Proceedings of the 2007 American Control Conference
Marriott Marquis Hotel at Times Square
New York City, USA, July 11-13, 2007

Effects of Sensor Placement and Errors on Path Following Control of a Mobile Robot-Trailer System

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Abstract— **In this paper, effects of sensor location and measurement errors on path tracking control of a mobile robot pulling a trailer are reported. A model for a two-wheeled robot towing a trailer is presented, followed by a state feedback controller designed to position the trailer along a path consisting of lines and circular arcs. An incremental encoder is used to measure the hitch angle between the robot and trailer. A combined GPS/IMU system is used to measure position and orientation of either the robot or the trailer (but not both). The choice of mounting location for the GPS/IMU system, and consequent effects on trailer path tracking performance are explored. The sensitivity of the system to hitch angle errors is also examined for both cases. It is shown that the system is less sensitive to errors in the hitch angle measurement when the GPS/IMU system is located on the trailer.**

I. INTRODUCTION

Motion planning and control for mobile robots pulling trailers has been extensively explored in recent research. Many methods have been developed to control a robot to follow a path, including control via approximate linearization, exact feedback linearization, full-state linearization via dynamic feedback, and time-varying feedback. A good overview of the various methods can be found in [1]. Many of these methods can be extended to control a mobile robot towing a trailer. Exact linearization was used in [2] to control a robot with a trailer along a straight line. That work was extended to allow the system to follow straight lines and circular arcs in [3]. Other methods for path following control of a mobile robot having one or more trailers are described in [4]–[7].

Much of the above research has been motivated by applications in the world of factory automation. Problems being addressed include obstacle avoidance and complex maneuvers, such as the backing of a trailer. There are some applications, however, where the precise control of the trailer's path is desirable. One such application is geophysical mapping, which requires that instruments mounted on a trailer accurately traverse a specified path. Such a system is being developed in conjunction with the U.S. Army Corps of Engineers and the Environmental Security Technology Certification Program (ESTCP), to be used in the task of locating and mapping unexploded ordnance for humanitarian benefit. Various types of electromagnetic sensors are typically towed on a trailer pulled by either a human or an all-terrain vehicle (ATV). Precise positioning of the towed sensor package with high repeatability between experiments is necessary to produce accurate maps of buried objects. This is not easily done when manually towing the sensor-laden trailer, but can be accomplished using a properly designed autonomous robotic system.

Autonomous trailer path control has been achieved in the agriculture industry, where control systems have been developed for tractors to precisely control the position of a towed implement. In [8], researchers designed a control system that utilizes differential GPS to control the position of an implement towed behind a tractor. The system used two separate GPS receiver antennas, one on the tractor and and one on the implement.

In this paper, a much lower cost system is presented. Here, a single GPS antenna is placed on either the robot or the trailer. An optical encoder is used to measure the orientation between the robot and the trailer (hitch angle sensor). The combination of instruments make it possible to precisely control the position of the trailer. A practical hitch angle sensor introduces imperfections such as quantization error and measurement bias. The authors have discovered that the location of the GPS receiver antenna becomes an important consideration when the hitch angle sensor is imperfect. In the remaining sections, the effects on path following control system performance will be compared with the GPS antenna mounted on either the robot or the trailer. Effects of hitch

Fig. 1. Left: Segway RMP 200ATV Right: Segway RMP 400

Fig. 2. Picture of Segway and Trailer

angle sensor quantization and bias are explored.

II. SYSTEM DESCRIPTION

The system under study is a Segway Robotics Mobility Platform (RMP) robot pulling a two wheeled trailer. (The Segway RMP family of robots were developed for mobile robotics research by Segway Inc., at the request of the Defense Advanced Research Projects Agency (DARPA) [9].) Two versions of the Segway RMP are being used as test platforms. The Segway RMP 200 ATV is a self-balancing, two-wheeled robot. The RMP 400 is a four-wheeled version of the RMP 200. Both units are shown in Fig. 1. A picture of the Segway RMP 400 attached to a trailer is shown in Fig. 2. A schematic representation of the robot and trailer is given in Fig. 3.

A. State Variable Models

The dynamics of the mobile robots used in this work can be modeled using three state variables to represent position and orientation in two dimensional space. In the case of a mobile robot pulling a trailer, a fourth state variable is added to describe the orientation between the robot and trailer. The kinematic model for a robot with trailer is given in [10]. A coordinate transformation from Cartesian coordinates to Universal Transverse Mercator (UTM) coordinates yields the fourth order nonlinear model

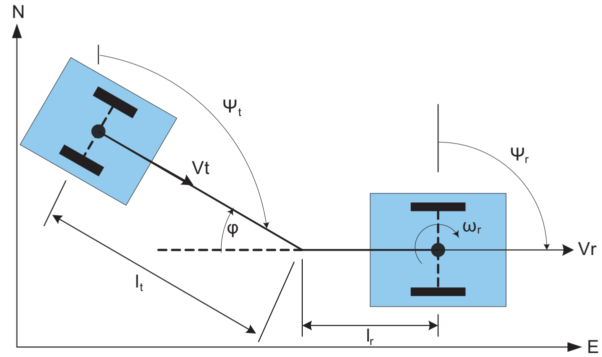

Fig. 3. Schematic Representation of Segway and Trailer

$$\begin{aligned} \dot{e}_r &= V_r\cos(\psi_r) \\ \dot{n}_r &= V_r\sin(\psi_r) \\ \dot{\psi}_r &= \omega_r \\ \dot{\varphi} &= -\frac{V_r}{l_t}\sin(\varphi) - \frac{l_r\omega_r}{l_t}\cos(\varphi) - \omega_r \end{aligned} \tag{1}$$

where (e_r, n_r) is the position of the robot in UTM coordinates, ψ_r is the robot's heading angle measured clockwise from north, and φ is the hitch angle (the angle between the robot and the trailer). The model has two inputs: a commanded angular velocity ω_r and a commanded linear velocity V_r. Under the assumption of zero wheel slip, the same model can be used for both the two-wheeled and four-wheeled robot configurations.

Since the goal is to control the position and orientation of the trailer, it is desirable to describe the model in terms of trailer variables rather than robot variables. The relationships between the positions, orientations, and velocities of the robot and trailer are given by:

$$\begin{aligned} e_t &= e_r - l_r\sin(\psi_r) - l_t\sin(\psi_r+\varphi) \\ n_t &= n_r - l_r\cos(\psi_r) - l_t\cos(\psi_r+\varphi) \\ \psi_t &= \psi_r + \varphi \\ V_t &= V_r\cos(\varphi) - l_r\omega\sin(\varphi) \end{aligned} \tag{2}$$

where (e_t, n_t) is the position of the trailer, ψ_t is the heading of the trailer, and V_t is the linear velocity of the trailer. Applying mapping (2) to the model (1) yields the dynamic model of the trailer under tow:

$$\begin{aligned} \dot{e}_t &= V_t\cos(\psi_t) \\ \dot{n}_t &= V_t\sin(\psi_t) \\ \dot{\psi}_t &= -\frac{V_r}{l_t}\sin(\varphi) - \frac{l_r\omega_r}{l_t}\cos(\varphi) \\ \dot{\varphi} &= -\frac{V_r}{l_t}\sin(\varphi) - \frac{l_r\omega_r}{l_t}\cos(\varphi) - \omega_r \end{aligned} \tag{3}$$

B. System Sensors

The position in UTM coordinates (e, n) and orientation ψ of either the robot or its trailer are provided by a NovAtel synchronized position attitude navigation (SPAN) system,

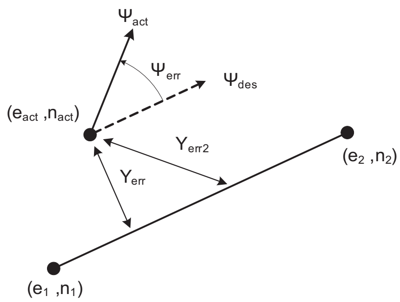

Fig. 4. Line Segment Errors

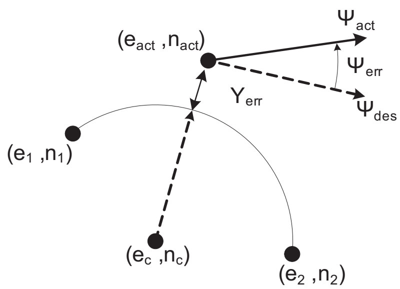

Fig. 5. Arc Segment Errors

hereafter referred to as the "positioning system." The SPAN system consists of a NovAtel OEM4 global positioning system (GPS) receiver and a Honeywell HG1700 inertial measurement unit (IMU). The system receives real-time kinematic (RTK) corrections (based on GPS carrier phase corrections) from a local base station to provide centimeter level accuracy (2 cm standard deviation) [11]. The hitch angle between the robot and the trailer is measured using a U.S. Digital E5S-1800 optical encoder yielding 1800 cycles per revolution (CPR).

III. Controller Design

A control law is desired that will cause the trailer to accurately follow a desired path. The path is defined as a series of line segments and circular arcs. Dubins has shown that the shortest continuously differentiable path between two points to be traveled by a nonholonomic vehicle consists of only lines and circular arcs of maximum curvature (which is determined by finding the minimum turning radius of the vehicle) [12] . Methods for creating these paths can be found in the literature [13].

The model (3) is rewritten in terms of errors from the path. These errors are shown graphically in Fig. 4 for line segments and Fig. 5 for arc segments.

The linearized dynamics of the error x are given by:

$$\dot{x} = \begin{bmatrix} \dot{y}_{terr} \\ \dot{\psi}_{terr} \\ \dot{\psi}_{rerr} \end{bmatrix} = \begin{bmatrix} V_r \psi_{terr} \\ \frac{-V_r}{l_t}(\psi_{terr} - \psi_{rerr}) - \frac{l_r \omega_r}{l_t} \\ \omega_r \end{bmatrix} \quad (4)$$

where y_{terr} and ψ_{terr} are the lateral and heading error of the trailer respectively, and ψ_{rerr} is the heading error of the robot. The order of the model (4) is reduced from the model (3) because the vehicle is moving at a fixed speed. Therefor, the linear velocity V_r is assumed to be constant and so is treated as a parameter rather than an input. A linear state feedback control law is then determined to be: $\omega_r = -Kx$. The gains K are chosen using standard pole placement techniques.

IV. Effects of Positioning System Location

For the control law given in the previous section, knowledge of both the robot and trailer states are required. Since only one GPS antenna is to be used, the positions and orientations of both the robot and trailer cannot be measured directly. There are two possible options: (a) place the positioning system on the robot and calculate the position and orientation of the trailer, or (b) place the positioning system on the trailer and calculate the position and orientation of the robot. In either case, the state of either the robot or the trailer must be calculated based on (2), using a measurement of the hitch angle. From a mathematical point of view, both cases yield identical information for the robot and trailer states.

There are several good arguments for placing the positioning system on the robot. Since the system input is the robot angular velocity ω_r, one can avoid problems associated with non-collocated actuator and sensor by placing the sensors on the robot. From the viewpoint of electrical wiring, it is more convenient to place the positioning system as close as possible to the electrical power source and control computer, both of which are mounted on the robot.

When considering where the positioning system should be placed, effects of potential errors in the hitch angle measurement should also be considered. In the discussions to follow, the authors show that the control system described in Sec. III is less sensitive to sensor errors and parameter variations by placing the positioning system on the trailer.

Errors in hitch angle measurement can arise from several sources, including quantization error, joint backlashes, and calibration errors caused by not having the encoder "home", or zero angle position properly aligned. A general model of hitch angle errors was examined by Park et al [14], [15]. Their conclusion was that hitch angle error could produce a constant lateral offset from the desired path, when the position and orientation of the robot were measured and that of the trailer were calculated from the hitch angle. Divelbiss and Wen also found that very careful calibration is essential if trailer state is to be estimated from hitch angle measurement [6]

Shown in Fig. 6 are the measurement errors produced in the error state x due to hitch angle error ε. With the positioning system located on the robot, the error variables as functions of the hitch angle error are:

$$\begin{aligned} y_{terr}(\varepsilon) &= l_t \sin(\varepsilon) \\ \psi_{terr}(\varepsilon) &= -\varepsilon \\ \psi_{rerr}(\varepsilon) &= 0 \end{aligned} \quad (5)$$

When the positioning system is located on the trailer, the hitch angle measurement error does not affect measurement of trailer lateral position and trailer heading. In this case, the only error that arises from hitch angle imperfection is the heading error of the robot. Consequently, the error variables as functions of hitch angle error ε are:

$$\begin{aligned} y_{terr}(\varepsilon) &= 0 \\ \psi_{terr}(\varepsilon) &= 0 \\ \psi_{rerr}(\varepsilon) &= -\varepsilon \end{aligned} \tag{6}$$

Hitch angle measurement error effects the system in both cases, but its effect on the controller output ω_r is much less for the second case. Examining (5) and (6), note that the hitch angle error ε impacts two state variables when the positioning system is on the robot. In contrast, only one state variable is affected when the positioning system is on the trailer. Furthermore, controller gains for the first two state variables (trailer lateral error and trailer heading error) are much larger than the gains for the third variable (robot heading error). Consequently

$$K_1 y_{terr} + K_2 \psi_{terr} \gg K_3 \psi_{rerr} \tag{7}$$

From (7), it can be argued that the system is less sensitive to hitch angle errors when the positioning system is on the trailer.

Another important point to note from (5) is that the error increases as the trailer tongue length increases when the positioning system is on the robot. Errors in hitch angle may not be an issue with robots pulling short trailers, but can have significant impact in systems with longer trailers, such as the one pictured in Fig. 2, or tractor-trailer systems commonly used for commercial freight transfer.

V. Simulation Results

For the simulation studies, an S-shaped path was created as shown in Fig. 8 and Fig. 9. As mentioned earlier in Sec. III, the reference paths consist of straight line segments joined by circular arcs. A 60 second simulation was run for various sensor configurations. The model parameters used in the simulations are given in Table I.

Controller gains were calculated using pole placement techniques. Closed loop poles are described by a pair of complex conjugate poles (natural frequency ω_n and damping factor ζ) and a third real pole with time constant τ_3. The controller gains for the three different values of trailer tongue length l_t are given in Table II.

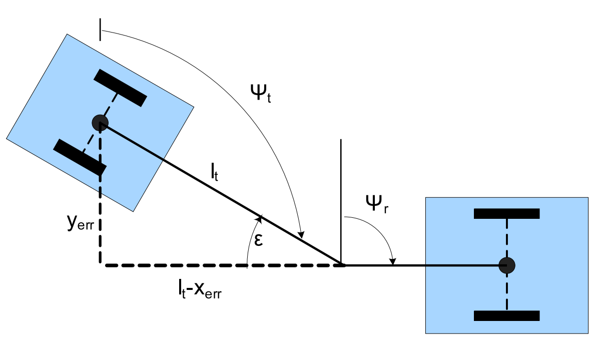

Fig. 6. Effect of Hitch Angle Error on Error States

TABLE I

Simulation model parameters

Parameter	Symbol	Value	Units
Robot speed	V_r	1	m/s
Robot tongue length	l_r	0.1	m
Trailer tongue length	l_t	[2, 6]	m
Robot angular rate limit	$\lvert\omega_r\rvert_{max}$	3	rad/s
Closed loop natural frequency	ω_n	π	rad/s
Closed loop damping factor	ζ	0.707	
Time constant of third closed loop pole	τ_3	0.5	s

TABLE II

Controller gains for various trailer tongue lengths

l_t	Controller gain, K
2	$K = [\ 9.87 \quad 19.7 \quad 5.4\ ]$
4	$K = [\ 19.7 \quad 44.5 \quad 5.8\ ]$
6	$K = [\ 29.6 \quad 69.6 \quad 5.9\ ]$

A. Effect of Quantization on Path Tracking Error

Fig. 7 shows the change in lateral mean square error between the trailer and the path as a function of quantization error for various trailer lengths. Lateral error of the trailer increases with quantization error when the positioning system is mounted on the robot (square, $\times$, and diamond curves). In contrast, increasing quantization has little effect on system performance when the positioning system is on the trailer (solid and dotted curves).The curves also show that sensitivity to quantization error increases with increases in the trailer tongue length l_t, especially when the positioning system is on the robot. Increasing tongue length has minimal effect, however, when the positioning system is on the trailer. For the experimental system currently being using, $l_t = 2m$ and the encoder has a resolution of 1800 CPR, which yields relatively good performance regardless of sensor placement.

The effects of hitch angle quantization can also be illus-

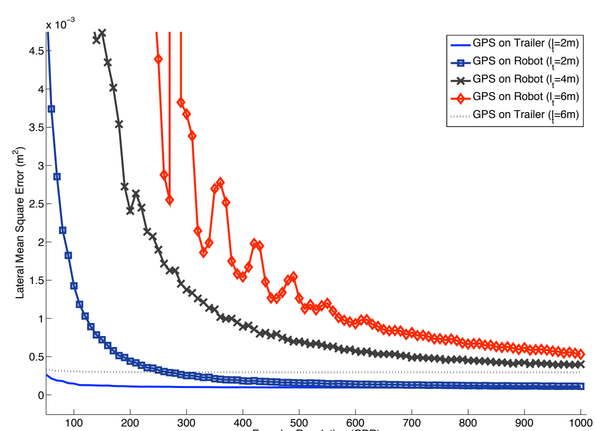

Fig. 7. Control System Performance for Varying Encoder Resolutions

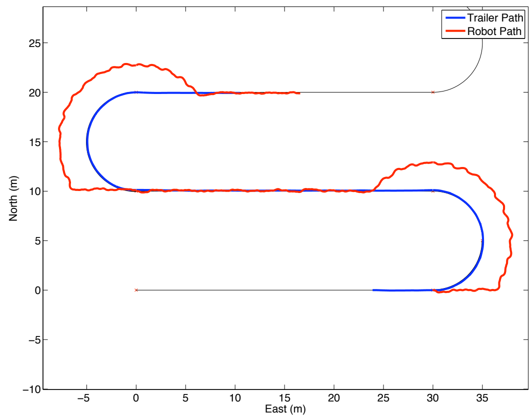

Fig. 8. Simulation Result: GPS on Robot & $l_t = 6$m

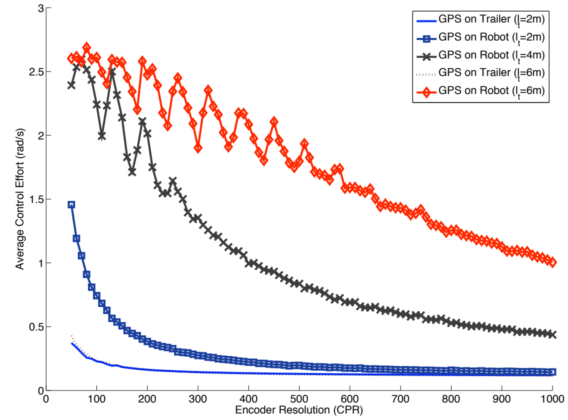

Fig. 10. Average Control Effort for Varying Encoder Resolutions

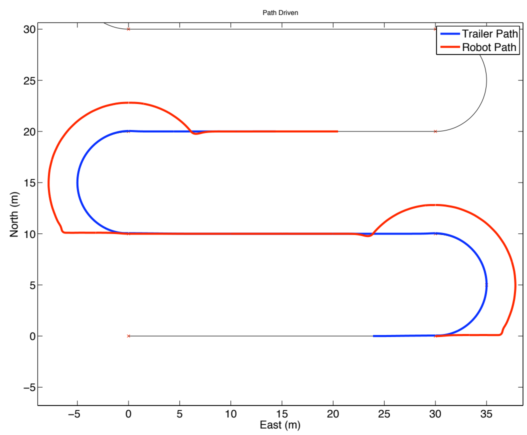

Fig. 9. Simulation Result: GPS on Trailer & $l_t = 6$m

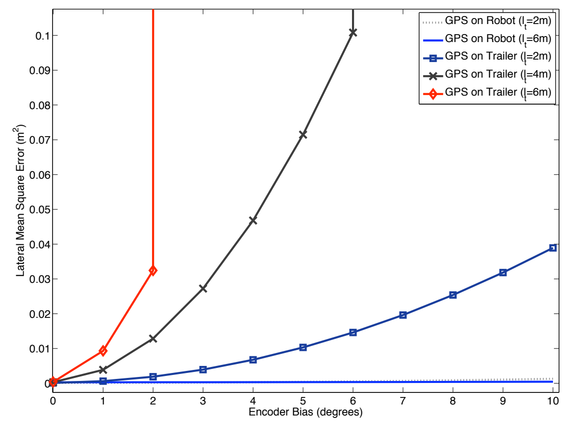

Fig. 11. Control System Performance for Varying Encoder Biases

trated by plotting the paths traversed by both the robot and the trailer, as shown in Fig. 8 and Fig. 9. (The simulations were run for 1min 40sec with an encoder resolution of 250 CPR and $l_t = 6$m.)

B. Control Effort

Placing the positioning system on the trailer not only reduces lateral error for a given encoder resolution, but also produces much smoother control responses. When the positioning system in on the robot, quantization error and a long tongue length result in a control effort that tends to resemble step changes. As the encoder resolution is decreased, increasingly larger step changes are produced in y_{terr} and ψ_{terr}, creating undesirably abrupt changes in robot angular rate. The tendency to saturate the controller output also grows with increasing tongue length l_t.This results in the system approaching the limits of stability. However, when the positioning system is located on the trailer, these quantization errors occurs only in robot heading error ψ_{rerr}, and therefore have less effect on the trailer path tracking error. Fig. 10 shows the average control effort as a function of encoder resolution, for various combinations of positioning system placement and trailer tongue length.

C. Effect of Hitch Angle Measurement Bias

The effects of a bias were also examined. A series of simulations were run with fixed, high encoder resolution (1800 CPR), but varying the measurement bias from 0° to 10°. The results are shown in Fig. 11.

Increasing encoder bias produces an effect very similar to increasing quantization error. When the GPS antenna is on the robot, large encoder biases result in large lateral error. When the antenna is moved to the trailer, there is very little effect on system performance as the bias increases.

VI. Conclusions

The authors have shown that the choice of sensor placement in a tractor-trailer path following control system greatly affects the sensitivity to measurement imperfections, specifically quantization and bias errors in the hitch measurement. The control system described here was designed for path following by the trailer, with measurements coming from a hitch angle sensor and GPS/IMU positioning system.

When implementing such a system, errors in the hitch angle measurement must be taken into account. Directly measuring the position and orientation of the robot, as is conventionally done, can lead to substantial lateral errors in the trailer's position. The system can be made more robust against hitch angle measurement errors by placing the positioning system on the trailer. Resulting error in estimating the robot heading angle have little effect on trailer path tracking performance.

VII. Acknowledgements

The authors express thanks for the support from the U.S. Army Corps of Engineers and the Environmental Security Technology Certification Program (ESTCP).

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