## 0.1 What is new in this course

Instead of computation, we emphasize more on **understanding** math. We will think about math objects and relations in axiomatic way.

**Tips:**

1. Understand definitions, theorems/corollaries, and examples first. Then understand the proofs.

2. Notations (pp487) and Index (pp513) are helpful to find the definitions of one word.

Roughly speaking:

**Def 0.1.1 (Algebraic Structure).** A set on which we define some operations (eg. $+$, $\cdot$, inverse). These operations follow certain rules.

**Ex 0.1.2.** $(\mathbf{R}, +)$, $(\mathbf{R}^+, \cdot)$, $(\mathbf{Z}, +)$, $(\mathbf{R}, +, \cdot)$, $(\mathbf{R}^2, +)$, vector spaces.

## 0.2 Sets and Relations

**Def 0.2.1.** Sets, elements, empty set, $a \in S$, $a \notin S$, subset, proper subset. Cartesian product. cardinality $|S|$.

**Ex 0.2.2 (Ex 0.5, p3, Cartesian product).**

**Def 0.2.3.** A **relation** between sets $A$ and $B$ is a subset $\mathcal{R}$ of $A \times B$.

$$(a, b) \in \mathcal{R} \iff a \in A \text{ has relation with } b \in B \iff a\mathcal{R}b.$$

**Def 0.2.4.** A **function** $\phi$ mapping $X$ into $Y$ is a relation between $X$ and $Y$, such that each $x \in X$ appears as the first member of exactly one ordered pair $(x, y)$ in $\phi$. We write $\phi : X \to Y$ and express $(x, y) \in \phi$ by $\phi(x) = y$. Set $X$ is called the **domain**. Set $Y$ is called the **codomain**. The **range** of $\phi$ is $\phi(X) = \{\phi(x) \mid x \in X\}$. Clearly $\phi(X) \subseteq Y$.

**Def 0.2.5.** A function $\phi : X \to Y$ is **one-to-one** (or **injective**) if $\phi(x_1) = \phi(x_2)$ only when $x_1 = x_2$. The function $\phi$ is **onto** $Y$ (or **surjective**) if the range of $\phi$ is $Y$ (i.e. $\phi(X) = Y$). If $\phi$ is one-to-one and onto, we call it a **bijection**.

**Ex 0.2.6.** Consider the following functions $f_i : \mathbf{R} \to \mathbf{R}$. (by graphs)

1. $f_1(x) := e^x$ is one-to-one, but not onto $\mathbf{R}$.
2. $f_2(x) := x \sin x$ is onto $\mathbf{R}$, but not one-to-one.
3. $f_3(x) := 2x - 1$ is one-to-one and onto. It is a bijection.

**Def 0.2.7.** A **partition** is to divide a set into several subsets. Each two subsets have no interception.

**Def 0.2.8.** An **equivalent relation** $\mathcal{R}$ on a set $S$ is one satisfying the three properties for $x, y, z \in S$:

1. (Reflexive) $x\mathcal{R}x$.
2. (Symmetric) If $x\mathcal{R}y$ then $y\mathcal{R}x$.
3. (Transitive) If $x\mathcal{R}y$ and $y\mathcal{R}z$ then $x\mathcal{R}z$.

Equivalent relation $\Longleftrightarrow$ Partition

**Ex 0.2.9.** Ex 0.20, p7. (Congruence Modulo $n$).

### 0.2.1 Homework (optional)

Section 0, p10, 29, 30, 32