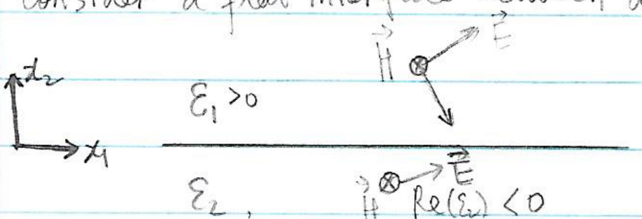
chap 3. Surface plasmon polariton

§ 3.1 Surface plasmon modes and dispersion curve

Consider a flat interface between a dielectric material and a metal:



For the TM polarization, $\vec{H} = (0, 0, u)$, $u(x) = \begin{cases} A_1 e^{i(\xi x_1 + \eta_1 x_2)} & x_2 > 0 \\ A_2 e^{i(\xi x_1 + \eta_2 x_2)} & x_2 < 0 \end{cases}$

$\frac{\partial u}{\partial x_2} = \begin{cases} i\eta_1 A_1 e^{i(\xi x_1 + \eta_1 x_2)} & x_2 > 0 \\ i\eta_2 A_2 e^{i(\xi x_1 + \eta_2 x_2)} & x_2 < 0 \end{cases}$ where $\xi^2 + \eta_1^2 = k^2 \varepsilon_1$, $\xi^2 + \eta_2^2 = k^2 \varepsilon_2$.

From the continuity of EM fields: $\begin{cases} n \times (\vec{H}_1 - \vec{H}_2) = 0 \\ n \times (\vec{E}_1 - \vec{E}_2) = 0 \end{cases}$

$$\Rightarrow \begin{cases} u(x_1, 0+) = u(x_1, 0-) \\ \frac{1}{\varepsilon_1} \frac{\partial u}{\partial x_2}(x_1, 0+) = \frac{1}{\varepsilon_2} \frac{\partial u}{\partial x_2}(x_1, 0-) \end{cases} \Rightarrow \begin{cases} A_1 e^{i\xi x_1} = A_2 e^{i\xi x_1} \\ \frac{1}{\varepsilon_1} i\eta_1 A_1 e^{i\xi x_1} = \frac{1}{\varepsilon_2} i\eta_2 A_2 e^{i\xi x_1} \end{cases}$$

$$\Rightarrow \eta_1 \varepsilon_2 = \eta_2 \varepsilon_1 \text{ or } \eta_1^2 \varepsilon_2^2 = \eta_2^2 \varepsilon_1^2 \Rightarrow (k^2 \varepsilon_1 - \xi^2) \varepsilon_2^2 = (k^2 \varepsilon_2 - \xi^2) \varepsilon_1^2$$

$$\Rightarrow (\varepsilon_1^2 - \varepsilon_2^2) \xi^2 = k^2 (\varepsilon_2 \varepsilon_1^2 - \varepsilon_1 \varepsilon_2^2) \Rightarrow (\varepsilon_1 + \varepsilon_2) \xi^2 = k^2 \varepsilon_1 \varepsilon_2 \Rightarrow \xi = \pm k \sqrt{\frac{\varepsilon_1 \varepsilon_2}{\varepsilon_1 + \varepsilon_2}}$$

Let $\varepsilon_1 = 1$, $Re\,\varepsilon_2 < 0$, $|Re\,\varepsilon_2| \gg |Im\,\varepsilon_2| \sim O(1)$. $\frac{\varepsilon_1 \varepsilon_2}{\varepsilon_1 + \varepsilon_2}$ lives in the first quadrant.

$\Rightarrow Re \sqrt{\frac{\varepsilon_1 \varepsilon_2}{\varepsilon_1 + \varepsilon_2}} > 1$, and $0 < Im \sqrt{\frac{\varepsilon_1 \varepsilon_2}{\varepsilon_1 + \varepsilon_2}} \ll 1$.

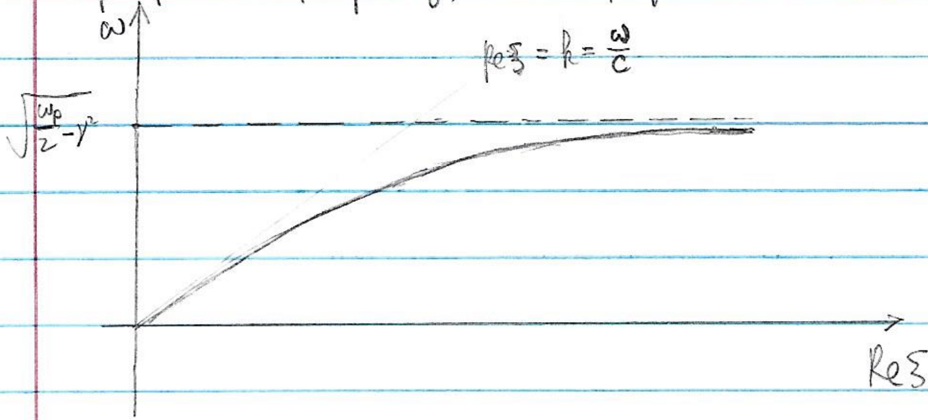
Therefore $|\xi| > k$, $u(x) = \begin{cases} A e^{i(\xi x_1 + \eta_1 x_2)}, & x_2 > 0 \\ A e^{i(\xi x_1 + \eta_2 x_2)}, & x_2 < 0 \end{cases}$ is localized field

that decays exponentially away from the interface, which is called

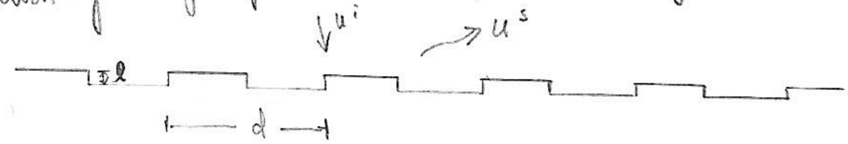
Surface plasmon mode.

Drude model for the diaelectric constant $\varepsilon_2(\omega) = 1 - \frac{\omega_p^2}{\omega^2 + i\gamma\omega}$

ω_p: plasma frequency, γ: damping constant.



§3.2 Excitation of Surface plasmon with periodic corrugations



Approximate the interface condition by impedance condition.

$u^i = e^{i(\xi x_1 - \eta x_2)}$

$u^s = \sum_{n=-\infty}^{\infty} u_n e^{i(\xi_n x_1 + \eta_n x_2)}$

$\frac{\partial u}{\partial x_2} + i\gamma^l(x_1) u = 0$

where l denotes the depth of groove

$$\gamma^l(x_1) = \sum_{n=-\infty}^{\infty} \gamma_n e^{i\frac{2\pi n}{d} x_1}$$ is impedance function

$$\frac{\partial u^s}{\partial x_2} = \sum_n i\eta_n u_n e^{i(\xi_n x_1 + \eta_n x_2)}, \qquad \frac{\partial u^i}{\partial x_2} = -i\varsigma e^{i\xi x_1}$$

$$\frac{\partial u}{\partial x_2} + i\gamma u = \frac{\partial u^i}{\partial x_2} + i\gamma u^i + \frac{\partial u^s}{\partial x_2} + i\gamma u^s$$

$$= -i\eta e^{i\xi x_1} + i\sum \gamma_n e^{i\frac{2\pi n}{d}x_1} e^{i\xi x_1} + \sum_n i\eta_n u_n e^{i\xi_n x_1} + i\sum_n \gamma_n e^{i\frac{2\pi n}{d}x_1} \sum u_n e^{i\xi_n x_1} = 0$$

$$\Rightarrow -i\eta e^{i\xi x_1} + i\sum_n \gamma_n e^{i\xi_n x_1} + i\sum_n \gamma_n e^{i\xi_n x_1} + i\sum_n \eta_n u_n e^{i\xi_n x_1} + i\sum_n \sum_m \gamma_m u_{n-m} e^{i\xi_n x_1} = 0$$

$$\Rightarrow -i\eta e^{i\xi x_1} + i\sum_n \left[\gamma_n + \eta_n u_n + \sum_m \gamma_m u_{n-m}\right] e^{i\xi_n x_1} = 0$$

$\underline{n \neq 0}$ $\quad \gamma_n + \eta_n u_n + \sum_m \gamma_m u_{n-m} = 0$

or $\quad (\eta_n + \gamma_0) u_n + \sum_{m \neq 0} \gamma_m u_{n-m} = -\gamma_n$

$\underline{n = 0}$ $\quad -\eta + \gamma_0 + \eta_0 u_0 + \sum_m \gamma_m u_{-m} = 0$

or $\quad (\gamma_0 + \eta_0) u_0 + \sum_{m \neq 0} \gamma_m u_{-m} = \eta - \gamma_0$

$$
\begin{bmatrix}
\eta_3+\gamma_0 & \gamma_1 & \gamma_2 & \cdots & & & & \\
\gamma_{-1} & \eta_2+\gamma_0 & \gamma_1 & \gamma_2 & \gamma_3 & \cdots & & \\
\cdots\ \gamma_{-2} & \gamma_{-1} & \eta_1+\gamma_0 & \gamma_1 & \gamma_2 & \cdots & & \\
\cdots & \gamma_{-2} & \gamma_{-1} & \eta_0+\gamma_0 & \gamma_1 & \gamma_2 & \cdots & \\
\cdots & & \gamma_{-2} & \gamma_{-1} & \eta_{-1}+\gamma_0 & \gamma_1 & \gamma_2 & \cdots \\
 & & & & & \ddots & &
\end{bmatrix}
\begin{bmatrix} \vdots \\ u_3 \\ u_2 \\ u_1 \\ u_0 \\ u_{-1} \\ u_{-2} \\ u_{-3} \\ \vdots \end{bmatrix}
=
\begin{bmatrix} \vdots \\ -\gamma_3 \\ -\gamma_2 \\ -\gamma_1 \\ \eta-\gamma_0 \\ -\gamma_{-1} \\ -\gamma_{-2} \\ -\gamma_{-3} \\ \vdots \end{bmatrix}
$$

$$\vec{u} = [\cdots\ u_3, u_2, u_1, u_0, u_{-1}, u_{-2}, u_{-3}, \cdots]^T \in l^2$$

$$\vec{b} = [\cdots\ -\gamma_3, -\gamma_2, -\gamma_1, \eta-\gamma_0, -\gamma_{-1}, -\gamma_{-2}, -\gamma_{-3}, \cdots]^T \in l^2$$

$$A\vec{u} = \vec{b} \qquad A: l^2 \to l^2$$

$$A = D + L = D(I + D^{-1}L)$$

If $\|L\| \ll 1$ s.t. $\|D^{-1}L\| < 1$, then $A^{-1} = (I + D^{-1}L)^{-1} D^{-1}$

e.g. when the depth of groove $l \ll 1$.

If $|\zeta_n(k_*) + \gamma_n|$ is small for certain n and k_* s.t. $\|D^{-1}\|$ is big, we call k_* the surface plasmon frequency.

In general, one can set $k_* = \mathrm{Re}\, k_n$, where k_n is root of

$$\eta_n(k_n) + \gamma_0 = 0 \quad \text{or} \quad \sqrt{k_n^2 - (\xi + \tfrac{2\pi}{d}n)^2} + \gamma_0 = 0 \qquad (*)$$

Now if one considers the flat interface such that $l = 0$, the dispersive curve is given by $\sqrt{k^2 - \xi^2} + \gamma_{00} = 0$, $(**)$ where γ_{00} is the impedance constant for flat interface.

A comparison of $(*)$ & $(**)$ implies that, up to a small perturbation, the dispersion curve for the periodic structure can be viewed as a periodic shift of the dispersion curve for the flat interface if $l \ll 1$.