Analysis of Trailer Position Error in an Autonomous Robot-Trailer System With Sensor Noise

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***Abstract*—A low cost solution for controlling trailer lateral position in a robotic tractor-trailer system is being considered. In this paper, several practical issues involved in the implementation of such a system are presented. Instruments for navigation and control consist of a single GPS receiver and a hitch-mounted sensor for the measurement of the angle between robot (tractor) and trailer. In previous work, the authors examined the effects of errors in the hitch-angle measurement on path tracking performance. This analysis will now be expanded to include the effects of noise in the GPS measurements. The relative merits of mounting the GPS receiver on the robot or trailer are considered.**

I. INTRODUCTION

In this paper, a low cost solution for precise path following of a robotic tractor-trailer system is being considered. Control of mobile robots, including mobile robots pulling trailers, has been the subject of much research. Several different control methods have been developed for such systems. A good overview of these methods can be found in [1]. These systems have many practical uses including factory automation as well as agricultural and military applications.

II. BACKGROUND

A robotic tractor-trailer system is currently being developed at Auburn University in conjunction with the U.S. Army Corps of Engineers and the Environmental Security Technology Certification Program (ESTCP) for the purpose of locating and mapping unexploded ordnance (UXO) for humanitarian benefit. Currently various types of magnetometers and electromagnetic sensors are towed by either a human or all-terain vehicle (ATV) for geophysical mapping of an area. It is proposed that safety, precision, and efficiency can all be improved by replacing the current methods with a low cost, portable, highly accurate robotic system. In order to accurately map buried UXO, precise positioning of the sensors as well as high repeatability between runs is necessary.

In the desired system a trailer carrying a geophysical sensor towed by the robot vehicle will autonomously follow a predefined path. The system will simultaneously collect data containing the position of the trailer as well as the output of the geophysical sensor being towed. This data allows a map of any metallic objects to be generated. An example of the output of the system from a recent test conducted in a field seeded with inert ordnance is shown in Fig. 1. The black lines show the path followed by the trailer/sensor, while the color indicates the output of the geophysical sensor. The sensor being towed in this test was an EM61-Mk1 electromagnetic sensor manufactured by Geonics Limited.

In order to maintain simplicity and reduce cost, the system is controlled using a single GPS receiver to provide the position of either the robot or the trailer and an encoder to measure the angle between the robot and trailer. From initial experiments, hitch angle bias and heading are thought to be the dominant sources of error in the system. In this paper, the effects of these two error sources will be considered, and suggestions will be made on how to minimize their effect.

III. SYSTEM DESCRIPTION

A. Robot Description

The system being studied is a Segway Robotic Mobility Platform (RMP) 400 pulling a two-wheeled trailer. The RMP400 is a four-wheeled, differential drive vehicle. A picture of the RMP400 attached to its trailer is shown in Fig. 2.

B. System Sensors

The position in UTM coordinates (e, n) and orientation ψ of either the robot or its trailer are provided by a NovAtel DL-4plus GPS receiver. The system receives real-time kinematic (RTK) corrections (based on GPS carrier phase corrections) from a local base station to provide centimeter level accuracy (2 cm standard deviation) [2]. The hitch angle between the

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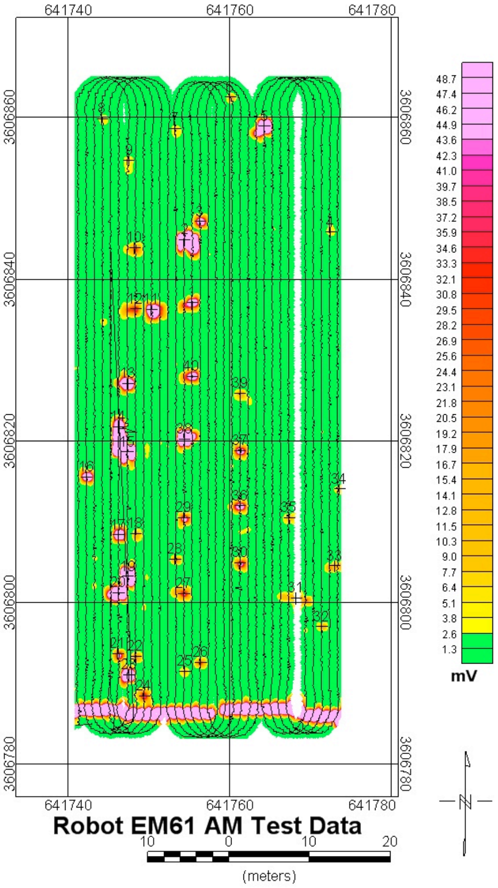

Fig. 1. Example Output of Geophysical Sensors

Fig. 2. Segway robot and geophysical instrument trailer

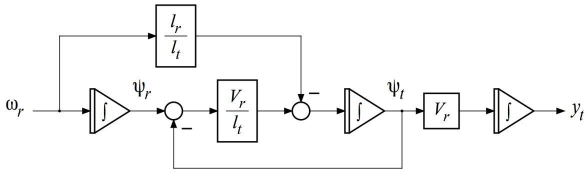

Fig. 3. Model of linearized error dynamics.

robot and the trailer is measured using a U.S. Digital E5S-1800 optical encoder yielding 1800 cycles per revolution (CPR).

IV. PLANT MODEL AND CONTROLLER DESIGN

The kinematic model of a mobile robot and trailer is given in [3]. A control law is desired that will cause the trailer to follow a path defined by a series of line segments and circular arcs.

The linearized error dynamics are given by

$$\begin{bmatrix}\dot{y}_t\\ \dot{\psi}_t\\ \dot{\psi}_r\end{bmatrix} = \begin{bmatrix}0 & V_r & 0\\ 0 & -V_r/l_t & V_r/l_t\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}y_t\\ \psi_t\\ \psi_r\end{bmatrix} + \begin{bmatrix}0\\ -l_r/l_t\\ 1\end{bmatrix}\omega_r \tag{1}$$

where y_t is the trailer lateral error, ψ_t and ψ_r are the trailer and robot heading errors, respectively, V_r is the forward velocity of the robot, l_r and l_t are the robot and trailer hitch lengths, respectively, and ω_r is the robot angular velocity (turn rate). A block diagram of the linearized error model is shown in Fig. 3.

The system is controlled using linear state feedback of the form:

$$\omega_r = -k_1 y_t - k_2 \psi_t - k_3 \psi_r \tag{2}$$

The gains are calculated using standard pole placement techniques. A more detailed description of the model and controller design being studied are given in the authors' previous work [4].

V. IMPACT OF MEASUREMENT NOISE AND BIAS

Knowledge of both the robot and trailer's position and orientation is required for control of the system. Since only one GPS receiver is being used, the position and orientation of robot and trailer cannot both be measured directly. One position must be measured, and the other estimated based on the measured hitch angle and the geometry of the system. The relationships between the positions, orientations, and velocities of the robot and trailer are given by:

$$\begin{aligned} e_t &= e_r - l_r \sin(\psi_r) - l_t \sin(\psi_r + \varphi)\\ n_t &= n_r - l_r \cos(\psi_r) - l_t \cos(\psi_r + \varphi)\\ \psi_t &= \psi_r + \varphi\\ V_t &= V_r \cos(\varphi) - l_r \omega \sin(\varphi) \end{aligned} \tag{3}$$

where (e_r, n_r) and (e_t, n_t) are the position of the robot and trailer, respectively, φ is hitch angle, and V_t is the linear velocity of the trailer.

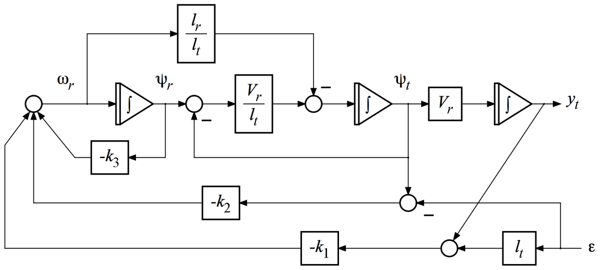

Fig. 4. Closed loop model with hitch angle bias - GPS on robot.

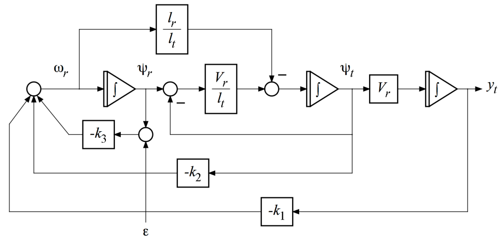

Fig. 5. Closed loop model with hitch angle bias - GPS on trailer.

Errors and noise in the sensors affect the quality of the estimate and thus affect the overall performance of the system. In this work, the effects of heading noise and hitch sensor bias on tracking performance are analyzed.

A. Effect of hitch angle sensor bias

Bias in the hitch angle sensor is a practical problem. It arises from the difficulty in obtaining perfect alignment between sensor, robot and trailer. This error affects the system differently depending on where the GPS receiver is mounted.

1) GPS on robot: From (3) it can be determined that when the GPS is mounted on the robot, the errors introduced by the hitch angle error are:

$$\begin{aligned} y_{terr}(\varepsilon) &= l_t \sin(\varepsilon) \\ \psi_{terr}(\varepsilon) &= -\varepsilon \\ \psi_{rerr}(\varepsilon) &= 0 \end{aligned} \tag{4}$$

Robot heading error ψ_r is measured directly and thus not affected by hitch angle bias ϵ when the GPS receiver is mounted on the robot. A bias does, however, affect the estimate of trailer position and orientation as described in (4). The effect of bias ϵ can be introduced in the state feedback (2) by replacing the trailer lateral error y_t and trailer heading error ψ_t by $y_t + l_t\epsilon$ and $\psi_t - \epsilon$, respectively. The block diagram model of the closed loop system is shown in Fig. 4. Applying Mason's Gain Rule to the diagram, followed by the Laplace transform final value theorem yields the dc gain between sensor bias and trailer lateral error:

$$\frac{y_t}{\epsilon} = \frac{k_2 - k_1 l_t}{k_1} \tag{5}$$

2) GPS on trailer: When the GPS unit is mounted on the trailer, the trailer lateral position measurement and trailer heading measurement are no longer affected by hitch angle sensor bias. The estimate of robot heading error will be biased, however, so ψ_r is replaced by $\psi_r + \epsilon$. The block diagram model for this case is shown in Fig. 5. Applying the same analysis approach as before yields a different dc gain between the hitch angle and the trailer lateral error:

$$\frac{y_t}{\epsilon} = -\frac{k_3}{k_1} \tag{6}$$

3) Mounting GPS receiver to minimize effect of hitch angle error: Bias in the hitch angle sensor affects trailer lateral error, regardless whether the GPS receiver is mounted on the robot or trailer. Comparing the dc gains (5) and (6), however, leads to the conclusion that mounting the GPS receiver on the trailer is favored under the condition

$$|k_3| < |k_2 - k_1 l_t| \tag{7}$$

The above result is consistent with that reported in the earlier work [4], where the controller tuning placed a greater weight on trailer lateral error y_t. On the other hand, placing the GPS receiver on the robot is favored for controller tunings that more heavily weight the robot heading error ψ_r.

B. Effect of heading noise

Mounting the GPS receiver on the trailer (instead of the robot) can reduce effects of hitch sensor calibration errors for certain controller tunings. Navigation instrument noise is another significant error, however, that must be considered. Next, the effect of noise in the heading measurement is considered. Because the navigation system consists of only a single-antenna GPS receiver, there is no way to measure the orientation or heading of the vehicle. Instead, the GPS course measurement, which is a measure of the direction of the instantaneous velocity of the vehicle, is used in place of a heading measurement. This measurement is noisy and can introduce errors caused by motion that is not in the direction of travel of the vehicle (such as pitching or rolling).

1) GPS on robot: When navigation instruments are placed on the robot, robot heading error is directly measured with noise w. The trailer heading error ψ_t and trailer lateral error y_t must be estimated from the kinematic model. Specifically, the tongue length l_t and hitch angle φ must be known. The estimated trailer heading error is given by:

$$\hat{\psi}_t = \psi_r + w - \varphi \tag{8}$$

where w is the heading noise, and φ is the hitch angle. Assuming small hitch angle φ, the trailer lateral error is estimated using:

$$\hat{y}_t = -l_t \hat{\psi}_t \tag{9}$$

Substituting (8) and (9) into the state feedback control (2) leads to the conclusion that heading noise contributes an error to the controller output ω_r, and the gain N_r on the heading noise is of the form

$$N_r = k_2 + k_3 - k_1 l_t \tag{10}$$

2) GPS on the trailer: With navigation instruments on the trailer, both the trailer heading error and trailer lateral position are measured directly. Robot heading error must be estimated from the relationship:

$$\hat{\psi}_r = \psi_t + w + \varphi \tag{11}$$

In this case, the gain N_t on the heading noise is of the form

$$N_t = k_2 + k_3 \tag{12}$$

3) Mounting GPS receiver to minimize effects of heading noise: Comparing (10) and (12) leads to the the conclusion that it is preferable to mount the GPS receiver on the robot for controller tunings where

$$\begin{aligned} |N_r| &< |N_t| \\ |k_2 + k_3 - k_1 l_t| &\leq |k_2 + k_3| \end{aligned} \tag{13}$$

VI. VALIDATION

A. *Simulation Results*

The model and analysis presented in the preceding sections were first validated using computer simulations. An S-shaped path consisting of several parallel line segments joined by 180° arcs as shown in Fig. 6 was created. Several simulations were run with various sensor configurations and errors and the results compared to those predicted in the previous sections. The model and controller parameters used in the simulations are given in Table I.

The variance of the course measurement given in Table I is calculated using:

$$\sigma_c = \frac{\sigma_v}{V} \tag{14}$$

where $\sigma_v (m/sec)$ is the variance of the GPS receiver's velocity measurement and $V(m/s)$ is the vehicle's speed. The velocity variance σ_v is defined in the receiver's specifications as $0.03 m/s$.

Simulations were run for various hitch angle biases from 0° to 15° for both the GPS mounted on the robot and the trailer. Zero mean, Gaussian noise with variance σ_c was added to the heading measurements. Hitch angle quantization effects were also included in the simulation. For each simulation, lateral root mean squared error (RMSE) and average error or bias were calculated. The predicted bias of the lateral error was also calculated for each case using (5) and (6). The results are given in Table II for the GPS on the robot and in Table III for the GPS on the trailer.

TABLE I

SIMULATION MODEL PARAMETERS

Parameter	Symbol	Value	Units
Robot speed	V_r	0.75	m/s
Robot tongue length	l_r	0.95	m
Trailer tongue length	l_t	2.11	m
Robot angular rate limit	$\|\omega_r\|_{max}$	3	rad/s
Trailer lateral error gain	K_1	1.89	
Trailer heading error gain	K_2	2.86	
Robot heading error gain	K_3	3.62	
GPS course variance	σ_c	2.29	deg.
Simulation duration	t_{sim}	200	sec.

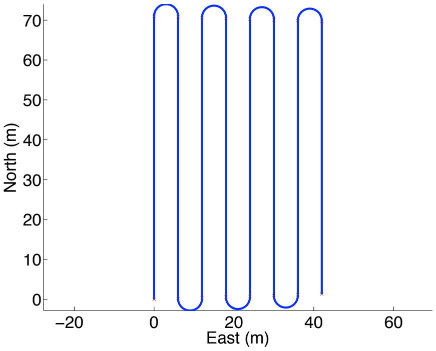

Fig. 6. Example Path

Comparing the third and fourth columns in Tables II and III, it can be seen that (5) and (6) very closely predict the bias in the lateral error that will be produced by a given hitch angle bias.

B. *Experimental Results*

Several experimental runs were made using the same path that was used for the simulations. Artificial hitch angle biases ranging from 0° to 15° were added to the system. A single GPS receiver on either the robot or the trailer and the hitch angle encoder were used for control of the system. A second antenna was placed on the trailer when the primary antenna was on the robot. This second antenna was not used for control, but only to measure the position and heading of the trailer for analysis purposes. For each run, lateral root mean squared error (RMSE) and average error or bias were calculated, just as was done for the simulations. The results are given in Table IV for the GPS on the robot and in Table V for the GPS on the trailer.

TABLE II

SIMULATION RESULTS (GPS ON ROBOT)

Bias(°)	RMSE(m)	Avg. Error(m)	Predicted Bias(m)
0	0.0447	0.0138	0.0
5	0.0700	-0.0595	-0.0521
10	0.1122	-0.1053	-0.1040
15	0.1578	-0.1508	-0.1561

TABLE III

SIMULATION RESULTS (GPS ON TRAILER)

Bias(°)	RMSE(m)	Avg. Error(m)	Predicted Bias(m)
0	0.0480	0.0216	0.0
5	0.1709	-0.1641	-0.1674
10	0.3362	-0.3296	-0.3344
15	0.4988	-0.4917	-0.5019

TABLE IV
EXPERIMENTAL RESULTS (GPS ON ROBOT)

Bias($^\circ$)	RMSE(m)	Avg. Error(m)	Predicted Bias(m)
0	0.0100	0.0034	0.0
5	0.0510	-0.0500	-0.0521
10	0.1030	-0.1021	-0.1040
15	0.1480	-0.1471	-0.1561

TABLE V
EXPERIMENTAL RESULTS (GPS ON TRAILER)

Bias($^\circ$)	RMSE(m)	Avg. Error(m)	Predicted Bias(m)
0	0.0707	0.0226	0.0
5	0.1900	-0.1771	-0.1674
10	0.3801	-0.3772	-0.3344
15	0.5572	-0.5551	-0.5019

As in the simulation results, comparing the third and fourth columns in Tables IV and V, further verifies that (5) and (6) closely predict the bias in the lateral error.

A section of an experimental run where the system was tracking a straight line segment with zero added bias for both the GPS on the robot and on the trailer are given in Figures 7 and 8 respectively. The effect of noise in the course measurement can be clearly seen in both figures. The uncertainty in the system's orientation essentially pivots or swings the estimate of the system about the GPS antenna position causing the drastic jumps in the estimated trailer or robot position. The magnitude of the position estimate error is a function of both the heading noise (σ_c) and the trailer and hitch lengths (l_r and l_t).

C. Summary

The simulation and experimental results provided are consistent with the analysis of hitch angle bias and heading noise presented in the previous sections. For the system under consideration, the transfer functions given in (5) and (6) accurately predict the bias in the lateral error for both the simulated and experimental results.

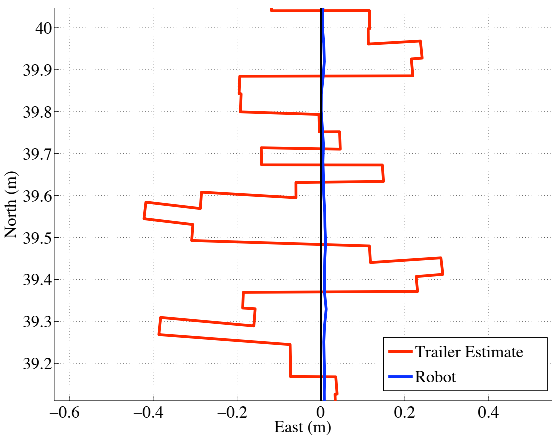

Fig. 7. Example Experimental Run (GPS on Robot)

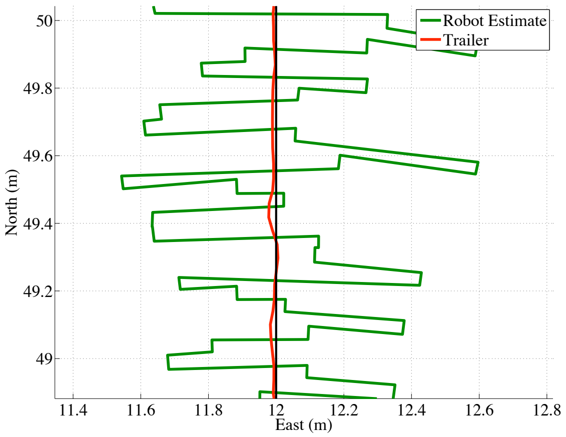

Fig. 8. Example Experimental Run (GPS on Trailer)

The guideline given for choosing the GPS sensor location in the presence of a hitch angle bias also correctly predicted the sensor location that produces the best tracking performance in terms of lateral mean square error. Substituting the parameters and gains from Table I into (7) results in $|3.62| > |-1.1279|$ which suggests that the effect of a hitch angle bias would be minimized by placing the GPS on the robot. When zero bias was used in simulation, the results for the GPS on the robot and on the trailer were almost identical. This is as expected, since in the absence of sensor imperfections, sensor placement should have no effect on system performance. As the bias increases, however, the average error when the GPS is on the robot is significantly smaller than the average error when the GPS is on the trailer, as is predicted.

The guideline for placing the GPS receiver to minimize the effect of heading noise also correctly predicted the correct GPS placement. Substituting the parameters and gains used into (13) results in $|2.492| < |6.48|$ which suggests that having the GPS on the robot is the better placement. It can be observed in both the simulation and experimental results that the lateral mean square error is approximately an order of magnitude larger when the GPS is placed on the trailer, which again agrees with the predicted results.

VII. CONCLUSIONS

The effects of both hitch angle sensor bias and GPS heading noise on a robotic tractor-trailer system have been considered. Transfer functions were derived that allow the bias in the trailer lateral error to be calculated for a given hitch angle bias. Equations were also provided that allow the effect on the control input of heading noise to be determined. From these it has been shown that certain controller tunings and vehicle parameters favor certain sensor placements. Guidelines were given for where the GPS receiver should be placed to minimize the effects of these errors based on those parameters and tunings.

The authors have shown that when implementing a robotic tractor-trailer system, measurement imperfections can have a significant impact on control system performance and should be taken into account. The analysis given can help to predict the effect various measurement errors will have on the system and suggest a sensor placement to minimize their effects.

VIII. ACKNOWLEDGEMENTS

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