1. Find the second-order partial derivatives of the function \( f(x, y) = x^3 - 2xy^2 + y^2 - 8. \)

   See \( \text{Cal. 8.2, Self-check Exercise 3 (p. 554), Solution at p. 557}. \)

   \[ f_x = 3x^2 - 2y^2, \quad f_y = 2y(1-2x). \]

   \[ \frac{\partial^2 f}{\partial x^2} = 6x \]

   \[ f_{x y} = -4y \]

   \[ f_{y x} = -4y \]

   \[ f_{y y} = 2(1-2x). \]

2. Let \( f(x, y) = xy + \frac{4}{x} + \frac{2}{y}. \)

   (a) Find the critical point(s) of the function;

   (b) Then use the second derivative test to classify the nature of each point;

   (c) Finally, determine the relative extrema of the function.

   \( \text{Cal Ex 8.3.0 \#13 (p. 567)} \)

   (a) \[ \begin{cases} f_x = y - \frac{4}{x^2} = 0 \Rightarrow x^2 y = 4 \Rightarrow x^3 y^3 = 8 \Rightarrow xy = 2 \\ f_y = x - \frac{2}{y^2} = 0 \Rightarrow xy^2 = 2 \end{cases} \]

   \[ \Rightarrow x = 2, \quad y = 1. \Rightarrow \text{critical point } (2, 1) \]

   (b) \[ f_{xx} = \frac{8}{x^3}, \quad f_{yy} = \frac{4}{y^3}, \quad f_{xy} = 1 \]

   \[ D(x, y) = \frac{\partial^2 f}{\partial x^2} \frac{\partial^2 f}{\partial y^2} - \left( \frac{\partial^2 f}{\partial x \partial y} \right)^2 \]

   \[ D(2, 1) = 3 > 0 \]

   \[ f_{xx}(2, 1) = 1 > 0. \]

   So by the second derivative test, \( f(x, y) \) has a relative minimum at \( (2, 1), \quad f(2, 1) = 2(1) + \frac{4}{2} + \frac{2}{1} = 6. \)
3. Find the domain: \( f(x, y) = \frac{1}{x} + \frac{1}{x-y} - e^{x+y} \)

The denominators \( x \neq 0 \) and \( x-y \neq 0 \) so the \( D = \{ (x, y) \mid x \neq 0, x+y \neq 0 \} \)

4. Find the maximum and minimum values of the function \( f(x, y) = xy \) subject to the constraint \( x^2 + y^2 = 16 \).

\[ g(x, y) = x^2 + y^2 - 16 = 0 \]
\[ F(x, y, \lambda) = f(x, y) + \lambda g(x, y) = xy + \lambda (x^2 + y^2 - 16) \]

Lagrangean Function

Solve \( \begin{cases} F_x = y + 2\lambda x = 0 \\
F_y = x + 2\lambda y = 0 \\
F_\lambda = x^2 + y^2 - 16 = 0 \end{cases} \)

\( \Rightarrow \begin{cases} y = -2\lambda x \\
x = -2\lambda y \\
x^2 + y^2 = 16 \end{cases} \)
\( \Rightarrow \begin{cases} x = \pm \frac{1}{2} \\
y = \pm 2 \lambda \frac{x}{2} \end{cases} \)

If \( \lambda = \frac{1}{2} \), then \( y = -x \) \( \Rightarrow \) \( (x, y) = (2\sqrt{2}, -2\sqrt{2}) \) or \( (-2\sqrt{2}, 2\sqrt{2}) \) \( \Rightarrow \) \( f(x, y) \) minimal \( \Rightarrow \) \( f(x, y) = 8 \)

If \( \lambda = -\frac{1}{2} \), then \( y = x \) \( \Rightarrow \) \( (x, y) = (2\sqrt{2}, 2\sqrt{2}) \) or \( (-2\sqrt{2}, -2\sqrt{2}) \) \( \Rightarrow \) \( f(x, y) = 8 \) maximal.

5. A sample of three apples taken from Cavallero's Fruit Stand are examined to determine whether they are good or rotten.

a. What is an appropriate sample space for this experiment?
b. Describe the event \( E \) that exactly one of the apples picked is rotten.
c. Describe the event \( F \) that the first apple picked is rotten.

\[ a. \text{ Sample space } S = \{G, R\} \quad \text{G = good, R = rotten} \]

\[ b. \text{ Sample space } S = \{GGG, GGR, GRG, GRR, RGG, RGR, RRG, PRR\} \]

\[ b. \text{ E = \{GGR, GRG, RGG\} } \]

\[ c. \text{ F = \{RRG, RGR, RRG, RRR\} } \]