MH3100 Quiz #3 (Take home; due noon Monday Sept. 11)

Name: The Prof

① Let a be a real number. Prove that $|a| = |-a|$. 12/12

(6 pts) Case 1 Assume $a > 0$.
Then $|a| = a$. Also, $-a < 0$, so $|-a| = -(-a) = a$
$\therefore |a| = |-a|$.

Case 2. Assume $a = 0$. Then $-a = 0$, so $|a| = 0 = |-a|$.

Case 3. Assume $a < 0$.
Then $|a| = -a$.
Since $a < 0$, $-a > 0$ so $|-a| = -a$
$\therefore |a| = |-a|$

There are no other cases, so $|a| = |-a|$ for any real # a.

QED.

Remark. This proof uses — and yours should use — the definition of absolute value:

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

② Let x be a real number. Prove that if $x \geq 2$, then $x^2 + 2x \geq 3x + 2$.

(6 pts) We try working backwards. This is not the proof yet.
We want $x^2 + 2x \geq 3x + 2$ (if $x \geq 2$).
or $x^2 - x - 2 \geq 0$
or $(x-2)(x+1) \geq 0$.
If $x \geq 2$, then both factors are ≥ 0, so this will be true.

Proof. Assume $x \geq 2$.
Then $x - 2 \geq 0$ and $x + 1 \geq 0$
$\therefore (x-2)(x+1) \geq 0$
$\therefore x^2 - x - 2 \geq 0$
$\therefore (x^2 - x - 2) + (3x + 2) \geq 3x + 2$
$\therefore x^2 + 2x \geq 3x + 2$.

QED.