MH3100   Quiz #2 $\frac{12}{12}$ Name: The Prof

① Consider the following statement:

"If f has a second derivative at x_0 and $f''(x_0) > 0$, then f is concave up at x_0."

(a) Rewrite it using logical connectives ($\wedge, \vee, \Rightarrow$, etc).

(2 pts) (f has a second derivative at x_0) $\wedge$ ($f''(x_0) > 0$) $\Rightarrow$ f is concave up at x_0.

(b) Write the converse of the statement.

(2 pts) If f is concave up at x_0, then f has a second derivative at x_0 and $f''(x_0) > 0$.

(c) Write the contrapositive of the statement.

(2 pts) If f is not concave up at x_0, then either f does not have a second derivative at x_0 or $f'(x_0) \leq 0$.

② Consider the formula $\forall x\,(x \geq 0 \Rightarrow \exists y\,(y^2 = x))$, where the universe of discourse is the set $\mathbb{R}$ of all real numbers.

(a) Write the formula as an English sentence.

(2 pts) OK answer: For every real number x, if $x \geq 0$ then there exists a real number y such that $x = y^2$.

Better: Every nonnegative real number is the square of some real number.

(b) Write a useful denial in symbols.

(2 pts) $\exists x\,(x \geq 0 \wedge \forall y\,(x \neq y^2))$

or $\exists x\,(x \geq 0 \wedge (\sim \exists y\,(x = y^2)))$

(c) Write the denial in English.

(2 pts) Best: There is a nonnegative real number that is not the square of any real number.

OK: There is a real number x that is ≥ 0, yet there is no real number y such that $x = y^2$.