MH 3100 Quiz #11 $\frac{12}{12}$ Name: The Prof

Take-home, due Wed. Dec. 6 at noon.

① (a) Find a specific number δ such that if x is within δ of 5, then $4x^2 - 15x$ is within $.001$ of 25.

(4 pts) By part (b), $\frac{.001}{29} \approx .000034$ should work.

Let $\delta = .00003$ or anything less.

(b) Give an ε-δ proof that $\lim_{x \to 5} (4x^2 - 15x) = 25$.

(Remark. You can do (a) without doing (b), but if you do (b) first, and correctly, it can help with (a).)

(8 pts) Proof. Let $\varepsilon > 0$.

Need to find δ such that, if $0 < |x-5| < \delta$ then $|(4x^2 - 15x) - 25| < \varepsilon$.

Note that $|4x^2 - 15x - 25| = |(x-5)(4x+5)|$.

Let's promise to make $\delta \le 1$.

Then $|x-5| < \delta \Rightarrow |x-5| < 1$
$\Rightarrow -1 < x-5 < 1$
$\Rightarrow 4 < x < 6$.

Then $|4x+5| < |4 \cdot 6 + 5| = 29$, and so

$|(x-5)(4x+5)| < |(x-5) \cdot 29|$.

We want $|(x-5) \cdot 29| < \varepsilon$ or $|x-5| < \frac{\varepsilon}{29}$.

Let $\delta = \min\{1, \frac{\varepsilon}{29}\}$, and suppose $0 < |x-5| < \delta$.

Then $|(4x^2 - 15x) - 25| = |(x-5)(4x+5)|$
$< |(x-5) \cdot 29|$
$< |\frac{\varepsilon}{29} \cdot 29| = \varepsilon$.

$\therefore$ $0 < |x-5| < \delta$ implies $|(4x^2 - 15x) - 25| < \varepsilon$.

$\therefore \lim_{x \to 5} (4x^2 - 15x) = 25$.

Q.E.D.