MH 3100 Quiz #9 12/12 Name: The Prof

(1) Define a relation R on the set $\mathbb{Z}$ of integers as follows:
xRy iff $x-y$ is divisible by 12.

(a) Show that R is an equivalence relation.

(8 pts) Check reflexive:
xRx iff $x-x=0$ is divisible by 12. Yes.

Check symmetric:
Does xRy imply yRx?
Suppose xRy. Then $x-y$ is divisible by 12.
So $-(x-y)=y-x$ is too. $\therefore yRx$.
Yes, R is symmetric.

Check transitive:
Does xRy and yRz imply xRz?
Suppose xRy and yRz.
Then $x-y$ and $y-z$ are divisible by 12.
$\therefore (x-y)+(y-z)=x-z$ is divisible by 12. $\therefore yRz$.
So yes, R is transitive.

(b) Name a positive integer (other than 17) and a negative integer that is in the equivalence class of 17.

(4 pts) $17/R = \{x : 17Rx\} = \{x : 17-x$ is divisible by 12$\}$
$= \{17+k\cdot 12 : k \in \mathbb{Z}\}$.

Possible answers:
$17-12=5$ or $17+12=29$
$17-24=-7$ or $17-36=-19$