposed in which the forced termination states are totally eliminated and also the dropping and blocking probabilities are reduced resulting in higher capacity for secondary users.

The rest of the paper is organized as follows. The system model is proposed in Section II. Section III discusses the Markov model and its analysis. Non-random channel access method is proposed in Section IV. Section V presents the results and Section V concludes the work.

II. SYSTEM MODEL AND ANALYSIS

In this section three different channel assignments are discussed and the system model explained and developed as these three methods are explained.

A. Random Channel Assignment

There are a total of N channels. Each channel is assumed to be of equal bandwidth. A channel can be accessed by a secondary user if it is not being occupied by any primary user. The primary users can occupy any channel and have the right to reclaim a channel at any time from secondary users. In the initial model it is assumed that both the primary users and secondary users access the channels randomly. This is explained with the help of Fig. 1. There are a total five channels of which two are occupied by PUs and one by an SU. When a new SU arrives as shown in Fig. 1a, it chooses a random free channel. A PU can choose any random channel and as shown in Fig. 1b, if it chooses a secondary occupied channel, the SU jumps to a different free channel. If there is no other channel available, the SU's service is dropped as shown in Fig. 1c. An SU cannot use a channel if it does not have an opportunity to do so as shown in Fig 1c.

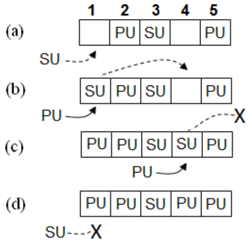

Figure 1. Random access in five channels.

There are four states in this model. The states are explained from the point of view of secondary users since the paper focuses on the capacity and channel utilization of secondary users and since primary users anyways have unrestricted usage of channels, study of their behavior is not of our interest.

Non-blocking state: A secondary user is considered to be in this state if it is completely serviced without being interrupted by a primary user on that channel.

Dropping state: When the primary user of a channel returns, the secondary user utilizing that channel should vacate. If there are no more free channels available then it is semi-serviced and its call is dropped.

Forced termination state or *Transition state*: This is the state during which the secondary user is shifting its channel due to the return of the licensed user into the previous channel. In this case there are free channels to shift to and so the secondary user performs a spectrum hand-off.

Blocking state: When all the channels are occupied by either primary users or secondary users, then an incoming secondary user does not have any opportunity for communication and it is considered to be completely blocked.

The Markov model for random assignment of channels with spectrum hand-off is explained using a sample system with 3 channels in Fig 2. The PUs and SUs are assumed to follow a Poisson arrival process with mean rates λ_p and λ_s respectively. They have a negative exponential service time distribution with mean rate $1/\mu_p$ and $1/\mu_s$ respectively. The numbers *i, j, k* represent the number of PUs, SUs and the type of state the secondary user is in respectively. Spectrum hand-off is accounted, for example, by letting the state (1, 1, 1) back to (1, 2, 0) and not dropping it. If it were dropped then it has to be sent to (1, 1, 0). P (i, j, k) denotes the steady-state probability of state. (i, j, k)

The balance equations for this model are given below.

For $i = 0, 0 \leq j \leq (N-1), k = 0,$

$$[j.\mu s + i\mu p + \lambda s + \lambda p].P(i,j,k) = \delta.\lambda s.P(i,j-1,k)$$
$$+(j+1).\mu s.P(i,j+1,k) + (i+1).\mu p.P(i+1,j,k)$$
$$\delta = 0\ for\ j = 0\ and\ \delta = 1\ for\ j \neq 0 \qquad (1)$$

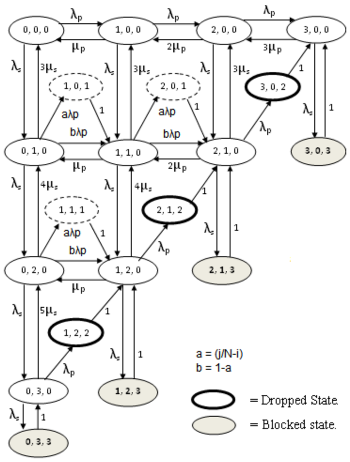

Figure 2. Markov model for cognitive network with spectrum hand-off.

For $i \neq 0, k = 0, i + j \leq (N-1),$

$$[j.\mu s + \lambda s + \lambda p + i.\mu p].P(i,j,k)$$
$$= \delta.\lambda s.P(i,j-1,k) + (j+1).\mu s.P(i,j+1,k)$$
$$+\left(\frac{(N-i-j)}{N-i}\right).\lambda p.P(i-1,j,k)$$
$$+(i+1).\mu p.P(i+1,j,0) + \delta.P(i,j-1,k+1)$$
$$\delta = 0\ for\ j = 0\ and\ \delta = 1\ for\ j \neq 0 \qquad (2)$$

For $k = i = 0, j = N,$

$$[j.\mu s + \lambda s + \lambda p].P(i,j,k) = \lambda s.P(i,j-1,k) + P(i,j,k+2) \qquad (3)$$

For $i \neq 0 \neq N, i + j = N, k = 0,$ (4)

$$[\lambda p + \lambda s + i\mu p + j\mu s].P(i,j,k) =$$
$$\lambda s.P(i,j-1,k) + P(i,j,k+2) + \left(\frac{(N-i-j)}{N-i}\right).\lambda p.P(i-1,j,k) +$$
$$P(i,j-1,k+1) + P(i,j,k+3)\lambda s.P(i,j-1,k) + P(i,j,k+2) +$$
$$\left(\frac{(N-i-j)}{N-i}\right).\lambda p.P(i-1,j,k) + P(i,j-1,k+1) + P(i,j,k+3)$$

For $j = 0, i = N, k = 0,$

$$[j.\mu p + \lambda s].P(i,j,k) =$$
$$P(i,j,k+3) + P(i,j,k+2) + \lambda p.P(i-1,j,k) \qquad (5)$$

For $i + j = N, k = 2,$

$$P(i,j,k) = \lambda s.P(i,j,k-2) \qquad (6)$$

For $k = 1, i + j \leq N, 1 \leq i \leq (N-1)$

$$P(i,j,k) = \lambda p.(\tfrac{j}{N-i})P(i-1,j+1,k-1) \qquad (7)$$

For $k = 1, i + j = N, i \neq 0$

$$P(i,j,k) = \lambda p.P(i-1,j+1,k-3) \qquad (8)$$

Equations (1) to (5) correspond to the non-blocking states. Equation (6) corresponds to the blocked states. Equation (7) corresponds to the transition states and (8) to the dropping states. The final equation is the sum of all probabilities which is,

$$\sum_{i=0}^{N}\sum_{j=0}^{N}\sum_{k=0}^{2} P(i,j,k) = 1$$

The dropping probability is given by the equation:

$$P_D = \sum_{i=1}^{N} \sum_{j=0, i+j=N}^{N} P(i,j,2)$$

The blocking probability is given by the equation:

$$P_B = \sum_{i=0}^{N} \sum_{j=0, i+j=N}^{N} P(i,j,3)$$

The graph in Fig. 3 shows the improvement in dropping probability after the spectrum hand-off is included over the model without spectrum hand-off as in [3]. This improvement is due to the fact that the SUs have an opportunity to shift from the reclaimed channel to another free channel.

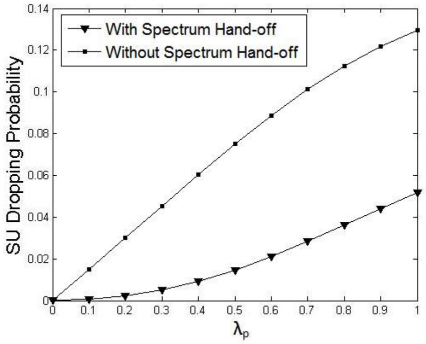

Figure 3. SU Drop probability with λ_p with and without spectrum hand-off.

B. Reservation Based Assignment

The transition states reduce the quality of service to the secondary users because there may be a delay involved in spectrum hand-off. Moreover, the PU's traffic will also be delayed if the SU takes a long time to scan and shift to a free channel. So [3] proposed a reservation based access method to reduce the transition/forced termination state probability at the cost of blocking probability. In this model, of 'N' total channels, 'R' channels are reserved for primary users and secondary users cannot access them. If the reserved channels are full the primary users will be assigned a channel randomly in the non-reserved *(N-R)* channels. But the model does not allow SUs to occupy the reserved channels if the non-reserved channels are full. Also, spectrum hand-off is not considered in this work. So we modify the system model such that SUs can access the reserved channels in case the non-reserved channels are fully occupied. In addition our model also allows spectrum hand-off capability. For example, N = 5 and R =3 in Fig. 4. As the primary users arrive, they are accommodated in the reserved channels and if there are more than three PUs, then they are assigned one of the channels in the rest of the two non-reserved channels as seen in Fig. 4b. The Markov model with R reservation channels is shown in Fig. 5. It should be observed that in comparison to Fig 2, some of the transition states in Fig 5 have been removed due to the reservation of R channels for PUs which cannot be accessed by SUs till all the *(N-R)* channels are occupied by SUs. A transition state exists in the model when either the SU or PU has crossed the boundary of reservation.

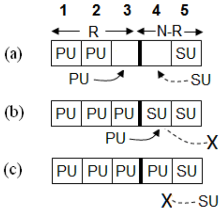

Figure 4. Reservation based access in five channels.

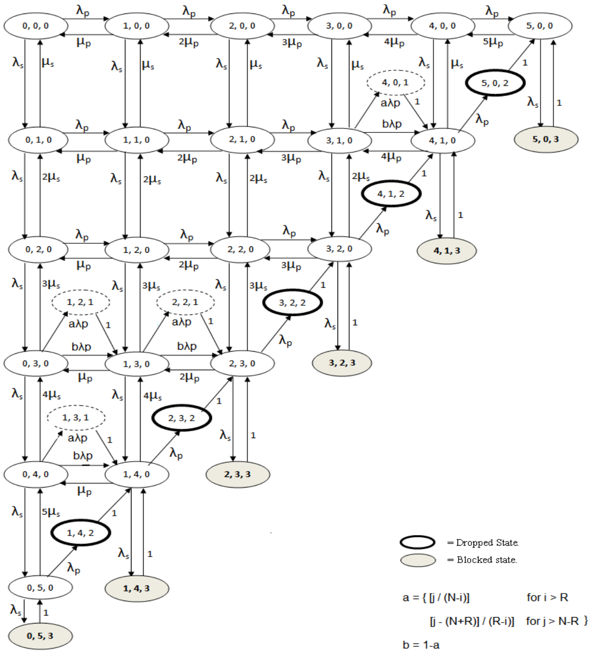

Figure 5. Non-random access in five channels.

Channel Utilization

Channel utilization is important when the incoming traffic is exceeding the number of available channels. SUs channel utilization (γ) is defined as the average number of occupied channels in all blocking states.

$$\gamma = \frac{\sum_{i=0}^{N} \sum_{j=0, i+j \neq N}^{N} (i+j) P(i,j,k)}{N}$$

By adding the capability of spectrum hand-off, the channel utilization in our model is higher compared to the model in [3]. The percentage improvement of γ over reservation-based model without spectrum hand-off [3] is plotted in Fig. 6. It is shown that there is nearly 20% improvement in average channel utilization. This is because although the secondary users are restricted to the *(N-R)* channels initially, they are allowed to occupy the free channels in the reserved slots if the *(N-R)* channels are full.

C. Non-Random Channel Assignment

We propose a simple non-random channel assignment to the primary and secondary users but still giving priority to the PUs. Suppose that the channels can be numbered from 1 to N. The incoming primary traffic will be assigned the first unoccupied channel starting from channel 1. If all the channels are full, the channel occupied by a SU who has been served the most is reclaimed to achieve fairness among the SUs. This is illustrated in Fig. 7. As the PUs arrive they are assigned channels starting from the first channel and SUs are assigned channels starting from the last channel as shown in Fig. 7a. In Fig. 7b, as all the channels are occupied, the SU in the channel 5 will be reclaimed to accommodate the incoming PU since it was the most serviced user in this example (Fig. 7c). Assigning channels in this manner will avoid all the transition states hence avoiding unwanted delays for spectrum hand-off. The Markov model for this system is shown in Fig. 8.

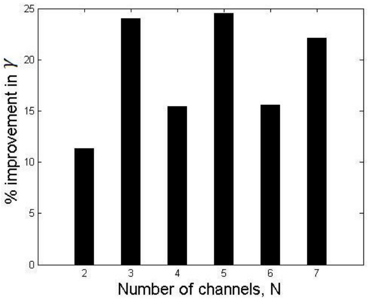

Figure 6. Channel Utilization of reservation-based assignment with spectrum hand-off over reservation-based assignement without spectrum hand-off.

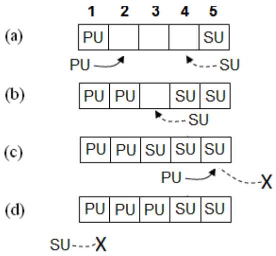

Figure 7. Non-random access in five channels.

All the balance equations given for the random assignment model apply here except equations (2) and (4). These equations have to be replaced by the following respectively to model the non-random channel assignment. The equations for blocking and dropping probabilities remain the same.

For $i \neq 0, k = 0, i + j \leq (N - 1)$,

$$[j.\mu s + \lambda s + \lambda p + i.\mu p].P(i,j,k) = \delta.\lambda s.P(i,j-1,k) + (j+1).\mu s.P(i,j+1,k) + \lambda p.P(i-1,j,k) + (i+1).\mu p.P(i+1,j,0) \qquad (2)$$

$\delta = 0$ for $j = 0$ and $\delta = 1$ for $j \neq 0$

For $i \neq 0 \neq N, i + j = N, k = 0$,

$$[\lambda p + \lambda s + i\mu p + j\mu s].P(i,j,k) = \lambda s.P(i,j-1,k) + P(i,j,k+2) + \lambda p.P(i-1,j,k) + P(i,j,k+3) \qquad (4)$$

III. RESULTS

In this section, the blocking and dropping state probabilities are compared for all the three models discussed previously. Primary user's traffic, λp is considered to vary from 0 to 0.5. This is a reasonable assumption because primary systems usually have stringent GoS requirements binding the traffic to low values. For the graphs in Fig. 9 and 10, $\lambda_s = 0.4$, $\mu_p = 0.4$and $\mu_s = 0.6$. Number of channels, N = 5 and R=3 for the reservation-based method. For graphs in Fig. 11 and 12, λp = 0.4 and λ_s varies from 0 to 0.5.

A. *Variation with λ_p*

The SU's dropping and blocking probabilities are plotted with the variation of λ_p in Fig. 9 and Fig 10. It can be observed that the Non-random channel assignment gives the lowest dropping probability and blocking probabilities for the SUs. The improvement in reservation-based method over the random method is due to the fact that some of the randomness in channel assignment is removed by reservation. But without the optimal value of reserved channels the dropping probability of reservation-based method will be higher than that of random method. The non-random method removes the randomness in channel assignment completely and as a result, the probability of call dropping and call blocking is further reduced. And moreover there is no problem of choosing an optimal value R in this algorithm unlike reservation based method of channel assignment [2, 3].

For a call dropping probability of 1% and a call blocking probability of 1.2% the random allocation method allows 0.4λ of traffic. With reservation it increases to approximately 0. 43λ and with a non random allocation it increases to 0.48λ. This shows an improvement of 20% for non random allocation over random channel allocation method.

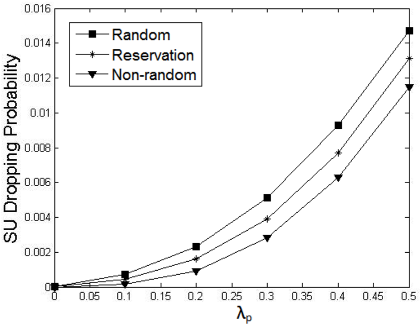

Figure 8. SU Dropping probability with the variation of λ_p.

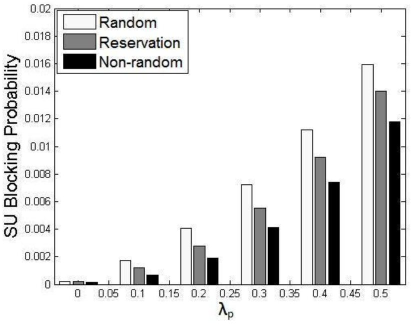

Figure 9. SU Blocking probability with the variation of λ_p.

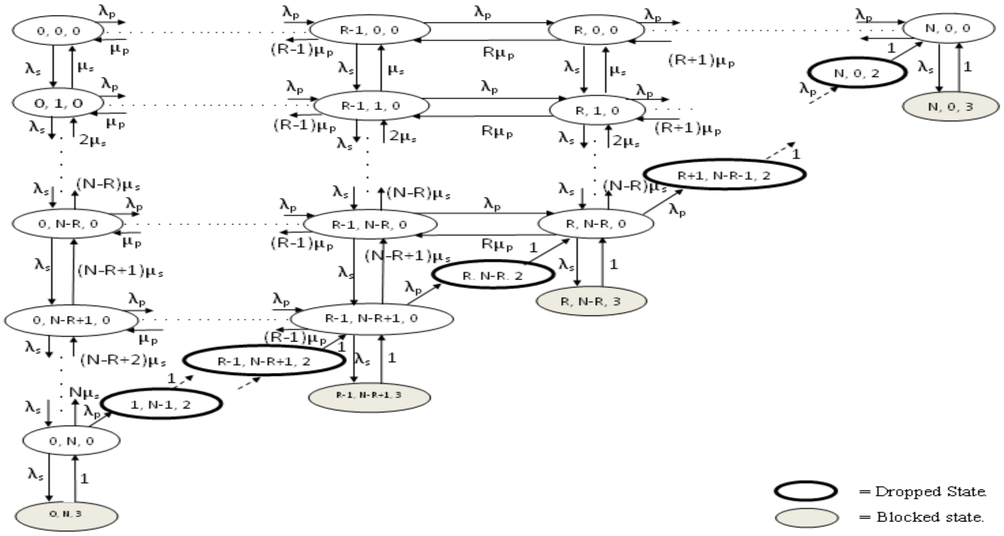

Figure 10. Markov model for non-random channel assignment method with spectrum hand-off.

B. *Variation with λ_s*

The SU's dropping and blocking probabilities are plotted with the variation of $\lambda_{s.}$ in Fig. 11 and Fig. 12 respectively. It can be observed that the Non-random channel assignment gives the lowest blocking probability for the SUs. This is again due to the fact that some of the randomness in channel assignment is removed by reservation.

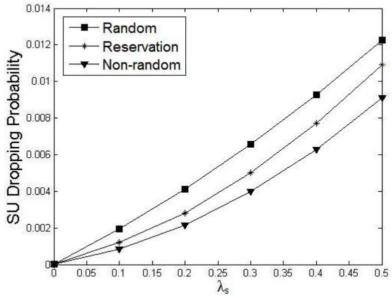

Figure 11. SU Dropping probability with the variation of $\lambda_{s.}$

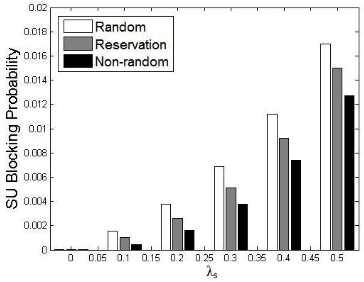

Figure 12. SU Blocking probability with the variation of $\lambda_{s.}$

IV. CONCLUSION

In this paper, the secondary user's capacity in the presence of unrestricted primary users is modeled using three dimensional Markov chains. Unlike in other models, spectrum hand-off has been included and the model is extended to reservation-based assignment system. A non-random channel assignment is proposed in-order to avoid the transition states and to decrease the dropping and blocking probabilities of the SUs. It is shown through the analysis that the non-random channel assignment gives a better result compared to the random channel assignment.

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